Week 3
Term 1
2023

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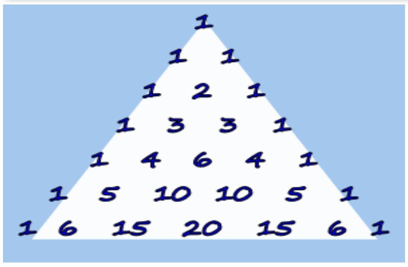
MM1 Learning Brief: Combinations and Binomial Expansions

Goals



By the end of this week, you should be able to:

- understand the notion of a combination as an unordered set of r objects taken from a set of n distinct objects
- use the notation $\binom{n}{r}$ and the formula $\binom{n}{r} = \frac{n!}{r!(n-r)!}$ for the number of combinations of r objects taken from a set of n distinct objects
- expand $(x+y)^n$ for small positive integers n
- recognise the numbers $\binom{n}{r}$ as binomial coefficients, (as coefficients in the expansion of $(x+y)^n$)
- use Pascal's triangle and its properties

Theoretical Components

Knowledge checklist:

- Use ^nP_r to count number of possible arrangements where order is important (permutations)
- Use ^nC_r to represent selections where order is not important (combinations)
- Make connections between the number of combinations and the number of permutations.
- Investigate patterns in Pascal's triangle and the relationship to combinations, establish counting principles and use them to solve simple problems involving numerical values for *n* and *r*.
- Use CAS to compute ^nC_r for a given n and a given r

Videos

Combinations -
https://www.youtube.com/watch?v=Ej_4oSipqes&feature=youtu.be

Binomial Expansion theorem -
https://www.youtube.com/watch?v=ajaAk1CP5pw

Online reading

http://www.mathsisfun.com/combinatorics/combinations-permutations.html

http://www.mathsisfun.com/pascals-triangle.html

Practical Components

Do the following questions.
Organise your solutions neatly in your exercise book.
Chapter 12 of Maths Quest 11 Mathematical Methods (pdf - Google Classroom). Make notes as needed.

Ex 12D Permutations using ^nP_r

- Q1 a,c,e,g,i,k then ALL even numbered questions

Ex 12G Combinations using ^nC_r

- ALL odd numbered questions

Complete the **Mathspace** task set for week 3.
https://mathspace.co/student/tasks/TopicCustomTask-600974/

Investigation

See notes and example on the following page.

Questions

1. Research and write down at least 3 interesting things about the number patterns in Pascals Triangle.

2. The 4th term in the expansion of $(3a^2 + \frac{p}{b})^4$ is $\frac{96a^2}{b^3}$, for some constant p. Find the value of p.

QFO

Quiz/Forum/Other

You will need a CAS calculator for this course – see link on Google Classroom. Please see Serene if you have any concerns.

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Pascal's triangle is a triangular array where each number is the sum of the two numbers above it (except for the edges, which are all "1"). It is named after the 17th century French mathematician, Blaise Pascal (1623 – 1662).

The triangle is constructed in the following manner: In row 0 (the topmost row), there is a unique nonzero entry 1. Each entry of each subsequent row is constructed by adding the two numbers above it. For example, numbers 1 and 3 in the third row are added to produce the number 4 in the fourth row.

$$
\begin{array}{c}
1\\
1 \quad 1\\
1 \quad 2 \quad 1\\
1 \quad 3 \quad 3 \quad 1\\
1 \quad 4 \quad 6 \quad 4 \quad 1\\
1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1\\
1 \quad 6 \quad 15 \quad 20 \quad 15 \quad 6 \quad 1\\
1 \quad 7 \quad 21 \quad 35 \quad 35 \quad 21 \quad 7 \quad 1
\end{array}
$$

The numbers in Pascal's Triangle have a special relationship with the coefficients of binomial expansions (binomial coefficients) and combinations.

$(a+b)^n$	*Binomial expansion*	*Pascals Triangle as combinations* ${}^nC_r = \binom{n}{r}$
$(a+b)^0 =$	1	$\binom{0}{0}$
$(a+b)^1 =$	$a+b$	$\binom{1}{0} \quad \binom{1}{1}$
$(a+b)^2 =$	$a^2+2ab+b^2$	$\binom{2}{0} \quad \binom{2}{1} \quad \binom{2}{2}$
$(a+b)^3 =$	$a^3+3a^2b+3ab^2+b^3$	$\binom{3}{0} \quad \binom{3}{1} \quad \binom{3}{2} \quad \binom{3}{3}$
$(a+b)^4 =$	$a^4+4a^3b+6a^2b^2+4ab^3+b^4$	$\binom{4}{0} \quad \binom{4}{1} \quad \binom{4}{2} \quad \binom{4}{3} \quad \binom{4}{4}$
$(a+b)^5 =$	$a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5$	$\binom{5}{0} \quad \binom{5}{1} \quad \binom{5}{2} \quad \binom{5}{3} \quad \binom{5}{4} \quad \binom{5}{5}$
		n is the row number (starting at row 0) and *r* is the element in the row (also starting at 0)

Consider the expansions above of $(a+b)^n$. Particularly note the following patterns.

- For each expansion to the power n, there are $n+1$ elements.
- For each term, the sum of the exponents is n.
- Powers of a decrease from left to right, from n down to 0
- Powers of b increase from left to right, from 0 up to n.
- The coefficients start at 1, end at 1, AND are the terms of the relevant row from Pascals triangle!

The pattern in the expansions observed is summarised in a formula called the **binomial theorem**.

Binomial Theorem

$$(a+b)^n = \binom{n}{0}a^nb^0 + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{n}a^0b^n$$

$$\text{where } \binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

Any particular term in the expansion of $(a+b)^n$ can be found using

$$\binom{n}{r}a^{n-r}b^r$$

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Example

What is the seventh term in the expansion of $(m-2n)^{12}$?

We need to construct the seventh term from this $\binom{n}{r} a^{(n-r)} b^r$ where n is 12 and r is 6.

The coefficient $\binom{n}{r}$ where n is 12 and r is 6 is $\binom{12}{6} = 924$.

The term will have both m and $(2n)$ components. The m component would be $m^{12-6} = m^6$

The $2n$ component would be $(2n)^6 = 64n^6$.

So putting that altogether will give us $924m^6 \times 64n^6 = 59\,136m^6n^6$.

Questions

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Student Reflection:

How did you go with this week's work?

What did you learn?

What did you find easy?

What do you need to work on?

Mathspace task score (if applicable):