

Learning Brief

Mathematical Applications
Year 11 Semester 1 - MAP1T

Goals

$$A = P \left(1 + \frac{r}{100} \right)^n$$

Unit goals

- Understand the concepts and techniques introduced in consumer arithmetic, algebra and matrices, and shape and measurement.
- Apply reasoning skills and solve practical problems.

This week the work is on

- Taxation
- Compound Interest

Theoretical Components

Resources:

For this week the theory work is in the *PDF file: Week 7 Notes & Exercises*

Knowledge Checklist

- Terminology associated with taxation; gross income, net income, taxable income, tax rates, deductions, Medicare levy
- Calculating income
- Calculating tax
- Compound interest – adding interest to interest
- The effect of varying the compounding period

Order

1. Look at the Investigation.
2. Complete the questions in the *Booklet*
3. Complete the Investigation.
4. Show your completed booklet to Steve and submit the Investigation for marking.

Practical Components

There are questions to be answered in the booklet *Week 7 Notes & Exercises*

Investigation

See next page. This week's investigation due Week 9 Friday.

Begin preparing for the test in **Week 8**.
Complete a summary sheet – double-side A4 handwritten-notes – to be handed up with the test.

Quiz

There is a revision quiz on mathspace



WHAT DO YOU GET TAXED ON?

We've already learnt about people earning wages and salaries. However, most people don't get to keep all of the money they earn. They must pay tax, which is a percentage of their income paid to the government based on their taxable income. Taxes are used to fund public projects like schools, hospitals and roads. Depending on where you work, each year you or your employer will have to lodge a tax return, so knowing how to calculate your taxable income is important.



You'll need to know the difference between your GROSS income and your NET income. Your gross income is all the money you earn. Your NET income is the amount you actually get paid AFTER tax is taken out. Think of this fishing analogy to help you remember: if you lifted your net out of the gross ocean, all the tax and deductions would drain away and all that would be left in the net is the big, juicy amount of money you get paid.

Terminology

Gross income: the amount of income you receive before tax and deductions.

Net income: the amount of income you receive after tax and deductions.

Taxable income: the income you are required to pay tax on.

Tax rates: The different tax amounts people are required to pay. This varies depending on your taxable income.

Deductions: expenses that are directly related to earning your income which you are allowed to subtract from your gross income. These include uniform expenses, vehicle and travel expenses, self-education expenses, as well as charitable donations.

Medicare levy: a tax of 1.5% of one's taxable income paid by most Australian taxpayers that is used to provide Australian residents with access to public health care.

Calculating Your Income

Each pay period (whether it's weekly, fortnightly, or monthly), your employer will take out a percentage of your income as tax, which is based on your gross income. However, you may have deductions that you can subtract from your gross income, which in turn will reduce your taxable income. If this is the case, and you have paid more tax than you needed to, you may be eligible for a tax return.

However, prior to lodging tax returns, it's also helpful to be able to calculate your net income, so you know how much money you will ACTUALLY receive each pay period. To do this, you need to subtract the amount of tax you are required to pay from your gross income. Then you can divide this net amount to find your weekly net wage, monthly net wage etc.

Examples

Q1. Calculate the net **weekly** wage of a carpenter who must pay 16% tax on an annual salary of \$38080. Give your answer correct to two decimal places.

Think: What would his annual net salary be? Remember that there are 52 weeks in a year.

Do: His annual net salary would be 100–16, or 84% of his gross income:

$$0.84 \times 38080 = \$31987$$

Then, to calculate his net weekly income:

$$\$31987.20 \div 52 = \$615.14$$

Q2. Sharon is a project manager with a taxable income of \$99192. Throughout the year, her employer has deducted \$512 per week in tax instalments.

The taxable income rates for individuals is provided in the table.

Income Bracket	Tax Payable
\$1 - \$18 200	Nil
\$18 201 - \$37 000	19 cents for each \$1 over \$18 200
\$37 001 - \$87 000	\$3 572 + 32.5 cents for each \$1 over \$37 000
\$87 001 - \$180 000	\$19 822 + 37 cents for each \$1 over \$87 000
\$180 001 and over	\$54 232 + 45 cents for each \$1 over \$180 000

a) Calculate the tax payable to the nearest cent.

Sharon's taxable income lies in the range of \$87001 - \$180000.

This means she needs to pay \$19822 plus an additional amount, found as follows.

