Week 3
Term 3
2022

HAWKER COLLEGE
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Learning Brief

MA4

Goals

This week:

- deseasonalise a time series by using a seasonal index, including the use of spreadsheets to implement this process
- implement the statistical investigation process to answer questions that involve the analysis of time series data

Theoretical Components

Resources:
For this week the theory work is in the *PDF file:* Week *3* Notes & Exercises

A brief (2 minutes) introduction on seasonal adjustment:
https://www.youtube.com/watch?v=ccgmdVsrVAw

This clip works through the steps in deseasonalising data:
https://www.youtube.com/watch?v=NhznWdDd9UY

Knowledge Checklist
- Seasonal trend
- Deseasonalising – removing seasonal variation
- Seasonal index
- Setting out
- Forecasting
- Re-seasonalise
- Graphing using a spreadsheet

Practical Components

There are questions to be answered in the booklet *Week 3 Notes & Exercises*.

Excel files:
- Seasonal Adjustment Example
- Sheet with Formulas

Investigation

On Google Classroom and attached to this week's work.

On-line Quiz

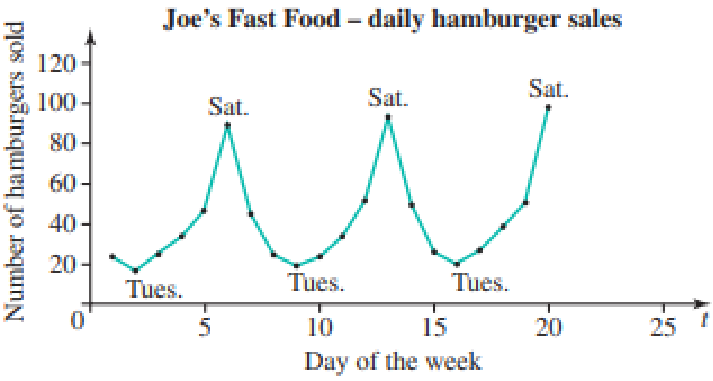
None

WEEK 3 NOTES & EXERCISES

Seasonal adjustment

A seasonal trend is similar to a cyclical trend where there are defined peaks and troughs in the time series data, except for one notable difference.

Seasonal trends have a fixed and regular period of time between one peak and the next peak in the data values. Conversely, there is a fixed and regular period of time between on trough and the next trough.



When fitting a straight line to a time series, it is difficult to find an effective linear equation for such data. As well, our work on smoothing indicated that seasonal data may not lead themselves to the techniques of moving average or median smoothing. We may just have to accept that the data vary from season to season and treat each record individually.

For example, the unemployment rate in Australia is often quoted as '6.8% - seasonally adjusted'. The Government has accepted that each season has its own time series, meaning each season is more or less independent of the other seasons.

How can we remove the effect of the season on our time series? The technique of seasonally adjusting, or 'deseasonalising'. Will modify the original time series, hopefully removing the season variation, and exposing any other trends (secular, cyclic, random) which may be 'hidden' by seasonal variation.

Deseasonalising time series

The process of deseasonalising time series data involves calculating seasonal indices. A **seasonal index** compares a particular season to the average season.

That is, a seasonal index measures by what factor a particular season is above or below the average of all seasons for the cycle. For example:

- Seasonal index = 1.3 means that season is 1.3 times the average season (that is, the figures for this season are 30% above the seasonal average). It is a peak or high season.
- Seasonal index = 0.7 means that season is 0.7 times the average season (that is, the figures for this season are 30% below the seasonal average). It is a trough or low season.
- Seasonal index = 1.0 means that season is the same as the average season. It is neither a peak nor a trough.

To deseasonalise the data, we divide each value by the corresponding seasonal index. That is:

$$\text{Deseasonalised figure or value} = \frac{\text{actual original figure or value}}{\text{seasonal index}}$$

The method of deseasonalising time series is best demonstrated with an example. Carefully observe the various steps, which **must** be performed in the order shown.

Example

Unemployment rates have been collected over a 5-year period and presented in this table. The data represents the percentage of the population unemployed. It is difficult to see any trends, other than seasonal ones, such as unemployment is lowest in Summer and highest in Autumn and Winter.

Original Series

Season	2005	2006	2007	2008	2009
Summer	6.2	6.5	6.4	6.7	6.9
Autumn	8.1	7.9	8.3	8.5	8.1
Winter	8.0	8.2	7.9	8.2	8.3
Spring	7.2	7.7	7.5	7.7	7.6

Steps:

1. Calculate the seasonal indices.
2. Deseasonalise the data using the seasonal indices.
3. Plot the original and deseasonalised data.
4. Comment on your results.

