Week 1 and 2
Term 3 2021

HAWKER COLLEGE
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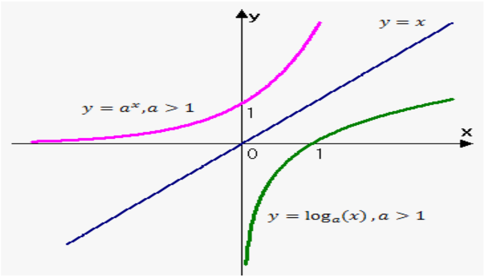
Learning Brief:MM4
Logarithmic Functions

Goals



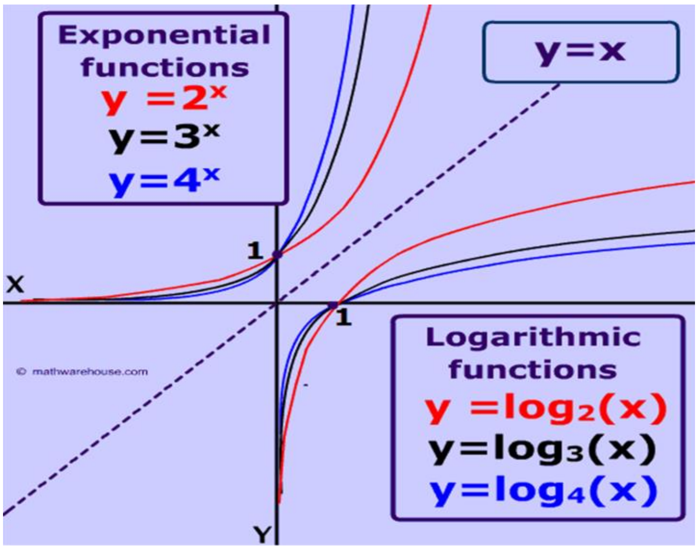
Welcome back, this fortnight:

- Review index laws and exponential functions
- Define logarithms definition and learn their algebraic properties
- Solving indicial and logarithmic equations using any base
- Sketch logarithmic functions showing necessary features

Theoretical Components

Resources:
Maths Quest Chapter 3: Exponential and logarithmic equations

- ***See PDF on Google Classroom***



Graphing Logarithmic Functions

https://www.youtube.com/watch?v=q9DhlR43P7A

https://www.youtube.com/watch?v=LqyA96oYtwE

See summary notes on the following page.

Practical Components

Do the following questions. Organise your solutions neatly in your exercise book.

3A The index laws
- Q's 1 – 3 (any 2 from each), 6, 7

3B Logarithmic laws
- Q's 1(a,d), 2 – 3 (1^{st} col), 7 – 9 (2 from each), 13, 14

3C Indicial equations
- Q's 1 – 5 (1 from each)

3D Logarithmic equations using any base
- Q's 1 – 5 (2 or 3 from each), 6, 7, 8 (a,c), 9, 10

Investigation

Watch the videos in the Theoretical Component on **Graphing Logarithmic Functions** and complete the questions on the following pages.

This investigation in ASSESSABLE.

(20 marks – see rubric)

QFO
Quiz/Forum/Other

No Mathspace this week 😊

Index laws

- $a^x \times a^y = a^{x+y}$
- $a^x \div a^y = a^{x-y}$
- $(a^x)^y = a^{xy}$
- $a^0 = 1$
- $a^{-x} = \frac{1}{a^x}$ and $\frac{1}{a^{-x}} = a^x$
- $a^{\frac{1}{y}} = \sqrt[y]{a}$ and $a^{\frac{x}{y}} = \sqrt[y]{a^x}$
- $a^x = y \Leftrightarrow \log_a y = x$

Logarithm laws

- $\log_a 1 = 0$
- $\log_a a = 1$
- $\log_a 0$ is undefined
- $\log_a mn = \log_a m + \log_a n$
- $\log_a \frac{m}{n} = \log_a m - \log_a n$
- $\log_a m^p = p \log_a m$
- $\log_b N = \frac{\log_a N}{\log_a b}$ (change-of-base rule)

remember

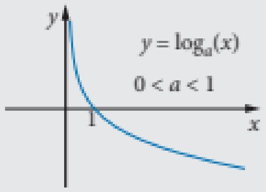
1. The equations $a^x = y$ and $2^x = 32$ are indicial equations.
2. Write numbers with the same base to help simplify problems. The most common ones to use are 2, 3 and 5.
3. If the base is the same equate the indices.
4. If the indices are the same equate the bases.
5. Use the Null Factor Law to solve quadratic equations.
6. A negative number cannot be expressed in index form, for example, –4 cannot be expressed with base 2.
7. $a^{2x} = (a^x)^2$
8. Take the logarithm of both sides of an equation or inequation using the same base.
9. Change the sign of an inequality when multiplying or dividing by a negative number.
10. $\log_a x > 0$ if $x > 1$
11. $\log_a x < 0$ if $0 < x < 1$

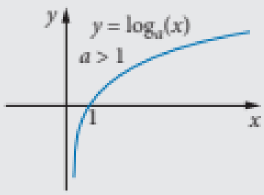
remember

1. In a logarithmic equation the unknown can be:
   (a) the number, $\log_2 x = 5$
   (b) the base, $\log_x 8 = 3$
   (c) the logarithm, $\log_2 4 = x$.
2. In the expression $\log_a x$, a is a positive real number other than 1.
3. Use laws of logarithms and indices to solve the equations.
4. Check that all solutions are valid.

