Week 6 & 7
Term 1
2021

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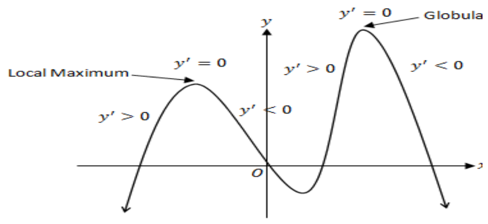
Learning Brief MM3: Further differentiation and applications

Goals

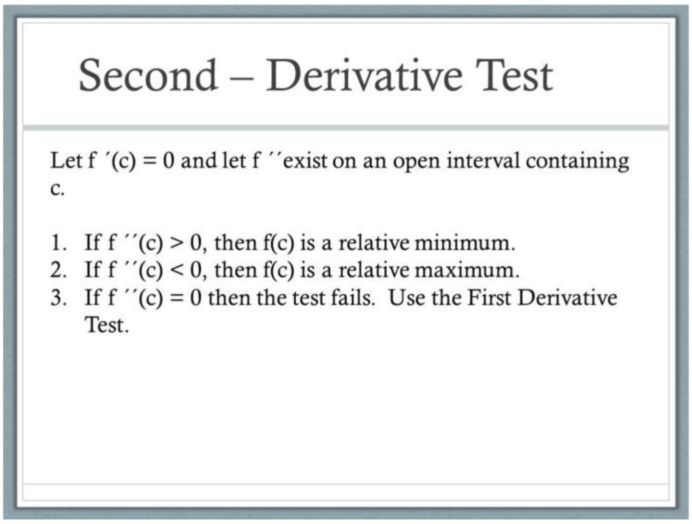


By the end of this week, you should be able to:

- Use derivatives to solve practical problems
- Identify turning points and points of inflection
- Find the second derivative
- solve optimisation problems from a wide variety of fields using first and second derivatives.

Theoretical Components

Resources:
Maths Quest Year 12 Chapter 8 (pdf on Google Drive)



Second – Derivative Test

Let f ′(c) = 0 and let f ′′exist on an open interval containing c.

1. If f ′′(c) > 0, then f(c) is a relative minimum.
2. If f ′′(c) < 0, then f(c) is a relative maximum.
3. If f ′′(c) = 0 then the test fails. Use the First Derivative Test.

Practical Components

Do the following questions.
Organise your solutions neatly in your exercise book.

Ex 8C: Maximum and minimum problems when the function is known
- Q's 4, 6, 9 – 11

Ex 8D: Maximum and minimum problems when the function is unknown
- Q's 1, 3, 6, 8, 10

Ex 8E: Rates of change
- Q's 3, 4, 6 – 8, 12 – 14

Complete the task on **Mathspace**

Investigation

This fortnight's investigation is **ASSESSABLE.**

See separate page

(20 marks – see rubric)

QFO
Quiz/Forum/Other

Remember to check-in with Jenny every lesson, and get your name marked off.

You have a test in week 8, worth 30% of your assessment. More details to come.

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MM3 2021 Investigation week 6/7

Read the following notes and example and complete the question.

An application of differentiation in the world of economics is that which involves marginal cost, marginal revenue and marginal profit.

All of these relate to an instantaneous rate of change (which we now know to be called the derivative) of some cost, revenue or profit functions.

Definitions

- Marginal Cost: the derivative of the cost function with respect to the production level
- Marginal Revenue: the derivative of the revenue function with respect to the production level
- Marginal Profit: the derivative of the profit function with respect to the production level

Example:

The revenue function for the production of watches is given by $R(n) = n[14 - (\frac{n}{1\,000}]$, and the cost function for the watches is given by $C(n) = 4n + 7\,000$

a) What is the profit function?

The profit is the total revenue (in) less the total costs (out). So $P(n) = R(n) = C(n)$

$$P(n) = n\left(14 - \frac{n}{1\,000}\right) - (4n + 7\,000)$$

$$P(n) = \left(14n - \frac{n^2}{1\,000}\right) - (4n + 7\,000)$$

$$P(n) = 14n - \frac{n^2}{1\,000} - 4n - 7\,000$$

$$P(n) = 10n - \frac{n^2}{1\,000} - 7\,000$$

b) What is the marginal profit function?

Marginal profit means to find $P'(n)$

$$P'(n) = 10 - \frac{n}{500}$$

c) What is the marginal profit for $2\,000$ watches? Interpret this result.

This means we need to find $P'(2000)$

$$P'(2000) = 10 - \frac{2\,000}{500} = 10 - 4 = 6$$

This means that the change in profit from making 2000 watches to 2001 watches is $6.

d) What is the optimum number of watches needed to maximise the profit?

So we need to maximise the profit function. The maximum profit happens when the marginal profit function is 0. This means that at that point, there is no extra profit to be made to make another watch.

Set $P'(n) = 0$ and solve.

$$P'(n) = 10 - \frac{n}{500}$$

$$10 - \frac{n}{500} = 0$$

$$10 = \frac{n}{500}$$

$$5\,000 = n$$

So to maximise profit we should aim to manufacture $5\,000$ watches.

Question:

The cost of producing x thousand pairs of shoes is given by $x^3 - 6x^2 + 15x$, measured in dollars. Each pair is sold for \$10.00 to a wholesaler.

a) Let $P(x)$ be the profit from selling x thousand pairs of shoes.
Determine the expression for marginal profit.

b) what is the optimum number of pairs of shoes needed to maximise the profit?