Step 1: Find the yearly averages over the four seasons for each year and put them in a table.

Year	2005	2006	2007	2008	2009
Average	$\frac{6.2+8.1+8.0+7.2}{4}$ = 7.3750	$\frac{6.5+7.9+8.2+7.7}{4}$ = 7.5750	7.5250	7.7750	7.7250

Step 2: Divide each term in the original time series by its yearly average.

Season	2005	2006	2007	2008	2009
Summer	$\frac{6.2}{7.3750}$ = 0.8407	$\frac{6.5}{7.5750}$ = 0.8581	$\frac{6.4}{7.5250}$ = 0.8505	$\frac{6.7}{7.7750}$ = 0.8617	$\frac{6.9}{7.7250}$ = 0.8932
Autumn	$\frac{8.1}{7.3750}$ = 1.0983	$\frac{7.9}{7.5750}$ = 1.0429	$\frac{8.3}{7.5250}$ = 1.1030	$\frac{8.5}{7.7750}$ = 1.0932	$\frac{8.1}{7.7250}$ = 1.0485
Winter	1.0847	1.0825	1.0498	1.0547	1.0744
Spring	0.9763	1.0165	0.9967	0.9904	0.9838

Step 3: Determine the seasonal averages from the second table using the same method as Step 1. These are called **seasonal indices**.

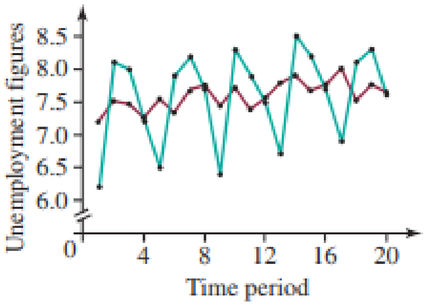
Season	Summer	Autumn	Winter	Spring
Seasonal index	0.8608	1.0772	1.0692	0.9927

Step 4: Divide each term in the original series by its seasonal index. This is the seasonally adjusted or deseasonalised time series. **Note:** Your answers may vary a little, depending upon how and when you rounded your calculations. It is best if you do not round your answers until the very last calculation.

You do not really see the effect of deseasonalising until you graph the original and deseasonalised data on the same graph.

Season	2005	2006	2007	2008	2009
Summer	$\frac{6.2}{0.8608}$ = 7.202	$\frac{6.5}{0.8608}$ = 7.551	7.435	7.783	8.015
Autumn	$\frac{8.1}{1.0772}$ = 7.520	$\frac{7.9}{1.0772}$ = 7.334	7.705	7.891	7.520
Winter	$\frac{8.0}{1.0692}$ = 7.482	$\frac{8.2}{1.0692}$ = 7.669	7.388	7.669	7.763
Spring	$\frac{7.2}{0.9927}$ = 7.253	$\frac{7.7}{0.9927}$ = 7.756	7.555	7.756	7.656

Step 5: Graph the original and the seasonally adjusted (deseasonalised) time series. **Note:** For this graph 1 is Summer 2005, 2 is Autumn 2005, 4 is Spring 2005 and 20 is Spring 2009.



There appears to be a slight upward trend in unemployment. Most but not all of the seasonal variation has been removed. However, by using least squares, we could more confidently fit a straight line to the deseasonalised data.

There is an Excel file attached to this week's work showing the trendline for the deseasonalised data and the least squares regression equation. Check Google Classroom for the file.

Forecasting with seasonal time series

In the example, we smoothed out the seasonal variation and are now able to see any secular trend more clearly. If there is an upward or downward secular trend, then a straight line equations can be calculated and used for making predictions into the future. Using the least squares regression methods of the deseasonalised data is always preferred.

REMEMBER

1. Deseasonalising a time series involves replacing the original time series with another one where most or all of the seasonal variation is removed. To deseasonalise data:
   (a) Average over all seasons for each year — these are the yearly averages.
   (b) Divide each point in the original time series by its corresponding yearly average.
   (c) Using this new series, average over all years for each season — these are the seasonal indices.
   (d) Divide each point in the original time series by its corresponding seasonal index.
2. To deseasonalise figures:

$$\text{Deseasonalised figure or value} = \frac{\text{actual original figure or value}}{\text{seasonal index}}$$

3. To seasonalise (predicted) figures:

$$\text{Seasonalised figure or value} = \text{deseasonalised figure or value} \times \text{seasonal index}$$

4. The sum of the seasonal indices is equal to the number of seasons.

Exercise 1

1. The price of sugar ($/kg) has been recorded over 3 years on a seasonal basis.

Season	2007	2008	2009
Summer	1.03	0.98	0.95
Autumn	1.26	1.25	1.21
Winter	1.36	1.34	1.29
Spring	1.14	1.07	1.04

   a. Compute the seasonal indices.
   b. Deseasonalise the data using the seasonal indices.
   c. Plot the original and deseasonalised data.
   d. Comment on your results.

