Week 11
Term 4
2024

HAWKER COLLEGE
Engage | Inspire | Achieve

MA2
Linear equations and their graphs

Goals

This week we are going to:

- Identify and solve linear questions
- Develop a linear formula from a word description

Say What? You need a challenge? OK

There were 124 students in the canteen. When $\frac{5}{8}$ of the boys and $\frac{4}{7}$ of the girls left the canteen, the total number of boys and girls who remained became 48. How many boys were there in the canteen at first?

Theoretical Components

Resources:
PDF file: Week 11 Notes and Exercises

Knowledge Checklist

- Solve one and two step equations
- Solve equations with pronumeral on either side of the equations
- Solve literal equations

Order

1. Read through the notes and examples
2. Work through the exercises
3. Complete the investigation at the end of the booklet.
4. Complete the reflection at the end of the booklet
5. Come and see your teacher and make sure you are up to date.

Practical Components

Work through the exercises and show the completed tasks to your teacher.

Be sure to ask for help as you need for the successful completion of all tasks.

Remember to regularly check Google Classroom for messages.

Investigation

Complete the task at the end of the booklet and submit your work for checking. 😊

QFO

MATHEMATICAL APPLICATIONS 2

WEEK 11 NOTES & EXERCISES

LINEAR EQUATIONS AND ALGEBRAIC MODELLING

A criminologist studying the effects of increasing police patrols on the streets of a big city noticed the following pattern.

No. of police	50	150	250	300
No. of crimes	3100	2800	2500	2350

Is it possible for the criminologist to find a general mathematical rule to fit this pattern? Algebra is the branch of mathematics that uses symbols called pronumerals to represent number patterns and relationships.

Algebraic modelling is the study of patterns and relationships occurring in nature, industry and society and the formulation of a mathematical rule or 'model' to describe such relationships.

An algebraic expression is a general statement involving pronumerals (also called variables). Pronumerals are letters of the alphabet that stand for numbers. An algebraic expression is made up of terms.

For example, $3x^2 - x + 10$ has 3 terms: $3x^2$, $-x$ and 10.

SOLVING EQUATIONS

An equation contains an algebraic expression and an equals (=) sign. For example, $3x - 4$ is an expression, but $3x - 4 = -13$ is an equation. To solve an equation, we find the value of the pronumeral that makes the equation true. The process of solving an equation requires the use of inverse (opposite) operations.

To solve an equation you can

- add the same number to both sides of the equation
- subtract the same number from both sides of the equation
- multiply each side of the equation by the same number.
- divide each side of the equation by the same number.

ONE-STEP EQUATIONS

The simplest equations are called one-step equations as they take one step to solve.

Example

Solve these equations **a** $x + 9 = 17$ **b** $p - 12 = 30$

Solution

a
$$x + 9 = 17$$
$$x + 9 - 9 = 17 - 9$$
$$x = 8$$

Check:

If $x = 8$, $x + 9 = 8 + 9 = 17$ ✓

b
$$p - 12 = 30$$
$$p - 12 + 12 = 30 + 12$$
$$p = 42$$

Check:

