

## Goals

Welcome back, this fortnight:

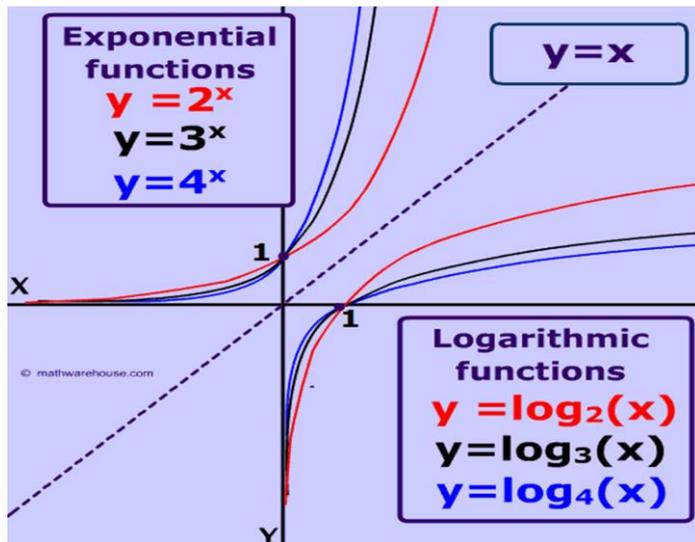
- Review index laws and exponential functions
- Define logarithms definition and learn their algebraic properties
- Solving indicial and logarithmic equations using any base
- Sketch logarithmic functions showing necessary features

## Theoretical Components

### Resources:

Maths Quest Chapter 3: Exponential and logarithmic equations

- See PDF on Google Classroom



### Graphing Logarithmic Functions

<https://www.youtube.com/watch?v=q9DhIR43P7A>

<https://www.youtube.com/watch?v=LqyA96oYtwE>

See summary notes on the following page.

## Practical Components

Do the following questions. Organise your solutions neatly in your exercise book.

### 3A The index laws

- Q's 1 – 3 (any 2 from each), 6, 7

### 3B Logarithmic laws

- Q's 1(a,d), 2 – 3 (1<sup>st</sup> col), 7 – 9 (2 from each), 10 – 12

### 3C Indicial equations

- Q's 1 – 5 (1 from each)

### 3D Logarithmic equations using any base

- Q's 1 – 5 (2 or 3 from each), 6, 7, 8 (a,c), 9

## Investigation

Watch the videos in the Theoretical Component on **Graphing Logarithmic Functions** and complete the questions on the following pages.

This investigation is ASSESSABLE.

(20 marks – see rubric)

**QFO**

Quiz/Forum/Other

No Mathspace this week ☹️

## Index laws

- $a^x \times a^y = a^{x+y}$
- $a^x \div a^y = a^{x-y}$
- $(a^x)^y = a^{xy}$
- $a^0 = 1$
- $a^{-x} = \frac{1}{a^x}$  and  $\frac{1}{a^{-x}} = a^x$
- $a^{\frac{1}{y}} = \sqrt[y]{a}$  and  $a^{\frac{x}{y}} = \sqrt[y]{a^x}$
- $a^x = y \Leftrightarrow \log_a y = x$

## Logarithm laws

- $\log_a 1 = 0$
- $\log_a a = 1$
- $\log_a 0$  is undefined
- $\log_a mn = \log_a m + \log_a n$
- $\log_a \frac{m}{n} = \log_a m - \log_a n$
- $\log_a m^p = p \log_a m$
- $\log_b N = \frac{\log_a N}{\log_a b}$  (change-of-base rule)

## remember

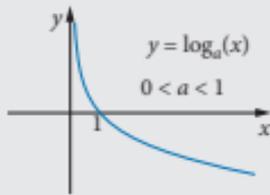
1. The equations  $a^x = y$  and  $2^x = 32$  are indicial equations.
2. Write numbers with the same base to help simplify problems. The most common ones to use are 2, 3 and 5.
3. If the base is the same equate the indices.
4. If the indices are the same equate the bases.
5. Use the Null Factor Law to solve quadratic equations.
6. A negative number cannot be expressed in index form, for example,  $-4$  cannot be expressed with base 2.
7.  $a^{2x} = (a^x)^2$
8. Take the logarithm of both sides of an equation or inequation using the same base.
9. Change the sign of an inequality when multiplying or dividing by a negative number.
10.  $\log_a x > 0$  if  $x > 1$
11.  $\log_a x < 0$  if  $0 < x < 1$

## remember

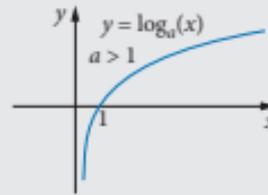
1. In a logarithmic equation the unknown can be:
  - (a) the number,  $\log_2 x = 5$
  - (b) the base,  $\log_x 8 = 3$
  - (c) the logarithm,  $\log_2 4 = x$ .
2. In the expression  $\log_a x$ ,  $a$  is a positive real number other than 1.
3. Use laws of logarithms and indices to solve the equations.
4. Check that all solutions are valid.

