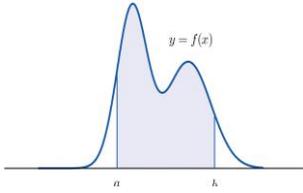


## Goals

By the end of this week, you should be able to:

- distinguish between discrete and continuous random variables (CRV)
- understand probability density functions and cumulative distributions for continuous random variables
- compute the central tendency and variability (spread) of continuous distributions

$P(a < X < b)$  = area of shaded region



## Theoretical Components

Resources:

- Maths Quest 12 Mathematical Methods, Chapter 12 (see pdf on Google Drive)

1.  $f(x) \geq 0$ , for all  $x$

2.  $\int_{-\infty}^{\infty} f(x) dx = 1$

3.  $P(a \leq X \leq b) = \int_a^b f(x) dx$

By now, you should be able to:

- Solving indicial and logarithmic equations using any base
- Investigate Euler's number
- Use natural logarithms to the base e. Learn notation used.
- Sketch logarithmic functions showing necessary features
- Solving indicial and logarithmic equations using base e
- Use natural logarithms to the base e.
- Using exponential and logarithmic modelling
- Use the chain rule for differentiation of exponential functions of the forms:  $y = e^x$ , and  $y = e^{f(x)}$
- Find the derivatives logarithmic functions of the forms;  
 $y = \ln(x)$  and  $y = \ln[f(x)]$ .
- Use derivatives to solve practical problems
- Integrate to give log functions
- Find an exact area under a given curve using definite integrals
- distinguish between discrete and continuous random variables (CRV)
- understand probability density functions and cumulative distributions for continuous random variables
- compute the central tendency and variability (spread) of continuous distributions

## Practical Components

Exercises 12A Q1 a,b Q2 a,b Q4  
Exercises 12B Q2,5,9,17  
Exercises 12C Q1 a, Q4 a, Q5,9,11  
Exercises 12D as many as you could

Revision:

Complete the chapter review for:

Chapter 3

Chapter 7

Chapter 12 (up to probability density function and application. No need to do the questions about normal distribution.)

## Investigation

$X$  is a random variable denoting the number of minutes in excess of two hours which a person takes to travel from one town to another. The probability density function is defined by:

$$f(x) = \begin{cases} k(10 + x), & -10 \leq x \leq 0 \\ k(10 - x), & 0 \leq x \leq 10 \\ 0, & \text{elsewhere} \end{cases}$$

(a) Find the value of  $k$

(b) Sketch the graph of  $f$

(c) Find the probability that:

(i)  $X$  is less than 5

(ii)  $X$  is less than 0, given that  $X$  is less than 5

(iii)  $-2 \leq x < 3$

**QFO**

Quiz/Forum/Other

**NO quiz for this week**



## Summary

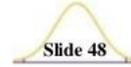
### Probability revision

- *Outcomes* are results of experiments.
- The set of all possible outcomes of an experiment is called the *sample space* and is denoted by  $\epsilon$ , and each possible outcome is called a *sample point*.
- A subset of the sample space is known as an *event*.
- The *union* (symbol  $\cup$ ) of two events  $A$  and  $B$  implies a combined event, that is, either event  $A$  or event  $B$  or both occurring. Common elements are written only once.
- The *intersection* (symbol  $\cap$ ) of two events  $A$  and  $B$  is represented by the common sample points of the two events.
- Venn diagrams involve drawing a rectangle that represents the sample space and a series of circles that represent subsets of the sample space. They provide a visual representation of the information at hand and clearly display the relationships between sets.
- The probability of an event occurring is defined by the rule:

$$\Pr(A) = \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$$

- The probability of an event occurring lies within the restricted interval  $0 \leq \Pr(A) \leq 1$ .
  - The individual probabilities of a particular experiment will sum to 1; that is,  $\sum p(x) = 1$ .
  - The addition rule of probability is defined by the rule  $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$ .
  - If two events  $A$  and  $B$  are mutually exclusive, then  $\Pr(A \cap B) = 0$  and therefore the addition rule becomes  $\Pr(A \cup B) = \Pr(A) + \Pr(B)$ .
  - If two events  $A$  and  $B$  are independent, then  $\Pr(A \cap B) = \Pr(A) \times \Pr(B)$ .
  - Karnaugh maps and probability tables summarise all combinations of two events (for example  $A$  and  $B$ ) and their complements (for example  $A'$  and  $B'$ ).
  - Conditional probability is defined by the rule  $\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$ , where  $\Pr(B) \neq 0$ . This can be transposed to  $\Pr(A \cap B) = \Pr(A|B) \times \Pr(B)$ .
  - Tree diagrams are useful tools in solving probability tasks as they display each of the possible outcomes along with their respective probabilities.
  - A combination is defined by  ${}^n C_r$ , that is, the number of selections of  $n$  different objects taken  $r$  at a time.
-

## Discrete and Continuous Random Variables - Revisited

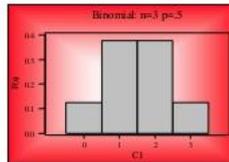


- **A discrete random variable:**

- counts occurrences
- has a countable number of possible values
- has discrete jumps between successive values
- has measurable probability associated with individual values
- probability is height

For example:  
Binomial  
 $n=3$   $p=.5$

$x$	$P(x)$
0	0.125
1	0.375
2	0.375
3	0.125
	1.000



- **A continuous random variable:**

- measures (e.g.: height, weight, speed, value, duration, length)
- has an uncountably infinite number of possible values
- moves continuously from value to value
- has no measurable probability associated with individual values
- probability is area

For example:  
In this case, the shaded area represents the probability that the task takes between 2 and 3 minutes.