If $p = 42$, $p - 12 = 42 - 12 = 30$ ✓

Example

Solve these equations **a** $3x = 18$ **b** $7x = 42$

Solution

a
$$3x = 18$$
$$\frac{3x}{3} = \frac{18}{3}$$
$$x = 6$$

Check: If $x = 6$, $3x = 3 \times 6 = 18$ ✓

b
$$7x = 42$$
$$\frac{7x}{7} = \frac{42}{7}$$
$$x = 6$$

Check: If $x = 6$, $7x = 7 \times 6 = 42$ ✓

Example

Solve these equations **a** $\frac{x}{2} = 5$ **b** $\frac{p}{8} = 6$

Solution

a
$$\frac{x}{2} = 5$$
$$2 \times \frac{x}{2} = 5 \times 2$$
$$x = 10$$

Check: If $x = 10$, $\frac{x}{2} = \frac{10}{2} = 5$ ✓

b
$$\frac{p}{8} = 6$$
$$8 \times \frac{p}{8} = 6 \times 8$$
$$p = 48$$

Check: If $p = 48$, $\frac{p}{8} = \frac{48}{8} = 6$ ✓

EXERCISE 1

1. Solve these equations by adding or subtracting.

a) $p + 8 = 16$ b) $d - 3 = 7$ c) $a + 16 = 31$

d) $f - 28 = 36$ e) $x + 21 = 48$ f) $r + 48 = 49$

2. Solve these equations by dividing.

a) $5m = 25$ b) $6p = 30$ c) $18b = 90$

d) $2x = -16$ e) $5q = -25$ f) $-9c = -72$

3. Solve these equations by multiplying.

a) $\frac{x}{7} = 2$ b) $\frac{d}{10} = 7$ c) $\frac{y}{3} = 12$

d) $\frac{m}{8} = 8$ e) $\frac{p}{3} = -4$ f) $\frac{z}{5} = -6$

TWO-STEP EQUATIONS

The techniques needed to solve two step equations are the same as for one step equations except two techniques need to be applied. Always add or subtract first to get the pronumeral on one side of the equation and the numbers on the other side. Then multiply or divide to find the answer.

Example

Solve these equations **a** $5x + 7 = 37$ **b** $8x - 9 = 79$

Solution

a

$$5x + 7 = 37$$
$$5x + 7 - 7 = 37 - 7 \quad \{\text{subtract } 7 \text{ from both sides}\}$$
$$5x = 30$$
$$\frac{5x}{5} = \frac{30}{5} \quad \{\text{divide both sides by } 5\}$$
$$x = 6$$

$$5x + 7 = 37$$
$$(5 \times 6) + 7 = 37$$
$$30 + 7 = 37$$
$$37 = 37$$
$\therefore$ correct ✓

b

$$8x - 9 = 79$$
$$8x - 9 + 9 = 79 + 9 \quad \{\text{add } 9 \text{ to both sides}\}$$
$$8x = 88$$
$$\frac{8x}{8} = \frac{88}{8} \quad \{\text{divide both sides by } 8\}$$
$$x = 11$$

$$8x - 9 = 79$$
$$(8 \times 11) - 9 = 79$$
$$88 - 9 = 79$$
$$79 = 79$$
$\therefore$ correct ✓

EXERCISE 2

1. Solve the following two-step equations.

a) $3x - 9 = 3$ b) $4p + 8 = 24$ c) $6x + 7 = 61$

d) $2x - 32 = 16$ e) $10c - 30 = 100$ f) $7t - 8 = 27$

g) $4q - 21 = 115$ h) $11s + 16 = 115$ i) $7p + 33 = 117$

2. Solve for x

a) $6 - x = -5$ b) $3 - 4x = 15$ c) $11 = 3 - 2x$

d) $-1 = 2x + 6$ e) $-15 = 3 - 6x$ f) $3x + 6 = 0$

3. Solve for x

a) $\frac{x}{2} + 3 = 8$ b) $\frac{x}{3} - 1 = 4$ c) $\frac{x}{10} - 6 = -1$

SOLVING EQUATIONS WITH PRONUMERALS ON BOTH SIDES

Example

a $5x + 2 = 3x - 5$ **b** $15 - 2x = 11 + x$

Solution

a

$$5x + 2 = 3x - 5$$
$$\therefore \quad 5x + 2 - 3x = 3x - 5 - 3x \qquad \{\text{subtracting } 3x \text{ from both sides}\}$$
$$\therefore \quad 2x + 2 = -5$$
$$\therefore \quad 2x = -7 \qquad \{\text{subtracting } 2 \text{ from both sides}\}$$
$$\therefore \quad x = -\tfrac{7}{2} \qquad \{\text{dividing both sides by } 2\}$$
$$\therefore \quad x = -3\tfrac{1}{2}$$

b

$$15 - 2x = 11 + x$$
$$\therefore \quad 15 - 2x + 2x = 11 + x + 2x \qquad \{\text{adding } 2x \text{ to both sides}\}$$
$$\therefore \quad 15 = 11 + 3x$$
$$\therefore \quad 15 - 11 = 11 + 3x - 11 \qquad \{\text{subtracting } 11 \text{ from both sides}\}$$
$$\therefore \quad 4 = 3x$$
$$\therefore \quad x = \tfrac{4}{3} = 1\tfrac{1}{3} \qquad \{\text{dividing both sides by } 3\}$$

