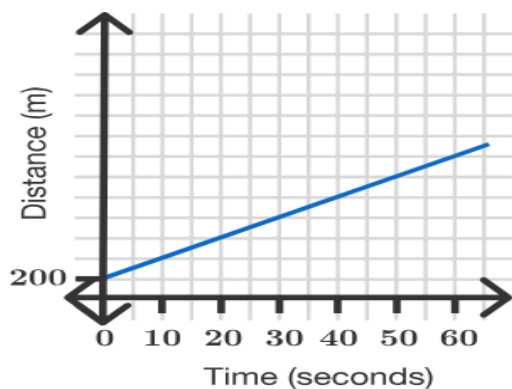


Goals

This week we are going to:

- construct straight-line graphs both with and without the aid of technology
- determine the slope and intercepts of a straight-line graph from both its equation and its plot
- interpret, in context, the slope and intercept of a straight-line graph used to model and analyse a practical situation
- construct and analyse a straight-line graph to model a given linear relationship; for example, modelling the cost of filling a fuel tank of a car against the number of litres of petrol required.
-



Theoretical Components

Resources:

PDF file: Week 12 Notes and Exercises

This clip shows how to sketch a linear graph using a table of values.

<https://www.youtube.com/watch?v=7sg8h0Y8oZk>

This link shows how to sketch a graph using the form $y = mx + b$

<https://www.youtube.com/watch?v=NAblGVxxJZo>

A good overview

<https://www.khanacademy.org/math/algebra-home/alg-linear-eq-func/modal/v/2-variable-linear-equations-graphs>

Knowledge Checklist

- Concept of gradient (including horizontal and vertical lines)
- Calculating gradient using rise over run
- Positive and negative gradient
- Table of values to draw a linear graph
- Using the y-intercept and gradient to sketch a linear function
- Linear relationship between two variables

Order

1. Read through the notes and examples
2. Work through the exercises
3. Complete the investigation at the end of the booklet.
4. Complete the reflection at the end of the booklet
5. Come and see your teacher and make sure you are up to date.

Practical Components

Work through the exercises and show the completed tasks to your teacher.

Be sure to ask for help as you need for the successful completion of all tasks.

Remember to regularly check Google Classroom for messages.

Investigation

Complete the task at the end of the booklet and submit your work for checking. 😊

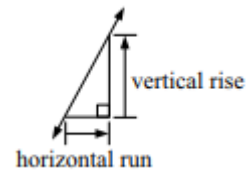
MATHEMATICAL APPLICATIONS 2

WEEK 12 NOTES & EXERCISES

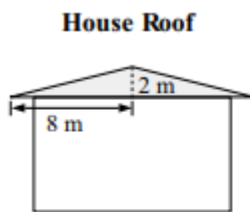
LINEAR GRAPHS – GRADIENT AND INTERCEPT

The words slope and gradient are used when talking about the steepness of a line. Gradient or slope is associated with house roofs, escalators, hills, etc. The steeper the slope the larger the gradient. In order to compare the slopes in different situations there is a standard approach.

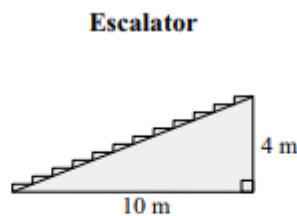
$$\text{i.e., slope} = \frac{\text{vertical rise}}{\text{horizontal run}}$$



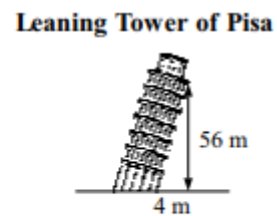
The following illustrations indicate slopes of varying amounts.



$$\begin{aligned} \text{slope} &= \frac{2}{8} \\ &= \frac{1}{4} \end{aligned}$$



$$\begin{aligned} \text{slope} &= \frac{4}{10} \\ &= \frac{2}{5} \end{aligned}$$



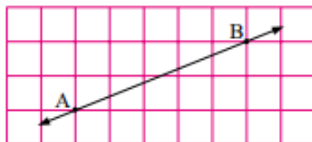
$$\begin{aligned} \text{slope} &= \frac{56}{4} \\ &= 14 \end{aligned}$$

Note: For a horizontal line the vertical rise is 0, therefore the slope is zero.