You need to show at least 4 of the calculations manually, for the intermediate steps use 4 decimal places. For the rest of the calculations, you can use a spreadsheet, but you will need to share it or attach it to your booklet.

Spreadsheet solution

Deseasonalising by hand can be long and tedious (and prone to error). A more efficient method is to use a spreadsheet.The spreadsheet solution for the unemployment example above will look like this:

Season	2005	2006	2007	2008	2009
Summer	6.2000	6.5000	6.4000	6.7000	6.9000
Autumn	8.1000	7.9000	8.3000	8.5000	8.1000
Winter	8.0000	8.2000	7.9000	8.2000	8.3000
Spring	7.2000	7.7000	7.5000	7.7000	7.6000

Step 1 Calculate the yearly averages.

Yearly ave.	7.3750	7.5750	7.5250	7.7750	7.7250

Step 2 Divide each value by the yearly average.

Season	2005	2006	2007	2008	2009	Seasonal indicies
Summer	0.8407	0.8581	0.8505	0.8617	0.8932	0.8608
Autumn	1.0983	1.0429	1.1030	1.0932	1.0485	1.0772
Winter	1.0847	1.0825	1.0498	1.0547	1.0744	1.0692
Spring	0.9763	1.0165	0.9967	0.9904	0.9838	0.9927

Step 3 Calculate the seasonal indices. (average of each season)

Step 4 Final seasonal adjustment (divide by seasonal indices).

Season	2005	2006	2007	2008	2009
Summer	7.2023	7.5508	7.4346	7.7831	8.0154
Autumn	7.5195	7.3338	7.7052	7.8908	7.5195
Winter	7.4820	7.6690	7.3884	7.6690	7.7625
Spring	7.2528	7.7564	7.5550	7.7564	7.6557

The formulas to generate these results are shown below.

Season	2005	2006	2007	2008	2009
Summer	6.2000	6.5000	6.4000	6.7000	6.9000
Autumn	8.1000	7.9000	8.3000	8.5000	8.1000
Winter	8.0000	8.2000	7.9000	8.2000	8.3000
Spring	7.2000	7.7000	7.5000	7.7000	7.6000

Step 1 Calculate the yearly averages.

Yearly ave.	=AVERAGE(E7:E10)	=AVERAGE(F7:F10)	=AVERAGE(G7:G10)	=AVERAGE(H7:H10)	=AVERAGE(I7:I10)

Step 2 Divide each value by the yearly average.

Season	2005	2006	2007	2008	2009	Seasonal indicies
Summer	=E7/E$13	=F7/F$13	=G7/G$13	=H7/H$13	=I7/I$13	=AVERAGE(E17:I17)
Autumn	=E8/E$13	=F8/F$13	=G8/G$13	=H8/H$13	=I8/I$13	=AVERAGE(E18:I18)
Winter	=E9/E$13	=F9/F$13	=G9/G$13	=H9/H$13	=I9/I$13	=AVERAGE(E19:I19)
Spring	=E10/E$13	=F10/F$13	=G10/G$13	=H10/H$13	=I10/I$13	=AVERAGE(E20:I20)

Step 3 Calculate the seasonal indices. (average of each season)

Step 4 Final seasonal adjustment (divide by seasonal indices).

Season	2005	2006	2007	2008	2009
Summer	=E7/$K17	=F7/$K17	=G7/$K17	=H7/$K17	=I7/$K17
Autumn	=E8/$K18	=F8/$K18	=G8/$K18	=H8/$K18	=I8/$K18
Winter	=E9/$K19	=F9/$K19	=G9/$K19	=H9/$K19	=I9/$K19
Spring	=E10/$K20	=F10/$K20	=G10/$K20	=H10/$K20	=I10/$K20

Notes about sheets

1. By adding/deleting columns, you could increase/decrease the number of years.
2. By adding/deleting rows between the seasons, you can increase/decrease the number of seasons.
3. The sum of the seasonal indices always equals to the number of seasons.
4. The $ sign in a formula 'locks' in the specific cell referenced.
5. Most of the cells were 'filled' by clicking and dragging.

Forecasting with seasonal time series

The prediction obtained using the deseasonalised data means the prediction has also be deseasonalised or smoothed out to the average season. As we have the relevant seasonal indices, we should be able to use it to remove the smoothing; that is to re-seasonalise the predicted value.

