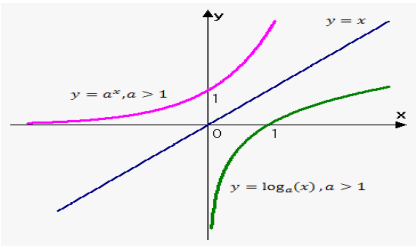
Week 3/4
Term 1
2021

SMM3
Logarithmic Functions and Further differentiation and applications

HAWKER COLLEGE
Engage | Inspire | Achieve

Goals



This week:

- Solving indicial and logarithmic equations using any base
- Investigate Euler's number
- Use natural logarithms to the base e. Learn notation used.
- Further graphing logarithmic functions, showing necessary features. Domain and range
- Using exponential and logarithmic modelling

Theoretical Components

STEP 1

What is base e?

Learn more about Euler's number.
Read through to get an insight on the number 'e':

http://www.mathopolis.com/questions/q.php?id=2011&site=1&ref=/numbers/e-eulers-number.html&qs=2011_2012_2013
Watch:
http://bit.ly/w8OiD

Graphing Logarithmic Functions

https://www.youtube.com/watch?v=q9DhlR43P7A

https://www.youtube.com/watch?v=LqyA96oYtwE

Domain and Range of Logarithmic Functions
https://mathspace.co/teach2/chapter/33888/1451/

Natural Logarithms
https://mathspace.co/teach2/chapter/33893/1455/

See summary notes on the last page.

Practical Components

STEP 2

Exercises 3E, 3F and 3G
Do as many as you need to become proficient at these skills and concepts.

Investigation

See next page

Quiz

Short quiz on mathspace

Week 3 & 4 Investigation

The following data is an extract from a planetary fact sheet published by NASA.

	Distance from sun, r $(10^6$ km)	Orbital period, P (days)
Mercury	57.9	88
Venus	108.2	224.7
Earth	149.6	365.2
Mars	227.9	687
Jupiter	778.6	4331
Saturn	1433.5	10 747
Uranus	2872.5	30 589
Neptune	4495.1	59 800
Pluto	5906.4	90 560

Physics suggests that the orbital period, P, and distance from the sun r, are related by a formula of the form:

$$P = Ar^k$$

(1) Use the data above to find the values of A and k. And then, graph the function with appropriate scale.

Things to think about before you start:

- the value of A and k cannot be found by use any pairs of data above as the data above doesn't form a proper logarithm graph. A straight-forward graph of P against r will probably show a curve from which it is hard to estimate values.
- Can you rearrange the function and convert it into a straight-line function?
- You will find it helpful to use graphing software to check your values.
- Use ln (natural logarithms base e)

(2) Use your results to predict the orbital period of Ceres, a dwarf planet in the asteroid belt with $r = 414.0$

remember

1. Euler's number e is an irrational number which is approximated to 2.718 (3 decimal places).
2. Evaluate e by using the 'e^x' button on the calculator.
3. The number e is an exact answer, whereas the calculator gives an approximation.
4. The laws of indices apply in the same way if e is the base.
5. Use the LN button to take the log of a number to base e. The LOG button means $\log_{10}$.
6. $\log_e x = \ln x$.
7. $e^x > 0$, that is, $e^x = -1$ has no real solution.

remember

1. The equations $a^x = y$ and $2^x = 32$ are indicial equations.
2. Write numbers with the same base to help simplify problems. The most common ones to use are 2, 3 and 5.
3. If the base is the same equate the indices.
4. If the indices are the same equate the bases.
5. Use the Null Factor Law to solve quadratic equations.
6. A negative number cannot be expressed in index form, for example, -4 cannot be expressed with base 2.
7. $a^{2x} = (a^x)^2$
8. Take the logarithm of both sides of an equation or inequation using the same base.
9. Change the sign of an inequality when multiplying or dividing by a negative number.
10. $\log_a x > 0$ if $x > 1$
11. $\log_a x < 0$ if $0 < x < 1$

REMEMBER

1. $f(x) = \log_a(x)$ is the inverse of $g(x) = a^x$ and they are therefore reflections of each other through the line $y = x$.
2. If $a > 1$, $f(x) = \log_a(x)$ has:
   - x-intercept $(1, 0)$
   - asymptote $x = 0$
   - domain $= R^+$
   - range $= R$.
3. The graph of $f(x) = \log_a(x + b) + c$ is obtained by translating the graph of $f(x) = \log_a(x)$ b units horizontally and c units vertically.