When line segments are drawn on graph paper we can easily determine the slope of the line segment by drawing horizontal and vertical lines to complete a right-angled triangle.

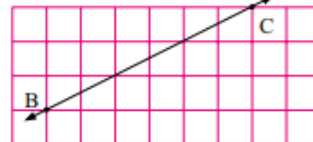
Examples

a Find the slope of AB.



$$\begin{aligned} \text{a slope of AB} &= \frac{\text{vertical rise}}{\text{horizontal run}} \\ &= \frac{2}{5} \end{aligned}$$

b Find the slope of BC.



$$\begin{aligned} \text{b slope of BC} &= \frac{\text{vertical rise}}{\text{horizontal run}} \\ &= \frac{3}{6} \\ &= \frac{1}{2} \end{aligned}$$

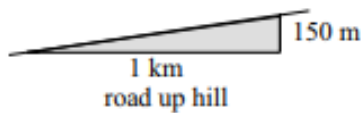
EXERCISE 1

1. Complete the sentences.

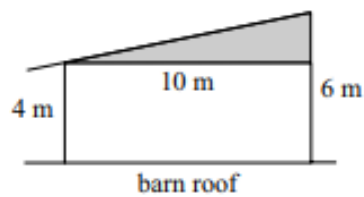
- The slope of a horizontal line is _____
- The slope of a vertical line is _____
- As the line segments becomes steeper their slopes _____

2. Find the slope of the following.

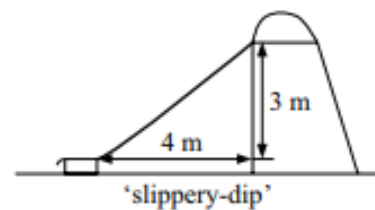
a)



b)



c)



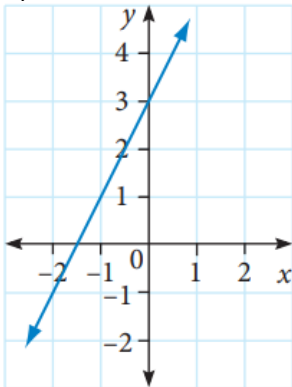
3. The following diagram represents a path crossing through the countryside.



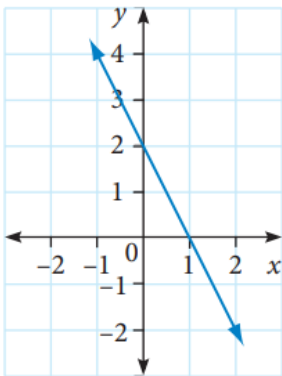
- Indicate the uphill parts of the path.
- Indicate the downhill parts of the path.
- Where is the steepest positive slope?
- Where is the steepest negative slope?
- Where is the slope equal to 0?

4. Find the gradient of these lines and the y-intercept (where they cross the vertical axis).

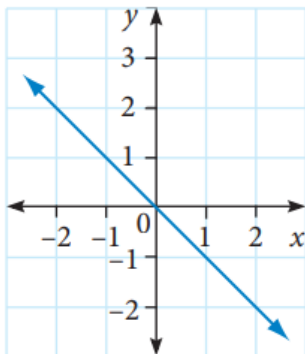
a)



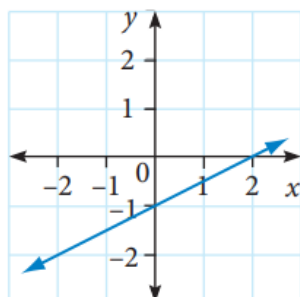
b)



c)



d)



LINEAR GRAPHS

The rules or equations for straight-line graphs may all be written in a similar form. The convention for writing the rule for straight line graphs is;

$$y = mx + b$$

This relationship connects every point on the straight-line graph.

x is the independent variable as any value may be used.
 y is the dependent variable as it depends on the value of x .
 m is the gradient.
 b is the intercept with the vertical axis, the y intercept in this case.

