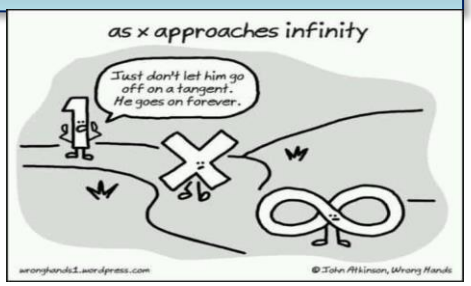
Week 9/10
Term 3
2023

HAWKER COLLEGE
Engage | Inspire | Achieve

MM2: Introduction to Differential Calculus

Goals



This fortnight we will endeavour to:

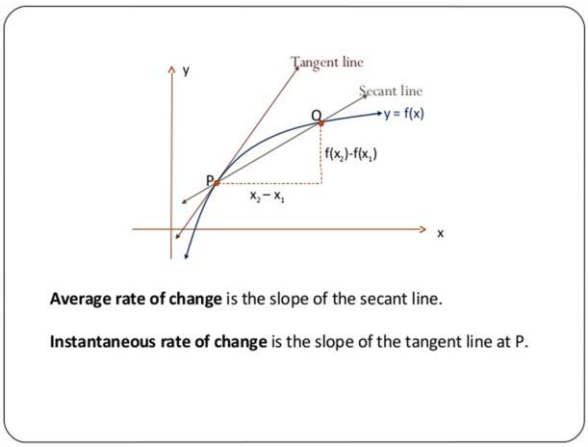
- understand average rates of change and use and apply gradients of tangents
- understand instantaneous rates of change and to compare average and instantaneous rates of change
- find equations of tangent lines and relate the gradient function to the original function
- informally find the rule for the gradient function
- finding limits of functions
- using limit theorems

Theoretical Components

Resources:
Maths Quest Year 11 Chapters 8 and 9

Knowledge Checklist:

- what is a rate?
- constant rates
- variable rates
- average rates of change
- instantaneous rates of change
- interpret graphs that illustrate rates of change
- equations of tangents
- **what is a limit?**
- **evaluating limits**
- what is a gradient function?
- what is the x-intercept of a gradient function?
- power rule
- finding gradient functions by sketching
- finding gradient functions by using the rule
- finding gradient functions using your CAS



Average rate of change is the slope of the secant line.

Instantaneous rate of change is the slope of the tangent line at P.

Practical Components

Maths Quest Year 11 **Chapter 8 – Rates of Change** (pdf – GC).

Exercise 8G Qs 1 a, b, c; 2; 4 Relating the gradient function to the original function

Exercise 8I Qs 1a, b, c; 2a, b, c; 5; Rates of change of polynomials

Maths Quest Year 11 **Chapter 9 – Differentiation** (see pdf – GC)

Exercise 9A Introduction to limits
Qs 1, 2, 3, 8 (a, c, f) 10, 12 (b, d)

Exercise 9B Limits of discontinuous, rational and hybrid functions
Qs 1, 2, 3, 7 (c), 8 (c), 9(b, f), 10(b, d, j)

Investigation

See the following page. This task is a continuation of the Investigation from the last brief.

QFO
Quiz/Forum/Other

Mathspace.co task:
Rates of change at a point and Limits
https://mathspace.co/student/tasks/TopicCustomTask-649004/

