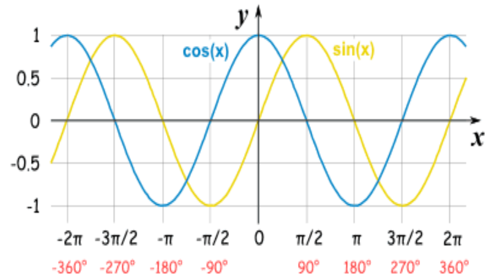
Week 3&4
Term 1
2021

HAWKER COLLEGE
Engage | Inspire | Achieve

Learning Brief MM3: Further differentiation and applications

Goals



By the end of this fortnight, you should be able to:

- establish the formulas $\frac{d}{dx}(\sin x) = \cos x$, and $\frac{d}{dx}(\cos x) = -\sin x$ by numerical estimations of the limits and informal proofs based on geometric constructions
- use trigonometric functions and their derivatives to solve practical problems.
- understand and use the product and quotient rules
- understand the notion of composition of functions and use the chain rule for determining the derivatives of composite functions
- apply the product, quotient and chain rule to differentiate functions such as xe^x, $\tan x$, $\frac{1}{x^n}$, $x\sin x$, $e^{-x}\sin x$ and $f(ax+b)$.

Theoretical Components

Resources:
Maths Quest Year 12 Chapter 7

Derivatives of Sine and Cosine functions
Read and make notes examples 23 and 24 from Chapter 7

1. If $f(x) = \sin x$ then $f'(x) = \cos x$
2. If $f(x) = \cos x$ then $f'(x) = -\sin x$
3. If $f(x) = \sin ax$ then $f'(x) = a\cos ax$
4. If $f(x) = \cos ax$ then $f'(x) = -a\sin ax$
5. If $f(x) = \sin[g(x)]$ then $f'(x) = g'(x)\cos[g(x)]$
6. If $f(x) = \cos[g(x)]$ then $f'(x) = -g'(x)\sin[g(x)]$

Product rule
Read and make notes example 25 from Chapter 7

Product rule

(a) If $y = uv$ then $\frac{dy}{dx} = u \times \frac{dv}{dx} + v \times \frac{du}{dx}$.

(b) If $f(x) = u(x) \times v(x)$ then $f'(x) = u(x) \times v'(x) + v(x) \times u'(x)$.

Quotient Rule
Read and make notes examples 27 and 28 from Chapter 7

Quotient rule

(a) If $y = \frac{u}{v}$ then $\frac{dy}{dx} = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$.

(b) If $f(x) = \frac{u(x)}{v(x)}$ then $f'(x) = \frac{v(x)u'(x) - u(x)v'(x)}{[v(x)]^2}$.

Practical Components

Do the following questions.
Organise your solutions neatly in your exercise book.

Ex 7G The derivatives of $\sin x$ and $\cos x$
- Q's 1 – 2 (a,b,f), 3, 6 – 7 (a,f), 8 (a,m,o,s), 11 (a,b)

Ex 7H The product rule
- Q's 1, 3, Q4 (j, n, r) 6

Ex 7I The quotient rule
- Q's 1 – 2, 3 (a,d,p), 4 – 7

Ex 7J Mixed problems on differentiation
- Q's 1 & 2 (b, c, e, g, j, m,n), 3 (a, d, f, i,j n, q, t, u)

Complete the **Mathspace** task set for week 4.

Investigation

See next page.

QFO
Quiz/Forum/Other

Complete the quiz on Mathspace.
Work on your assignment (due week 5).
Remember to check-in with Jenny each lesson and get your name marked off.

2021 MM3 Weeks 3 & 4 Investigation

Parametric Differentiation: In many later situations, a curve will be specified by two equations giving x and y in terms of some third variable t, called a *parameter*. For example,

$$x = 2t, \qquad y = t^2$$

specifies the parabola $y = \frac{1}{4}x^2$, as can be seen by eliminating t from the two equations. In this situation it is very simple to calculate dy/dx directly using *parametric differentiation*. The formula below is another version of the chain rule, because 'the dt's just cancel out'.

13	PARAMETRIC FUNCTIONS: $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt}$

WORKED EXERCISE: In the example above, $\dfrac{dy}{dx} = \dfrac{2t}{2}$

$$= t.$$

Question

If $x = \frac{1}{\sqrt{5t^2-2t}}$ and $y = e^{2t^2} - e^{-7}$, calculate $\frac{dy}{dx}$ using the method above.

Give your answer in the simplest form.