Example

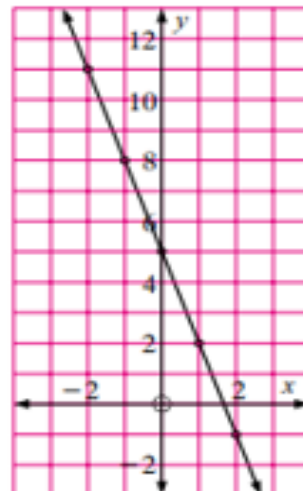
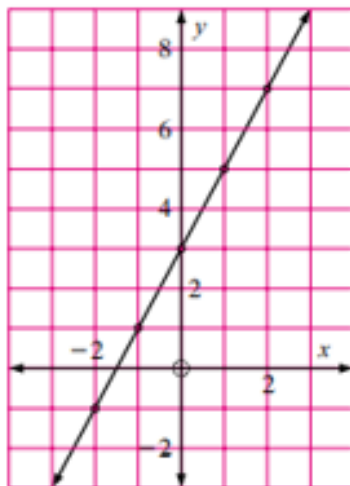
Draw the graph modelled by the equation a) $y = 2x + 3$ b) $y = -3x + 5$

Use a table of values

x	-2	-1	0	1	2
y	-1	1	3	5	7

x	-2	-1	0	1	2
y	11	8	5	2	-1

Plot the points and draw a line through them



The equation of a straight line can (usually) be written as $y = mx + b$ where m is the gradient and b is the y -intercept. Therefore, any equation involving x and y which can be rearranged into that form describes a straight line.

EXERCISE 2

1. Write the equation of the line for each of the graphs from page 4. $y = mx + b$

a)

b)

c)

d)

2. Which of the following describe or determine a straight line? You need to justify your answer in each case.

In some of these drawings a graph may be the best solution. You could use graphing software such as *Desmos*.

The equation of a straight line can (usually) be written as $y = mx + b$ where m is the gradient and b is the y-intercept. Therefore, any equation involving x and y which can be rearranged into that form describes a straight line.

Equation	Is it a straight line?	Explanation/give sketch
a) $4x - 2y = 6$		
b) $y = 2$		
c) $y = \frac{3}{2}x$		

d) $y = 3 - 2x$		
e) $y = x^2 + 2$		
f) $x = -2$		
g) the point (0, -1) and the constant gradient 3		
h) $x = 7y + 5$		
i) $y - 8 = 3(x - 3)$		

j) $y^2 = x^2$		
k) $\frac{1}{3}y - x + \frac{1}{3} = 0$		
l) $xy = 1$		
m) $y^2 - 4xy + 4x^2 = 0$		
n) the points $(\frac{1}{2}, 2)$, $(1, 1)$ and $(\frac{3}{2}, 0)$		

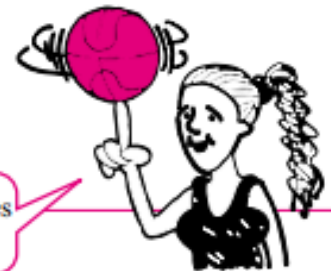
DIRECT VARIATION

The gradient and intercept of linear graphs have meaning in practical situations. A relationship that gives a straight line when graphed is a direct relationship between the two variables.

Example

At the athletics carnival Celine runs the 100m in 16 seconds. It is assumed that she runs at a constant rate.

- Sketch a straight-line graph to model Celine's run.
- Find the gradient.
- Explain the meaning of the gradient.

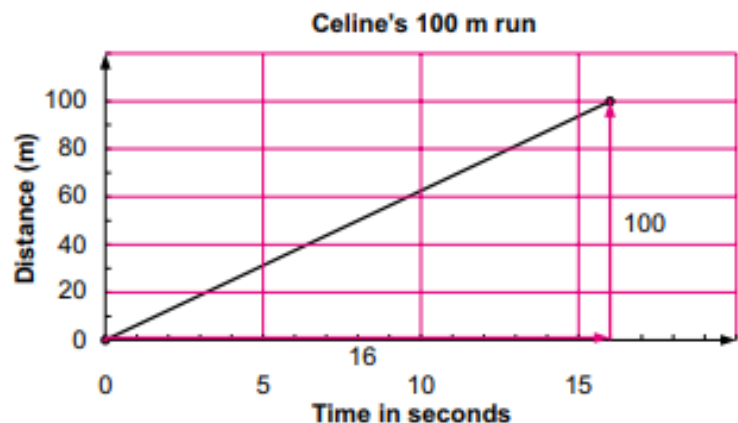


- Use two points to draw the graph. They are (16, 100) and (0, 0).