EXERCISE 3

1. Solve the following equations with integer solutions.

a) $6x + 2 = 2x + 18$ b) $3x + 7 = 12 - 2x$

c) $5x - 9 = 11 + 7x$ d) $9 - 2x = 3 - 5x$

2. Solve the following equations with fraction solutions.

a) $6x + 7 = 4x - 2$

b) $2x - 3 = 5x + 7$

c) $11a - 7 = 5a + 15$

d) $9x - 4 = 3 + 5x$

SOLVING LITERAL EQUATIONS

A literal equation is an equation whose solution will be expressed in terms of another variable rather than a number.

For example:

- $2x + 3 = 13$ is a linear equation in one unknown, whose solution for x is the number 5.
- $x + 2y = 6$ is a literal equation in two unknowns, whose solution for x is $6 – 2y$

Example

If $2a – 4b = 8$, find an expression for a

Steps

- Add 4b to both sides of the equation

  $2a – 4b + 4b = 8 + 4b$ gives

  $2a = 8 + 4b$
- Divide both sides by 2 which gives

  $a = 4 + 2b$

If $3x + 2y = 4$ find expressions for both x and y

For y

- Subtract $3x$ from both sides of the equation

  $3x + 2y – 3x = 4 - 3x$ which gives

  $2y = 4 – 3x$
- Divide both sides by 2 which gives

  $y = 2 - \frac{3}{2}x$

For x

- Subtract $2y$ from both sides of the equation

  $3x + 2y – 2y = 4 – 2y$ which gives

  $3x = 4 – 2y$
- Divide both sides by 3 which gives

  $x = \frac{4}{3} - \frac{2}{3}y$

EXERCISE 4

1. Find an expression for x

a) $x + 7y = 22$

b) $x - 4y = 11$

c) $5y = x - 12$

d) $4y = 2x + 6$

e) $3x + 5 = 2y$

f) $5y + 2x = 10$

2. Find an expression for y

a) $7x + 5y = 40$

b) $mx + y = 9$

c) $ny + mx = d$

d) $sx - ty + a = 0$

WEEK 11 INVESTIGATION

Solve the following equations. **Explain** each step of working out involved.

1. $5(x+1)-2(x-2)=7$

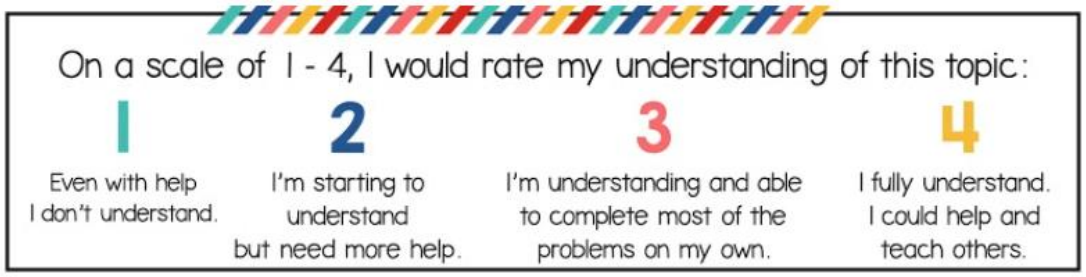
2. $\frac{4x+1}{3}=\frac{1}{2}$

3. $\frac{x}{5}-3=\frac{3x}{8}$

MARKING RUBRIC

CRITERIA	EXPECTATIONS	MARKS
Practical	Student completes practical work of the brief to an acceptable standard set by the teacher.	
*Completion of practical work is a **prerequisite** to submitting your investigation.* Your teacher reserves the right not to accept submission of your weekly/fortnightly investigation if the supporting work is incomplete.		
Investigation	Student response is correct.	**/2**
	Student response shows clear and logical working out.	**/2**
	Student response includes appropriate units and correct rounding, where relevant.	**/2**
	Student response states a conclusion which answers the question. • *Check:* Does your answer make sense in the given context?	**/2**
	Submission Guidelines	
Timeliness	Student submits the exercises and investigation by the set deadline.	**/2**
	TOTAL	**/10**

Student Reflection:



Written reflection (optional): What was interesting? What did you find easy? What do you need to work on? Any other comments?