MM2 INVESTIGATION Weeks 9/10

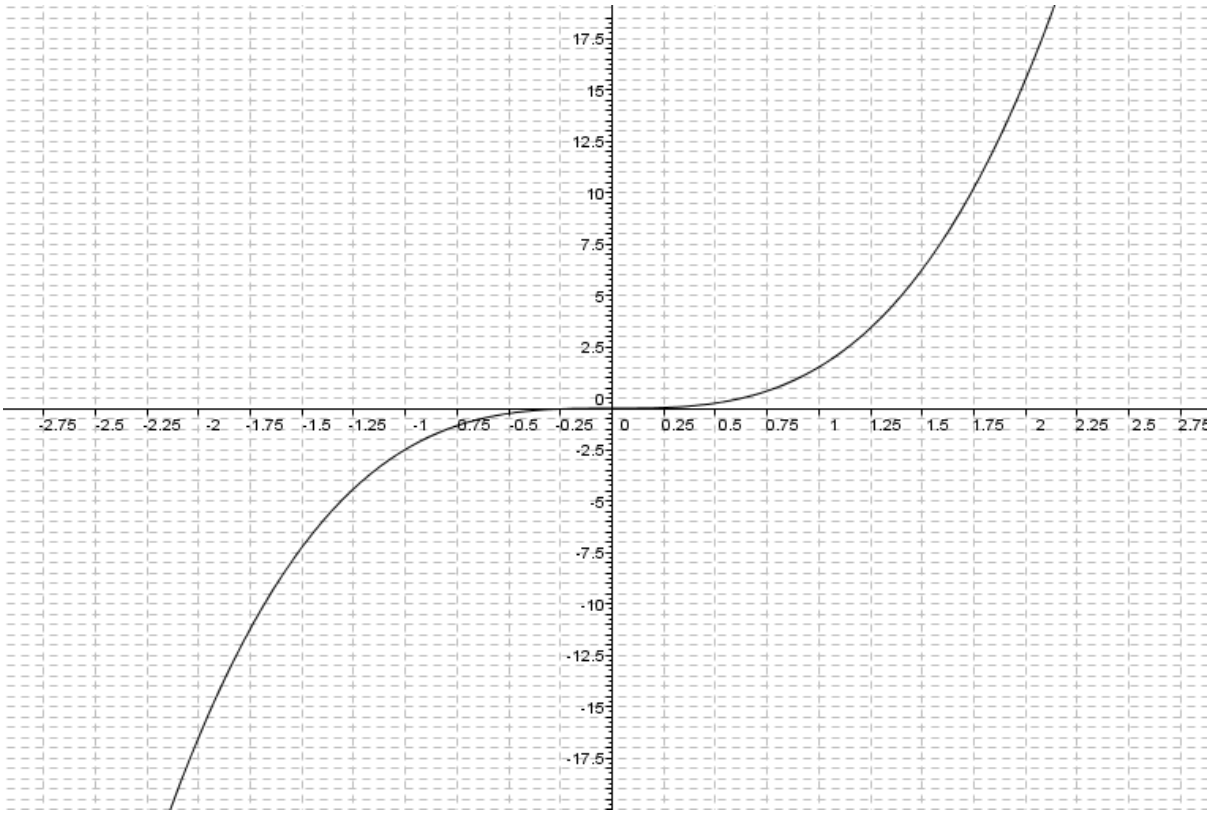
Consider the function: $y = x^3 + 3$. Compare it to the function from the Week 7-8 investigation. What does the +3 do to this function?

X	-2	-1	-0.5	0	0.5	1	2
Y	-5			3			
Gradient at x				0			

Will it change the gradient at the points evaluated above from the graph of : $y = x^3$? Why/Why not?
Write a statement about curves of the form $y = x^3 + k$, and what the gradient function would be like. What does the k do? Explain how you know this.

Now consider a function of the form $y = 2x^3$
What does the "2" in this function "do"?

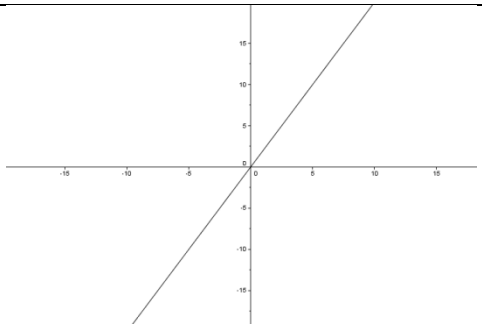
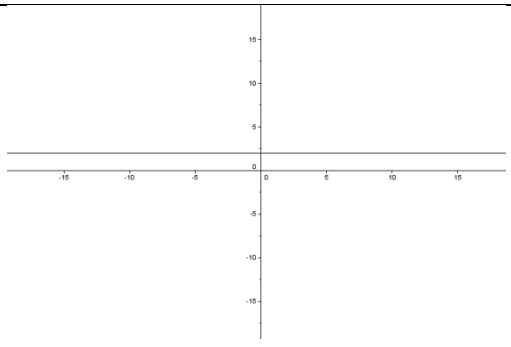
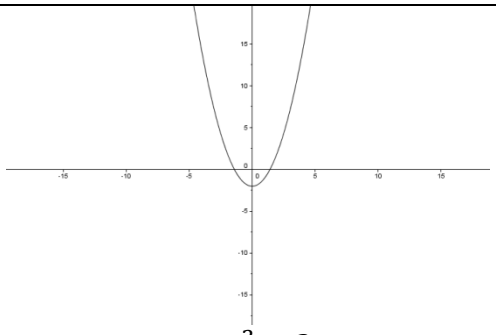
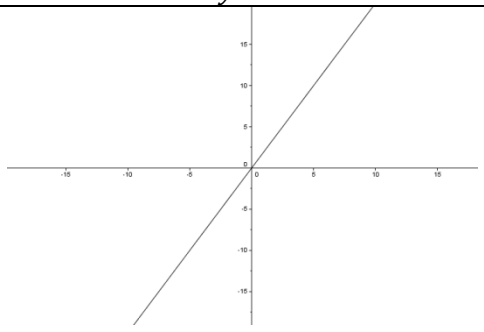
What do you think it will do to the gradient?
(as you did before, find some points and then find the gradient of the tangents at those points)