This is the graph.

- Draw in a triangle.

$$\begin{aligned}\text{gradient} &= \frac{\text{rise}}{\text{run}} \\ &= \frac{100}{16} \\ &= 6.25\end{aligned}$$



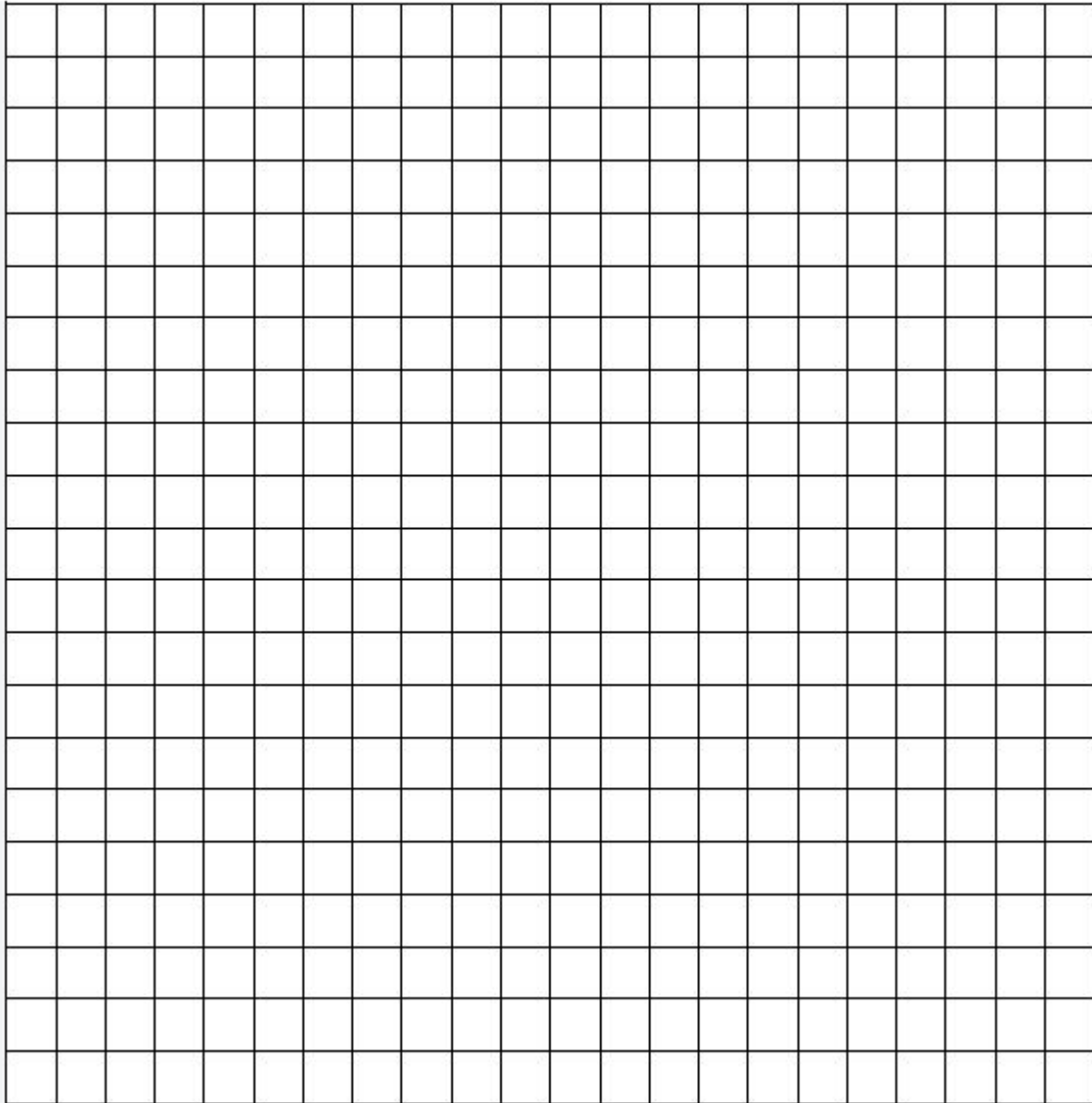
The units for the gradient in this case are m/s.

- The gradient is Celine's speed in metres per second. She runs at an average speed of 6.25 m/s.