The formula for re-seasonalised is:

$$\text{Reseasonalised figure or value} = \text{Deseasonalised figure or value} \times \text{seasonal index}$$

Example

Using the data from the example on unemployment figures, predict the unemployment rate for the Summer of 2010.

Solution

From the graph of seasonal and deseasonalised unemployment data, the regression formula is $y = 0.0227x + 7.3564$. When using the equation, Summer 2005 = 1, Autumn 2005 = 2, Winter 2005 = 3, etc. This means Summer 2010 = 21.

Substituting $y = 0.0227 \times 21 + 7.3564 = 7.8\%$ unemployment rate.

This number is also deseasonalised, as it came from the deseasonalised data.

We can re-seasonalise it by using the re-seasonalise formula.

$\text{Reseasonalised} = 7.8 \times 0.8608 = 6.7\%$

The deseasonalised figure seems high but the re-seasonalised data more closely resembles the original data.

IMPORTANT

On the Google Classroom, there is a file called 'Sheet with Formulas' which is a formatted spreadsheet with no data but containing the formulas necessary to answer the questions below and the investigation.

Make a copy of this file for you to work on. By now, you should know how to use the appropriate data to produce the graphs. You can also get a copy of this file from your teacher. You can try some formatting to improve the appearance of the spreadsheets for the questions below and the investigations.

Exercise 2

1. It is a known problem for young adults (18 – 25) to find work; these problems are different from those facing older people. The youth unemployment statistics are recorded separately form the overall figures.

Season	**2005**	**2006**	**2007**	**2008**	**2009**
Summer	7.6	7.7	7.8	7.7	7.9
Autumn	10.9	11.3	11.9	12.6	13.1
Winter	11.7	12.4	12.8	13.5	13.9
Spring	9.9	10.5	10.8	11.4	11.9

Using the youth unemployment figures for the five years given in the table:

a. Deseasonlise the time series.
b. Plot the original and deseasonalised time series.
c. Comment on your results.
d. Use the regression equation to predict youth unemployment rate in the Spring of 2010. You will need to reseasonalise.

Make sure you attach your answers to the booklet.

2. Data on the total seasonal rainfall (in mm) have been accumulated over a 6 year period.

Season	**2004**	**2005**	**2006**	**2007**	**2008**	**2009**
Summer	103	97	95	117	118	120
Autumn	93	84	82	100	99	98
Winter	143	124	121	156	155	151
Spring	123	109	107	125	122	124

a. Deseasonlise the time series.
b. Plot the original and deseasonalised time series.
c. Comment on your results.
d. Predict the rainfall for the Winter of 2010. You will need to reseasonalise.

Make sure you attach your answers to the booklet.

2022 MA4 Week 3 Investigation

Task 1

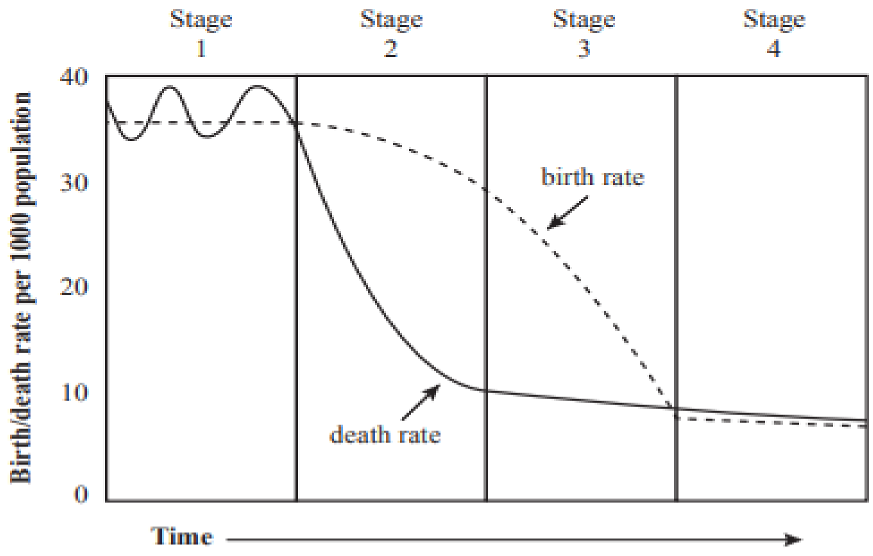
It is possible to seasonally adjust a time series for other than the usual 4 seasons. Consider a fine dining restaurant that wishes to study its customer patterns on a daily basis. In this case a ‘season’ is a single day and there are 7 seasons in a weekly cycle. Data recorded is total revenue each day shown in the table.