## MM4 Investigation weeks 1 & 2

### Graphs of logarithmic functions of the form $y = \log_a(x)$



Increasing logarithmic function



Decreasing logarithmic function

The graphs of exponential functions of the form  $y = \log_a(x)$  have a zero of 1.

For increasing functions, as  $x \rightarrow 0$ ,  $y \rightarrow -\infty$ .

For decreasing functions, as  $x \rightarrow 0$ ,  $y \rightarrow \infty$ .

The  $y$ -axis is a vertical asymptote.

## INVESTIGATION Transforming logarithms

- a** Consider the function  $f(x) = \log_2(x)$
- State the domain and range.
  - What is the zero for this function?
  - What is the asymptote for this function?
  - Find  $f(2)$  and explain why this value is so important for this function.

- b** Now consider the function  $f(x) = \log_2(x + 3)$ .

Use your CAS calculator or computer software to draw the graph.

- State the domain and range.
- What is the zero for this function?
- What is the asymptote for this function?
- Find  $f(-1)$  and explain why this value is so important for this function.

- c** Now consider the function  $f(x) = \log_2(x) + 4$ .

Use your CAS calculator or computer software to draw the graph.

- State the domain and range.
- What is the zero for this function?
- What is the asymptote for this function?
- Find  $f(2)$  and explain why this value is so important for this function.

From your investigation above, complete the following summary using the descriptions provided to fill in the blanks, then complete the multiple-choice questions.

upwards	horizontal	$a^{-b}$	right	$1 - c$	vertical	left	down	$-c$
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1. The graph of  $y = \log_a(x) + b$  is a \_\_\_\_\_ translation of  $y = \log_a(x)$  by  $b$ . For  $b$  positive, the translation is \_\_\_\_\_, and for  $b$  negative it is \_\_\_\_\_. The asymptote is still the vertical axis, but the zero becomes (\_\_\_\_, 0).

2. The graph of  $y = \log_a(x + c)$  is a \_\_\_\_\_ translation of  $y = \log_a(x)$  by  $c$ . For  $c$  positive, the translation is to the \_\_\_\_\_, and for  $c$  negative it is to the \_\_\_\_\_. The zero becomes (\_\_\_\_, 0) and the vertical asymptote becomes  $x =$  \_\_\_\_\_

**3** The graph of  $y = \log_5(2x)$  could be:

A

B

C

D

E

**4** The graph of  $f(x) = \log_3(x + 4)$  could be:

A

B

C

D

E

**5** The rule for the graph at right is:

A  $y = \log_3(x - 2) + 1$

B  $y = \log_3(x + 2) + 1$

C  $y = \log_2(x - 2)$

D  $y = \log_3(x - 2)$

E  $y = \log_2(x - 2) - 1$



## Marking Rubric

Name: \_\_\_\_\_

CRITERIA	EXPECTATIONS	POSS	MULT	GIVEN	TOTAL
<b>Practical</b>	Student completes practical work, including exercises and Mathspace task (if applicable), of the brief to an acceptable standard set by the teacher.	2	3		/6
<b>Investigation</b>	Student completes the investigation task of the brief to an acceptable standard set by the teacher.	2	2		/4
<b>Communication and Reasoning</b>	Student responses are accurate and appropriate in presentation of mathematical ideas in different contexts, with clear and logical working out shown.	4	-		/4
<b>Knowledge and Application</b>	Student submitted work selects and applies appropriate mathematical modelling and problem solving techniques to solve practical problems, and demonstrates proficiency in the use of mathematical facts, techniques and formulae.	4	-		/4
	<b>Submission Guidelines</b>				
<b>Timeliness</b>	Student submits the exercises, Mathspace task (if applicable) and investigation by the set deadline. See scoring guidelines for specific details.	2	-		/2
		<b>FINAL</b>			<b>/20</b>

### Student Reflection:

How did you go with this week's work?

What did you learn?

What did you find easy?

What do you need to work on?

**Mathspace task score (if applicable):**