MM4 Investigation weeks 1 & 2

Graphs of logarithmic functions of the form $y = \log_a(x)$





Increasing logarithmic function

Decreasing logarithmic function

The graphs of exponential functions of the form $y = \log_a(x)$ have a zero of 1.

For increasing functions, as $x \to 0$, $y \to -\infty$.

For decreasing functions, as $x \to 0$, $y \to \infty$.

The y-axis is a vertical asymptote.

INVESTIGATION Transforming logarithms

a Consider the function $f(x) = \log_2(x)$

- i State the domain and range.
- ii What is the zero for this function?
- iii What is the asymptote for this function?
- iv Find $f(2)$ and explain why this value is so important for this function.

b Now consider the function $f(x) = \log_2(x + 3)$.

Use your CAS calculator or computer software to draw the graph.

- i State the domain and range.
- ii What is the zero for this function?
- iii What is the asymptote for this function?
- iv Find $f(-1)$ and explain why this value is so important for this function.

c Now consider the function $f(x) = \log_2(x) + 4$.

Use your CAS calculator or computer software to draw the graph.

- i State the domain and range.
- ii What is the zero for this function?
- iii What is the asymptote for this function?
- iv Find $f(2)$ and explain why this value is so important for this function.

1. From your investigation complete the following summary, using the descriptions provided to fill in the blanks.

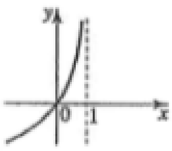
upwards	horizontal	a^{-b}	right	$1-c$	vertical	left	down	$-c$

- The graph of $y = log_a(x) + b$ is a __________ translation of $y = log_a(x)$ by b. For b positive, the translation is __________, and for b negative it is __________. The asymptote is still the vertical axis, but the zero becomes (____, 0).

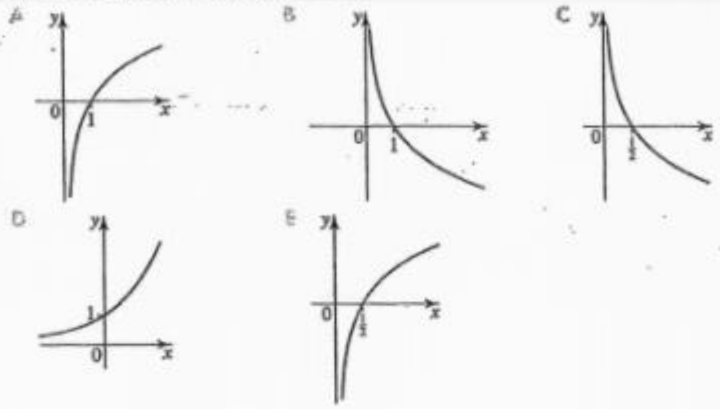
- The graph of $y = log_a(x + c)$ is a __________ translation of $y = log_a(x)$ by c. For c positive, the translation is to the __________, and for c negative it is to the __________. The zero becomes (____, 0) and the vertical asymptote becomes x $=$ ____

2 The rule for the graph at right is:

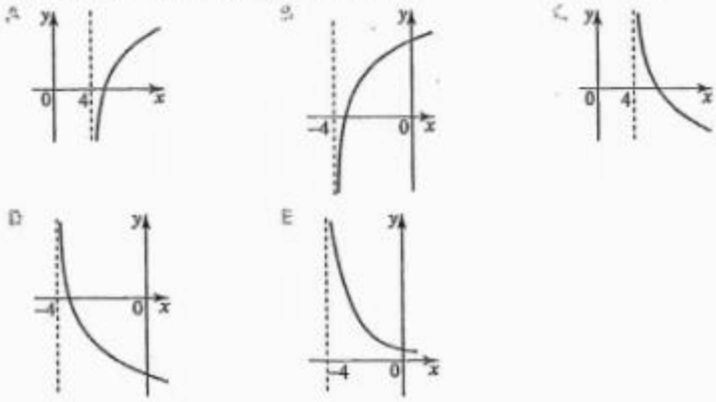
A $y = \log_{10}(1-x)$
B $y = \log_{10}(x-1)$
C $y = {}^-\log_{10}(1-x)$
D $y = {}^-\log_{10}(x-1)$
E $y = {}^-\log_{10}(x+1)$



3 The graph of $y = \log_5(2x)$ could be:



4 The graph of $f(x) = {}^-\log_3(x+4)$ could be:



5 The rule for the graph at right is:

A $y = \log_3(x-2)+1$
B $y = \log_3(x+2)+1$
C $y = \log_2(x-2)$
D $y = \log_3(x-2)$
E $y = \log_2(x-2)-1$