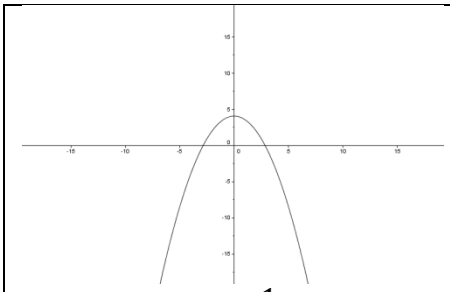
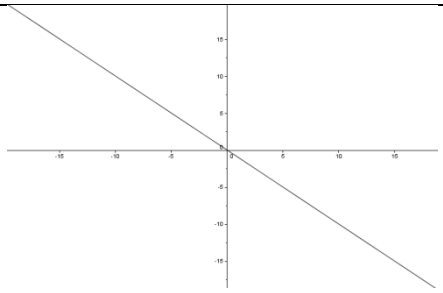
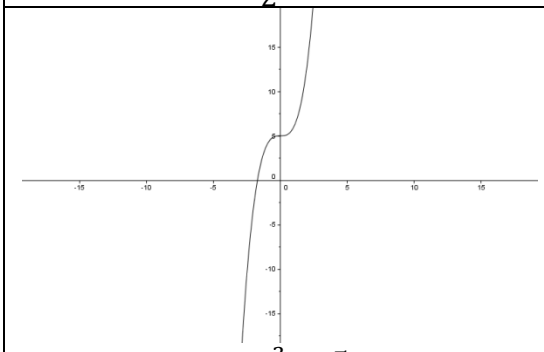
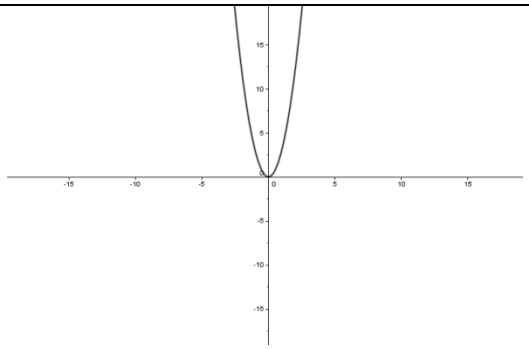
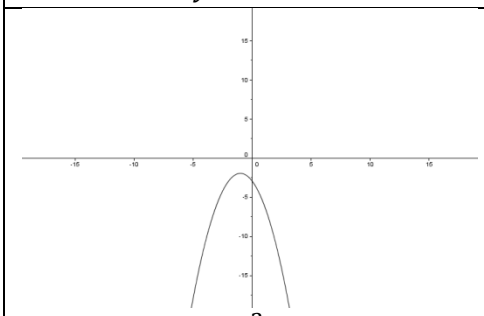
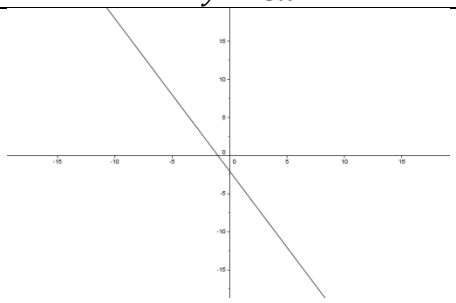
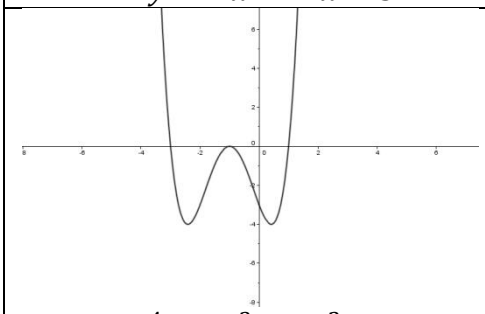
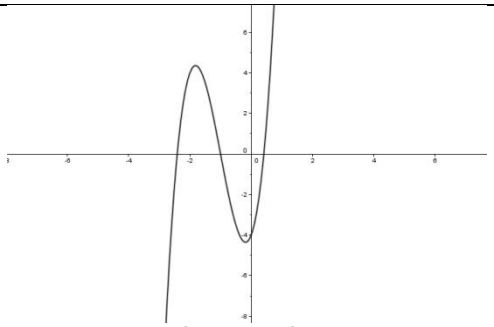


X	-2	-1	-0.5	0	0.5	1	2
Y							
Gradient at x							

What can you deduce about the "2" in relation to the value of the gradient compared to the value of the gradients in the curve of $y = x^3$?

Here are the graphs of some functions, and their matching "gradient functions".
Can you see the correlation between the value of the gradient, whether it is positive or negative, and the original curve? What is special about the x-intercepts of the gradient function?

FUNCTION	GRADIENT FUNCTION
 $y = 2x$	 $y = 2$
 $y = x^2 - 2$	 $y = 2x$

 $y = -\frac{1}{2}x^2 + 4$	 $y = -x$
 $y = x^3 + 5$	 $y = 3x^2$
 $y = -x^2 - 2x - 3$	 $y = -2x - 2$
 $y = x^4 + 4x^3 + 2x^2 - 4x - 3$	 $y = 4x^3 + 12x^2 + 4x - 4$

Can you see a pattern?

Develop a rule for finding the gradient function of $y = ax^n$.