Season	**Week 1**	**Week 2**	**Week 3**	**Week 4**	**Week 5**
Monday	1036	1089	1064	1134	1042
Tuesday	1103	1046	1085	1207	1156
Wednesday	1450	1324	1487	1378	1408
Thursday	1645	1734	1790	1804	1789
Friday	2078	2204	2215	2184	2167
Saturday	2467	2478	2504	2526	2589
Sunday	1895	1786	1824	1784	1755

1. Modify the provided spreadsheet (Sheet with Formula) so that the data can be deseasonalised. Print out or attach two spreadsheets, one with data and one with the formulas.
2. Graph the original and deseasonalised data showing the trendline for the deseasonalised data and the linear regression formula.
3. Use the formula to predict the income for Thursday of Week 6.
4. Reseasonalise this figure to give a more realistic prediction.
5. Comment on any trend.

Task 2

The following graph presents a theory of demographic transition: how a country's population changes as its economy develops.



40 According to the theory, at which stage of economic development does a country experience the most rapid rise in population?

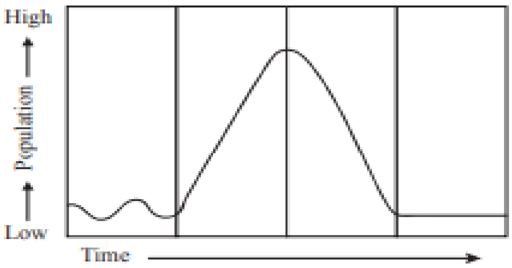
A Stage 1
B Stage 2
C Stage 3
D Stage 4

41 In which two stages of development is a country's population most stable?

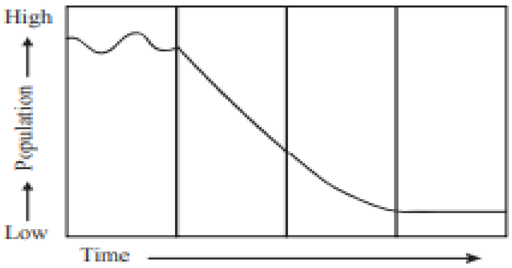
A Stage 1 and Stage 2
B Stage 2 and Stage 3
C Stage 3 and Stage 4
D Stage 1 and Stage 4

Which one of the following graphs most accurately illustrates the change in the size of a country's population as it moves from Stage 1 to Stage 4?

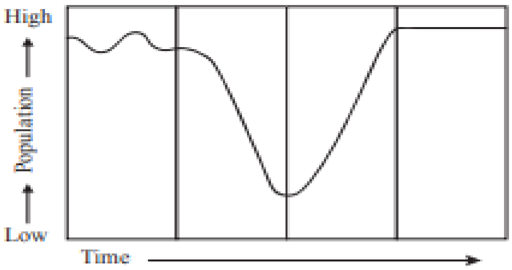
A



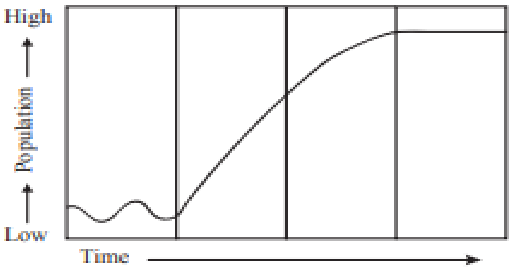
C



B



D



MARKING RUBRIC **Week 3** **Name:**

CRITERIA	EXPECTATIONS	POSS	MULT	GIVEN	TOTAL
Practical	Student completes practical work, including exercises and any Mathspace and/or other tasks, of the brief to an acceptable standard set by the teacher.	**2**	**3**		**/6**
Investigation Task	Student completes the investigation task of the week to an acceptable standard set by the teacher.	**2**	**2**		**/4**
Communication and Reasoning	Student responses are accurate and appropriate in presentation of mathematical ideas in different contexts, with clear and logical working out shown.	**4**	**-**		**/4**
Knowledge and Application	Student submitted work selects and applies appropriate mathematical modelling and problem-solving techniques to solve practical problems and demonstrates proficiency in the use of mathematical facts, techniques and formulae.	**4**	**-**		**/4**
	Submission Guidelines				
Timeliness	Student submits the practical work, including exercises and any Mathspace and/or other tasks, and investigation by the set deadline. See scoring guidelines for specific details.	**2**	**-**		**/2**
				FINAL	**/20**

Student Reflection: How did you go with this week's work? What was interesting? What did you find easy? What do you need to work on?