EXERCISE 3

1. Christopher runs 100 metres in 20 seconds.

a) Draw a graphical model of Christopher's run.

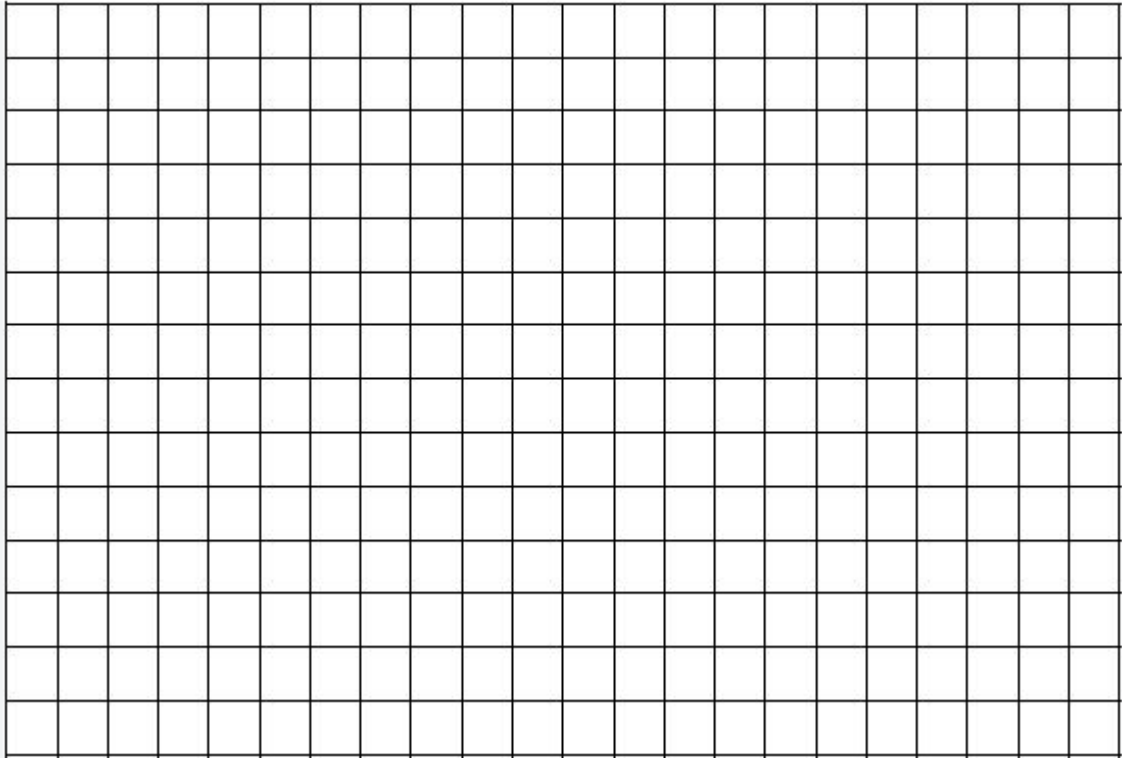


b) How long would it take to run 30 metres?

c) Find the gradient and hence Christopher's speed in metres per second.

2. Craig drives 200 km in 4 hours.

a) Draw a graphical model of Craig's drive.



b) How far does Craig drive in $1\frac{1}{2}$ hours?

c) Find the gradient.

d) What are the units of the gradient in this case?

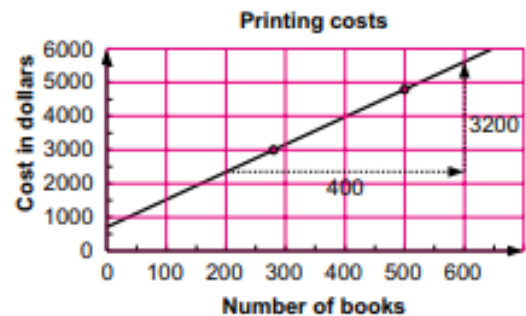
e) Expand the graph to 1000 km. This is called extrapolation. Is the graph still accurate as a model for Craig's drive? Explain.

LINEAR MODELLING

Example

Use this graph of printing costs to find

- a the cost to print 500 books
- b the number of books that can be printed for \$3000.
- c Find the gradient. What is its meaning?
- d Find the y intercept. What is its meaning?



Solutions

- a 500 books would cost about \$4800.
- b \$3000 would buy about 280 books
- c Draw in a triangle to find the gradient.

$$\text{gradient} = \frac{3200}{400} = 8$$

The gradient is 8. The units are dollars per book.
This means that the gradient is the cost per book.

- d The y -intercept is approx 800. This means the cost to print 0 books is \$800. This is the set up cost.

EXERCISE 4

1. Ruth exhibits her art work at Gino's Art Gallery. On all works sold through the gallery, Gino charges a commission calculated using a linear formula. In the past Gino received \$75 commission on a \$1000 painting and \$125 on a \$3000 sculpture. On this basis, Ruth expects to be charged \$200 on her most recent sale, a \$4000 sculpture. What commission does Gino actually receive on this sale?

2. Here is a graph modelling taxi charges.



a) Find the cost of travelling 25 km.

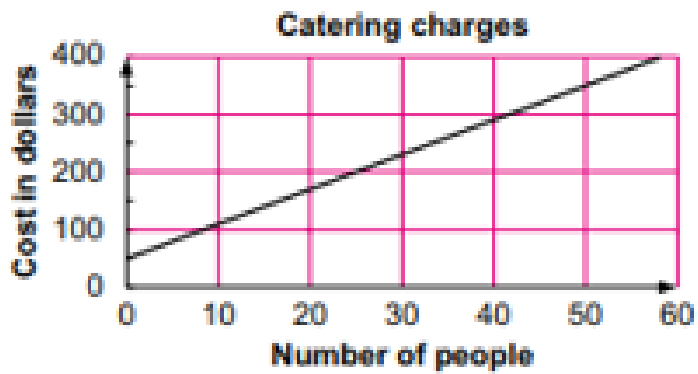
b) How far can you travel for \$15?

c) Find the gradient (m). What is the meaning of the gradient?

d) Find the intercept on the vertical axis (b). What is its meaning?

e) Write an equation for the model in the form of $C = b + md$
where C is the cost in dollars (\$) and d is the distance in kilometres (km).

3. Here is a graph modelling catering charges.



- a) How much would it cost for 35 people?
- b) How many people could eat for \$300?
- c) Find the gradient. What is the meaning of the gradient?
- d) Find the intercept on the vertical axis. What is its meaning?
- e) Write an equation for the model.

WEEK 12 INVESTIGATION

Here are the equations of 12 straight lines.

$y = 4x + 4$	$4y = x + 3$	$y = 8x - 3$	$y + 4x + 6 = 0$
$3y = 2x - 8$	$y + 6x = 11$	$y + 8x = 6$	$2y + 8 = 3x$
$2y + x = 4$	$2y = 8x + 3$	$y = 6x - 4$	$y + x + 8 = 0$

1. Rewrite each equation in the form $y = mx + b$

2. These 12 straight lines can be divided up into six pairs, each pair matching one of the following descriptions. Sort them into the correct pairs and complete the final description.

- These lines are parallel.

- These lines are perpendicular.

- These lines have the same y-intercept.

- These lines have the same x-intercept.


- These lines both go through the point (1,5).

- These lines ...

MARKING RUBRIC

CRITERIA	EXPECTATIONS	MARKS
Practical	Student completes practical work of the brief to an acceptable standard set by the teacher.	
<p><i>Completion of practical work is a prerequisite to submitting your investigation.</i></p> <p>Your teacher reserves the right not to accept submission of your weekly/fortnightly investigation if the supporting work is incomplete.</p>		
Investigation	Student response is correct.	/2
	Student response shows clear and logical working out.	/2
	Student response includes appropriate units and correct rounding, where relevant.	/2
	Student response states a conclusion which answers the question. <ul style="list-style-type: none"> • <i>Check:</i> Does your answer make sense in the given context? 	/2
Submission Guidelines		
Timeliness	Student submits the exercises and investigation by the set deadline.	/2
TOTAL		/10

Student Reflection:



On a scale of 1 - 4, I would rate my understanding of this topic:

1	2	3	4
Even with help I don't understand.	I'm starting to understand but need more help.	I'm understanding and able to complete most of the problems on my own.	I fully understand. I could help and teach others.

Written reflection (optional): What was interesting? What did you find easy? What do you need to work on? Any other comments?

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