\$99192 exceeds \$87000 by \$12192. This amount is taxed at 37 cents in the dollar.

$$\$12192 \times 0.37 = \$4511.04$$

Thus the total tax payable is $\$19822 + \$4511.04 = \$24333.04$

b) Calculate the Medicare Levy, which is 2% of taxable income.

The taxable income is \$99192. We can find 2% of this amount.

$$\frac{2}{100} \times 99192 = \$1983.84$$

Medicare Levy is \$1983.84

c) Is Sharon entitled to a refund? Sharon needs to pay the total of her tax and the Medicare Levy but she has already paid \$512 each week.

Thus she has paid $\$512 \times 52 = \26624 already.

Her tax and Medicare Levy come to $\$24333.04 + \$1983.84 = \$26316.88$

Now $\$26624 - \$26316.88 = \$307.12$

Thus she is entitled to a refund of \$307.12

Exercise Set 1

Q1. For a gross income of \$52000, calculate the net income when tax is \$11100 and superannuation is \$2500.

Q2. If tax is stated as being 15.9 cents in the dollar, what percentage tax is this?

Q3. Medicare Levy is charged at 1.5% of taxable income. Calculate the Medicare levy on a taxable income of \$60000.

Q4. Calculate the net weekly wage of a carpenter who must pay 16% tax on an annual salary of \$38080. Give your answer correct to two decimal places.

Q5. Calculate the percentage of gross income going to taxes, correct to 2 decimal places, when:
(Note: Don't forget to include the percentage symbol where required)

a) gross income is \$56000 and income tax is \$15000

b) gross income is \$49000, income tax is \$13000.

Q6. A self-employed tradesman must put aside 9% of each fortnightly wage towards superannuation and pay 10% GST to the government. If he earns \$1067 in a fortnight, what will his net income be after these deductions? Give your answer correct to two decimal places. Note: Do not use repeated percentage change.

Q7. Amelia's employer deducts a percentage of her income in tax. She sees the fortnightly amount deposited into her account is \$1964 and knows that she earns an annual gross income of \$63830.00 .

a) Use the amount deposited in her account to calculate her net annual income.

b) Hence, calculate the rate at which she is being taxed. Give your answer to the nearest whole percentage.

For the following questions use the tax table provided below:

Income thresholds	Rate	Tax payable on this income
\$0 – \$18,200	0%	Nil
\$18,201 – \$37,000	19%	19c for each \$1 over \$18,200
\$37,001 – \$90,000	32.5%	\$3,572 plus 32.5% of amounts over \$37,000
\$90,001 – \$180,000	37%	\$20,797 plus 37% of amounts over \$90,000
\$180,000 and over	45%	\$54,096 plus 45% of amounts over \$180,000

Source: ATO

Q8. Calculate the tax payable, to the nearest cent, on a taxable income of:

a) \$5347

b) \$22968

c) \$78671

Q9. Dave is a teacher who earns an annual salary of \$78600. He earns an additional \$4660 per year from private tutoring. He is allowed various tax deductions (for resource books etc.) amounting to \$852 and he has paid tax instalments of \$8430 during the year. Find:

a) His total gross income.

b) His taxable income.

c) Tax payable using the table.

d) the tax still owing.

Q10. Sally earns an annual income of \$50000. With tax deductions of \$1300 and having already paid \$4900 in tax instalments throughout the year, calculate her:

a) taxable income

b) tax payable to the nearest cent.

c) Medicare levy (1.5% of taxable income).

d) Amount due after levies and tax have been already paid.

Compound interest

In reality, calculating interest is not so ‘simple’ and straightforward. Simple interest is used only when the interest earned is collected by the investor and not added to the investment, such as in a term deposit account. With most accounts, however, the balance plus the interest becomes the new balance on which the interest is calculated next time. In other words, the interest will increase because you also earn ‘interest on your interest’. This is called **compound interest**. Compound means ‘to combine’.

The effect of compounding (which oil billionaire J. P. Getty called the ‘eighth wonder in the world’ and theoretical physicist Albert Einstein described as ‘the driving force of the Universe’) is a secret of financial wealth creation.

As mentioned previously, when we are dealing with simple interest, the interest is the same for each time period. The difference compounding makes can be seen in the following illustration.

Let us consider an amount of \$1000, to be invested for a period of 3 years at an interest rate of 10% p.a. We will compare the interest earned using (i) simple interest, and (ii) compound interest.

Simple interest	Compound interest
Initial principal, $P = \$1000$	Initial principal, $P = \$1000$
Rate of interest, $R = 10\%$	Rate of interest, $R = 10\%$
Interest for Year 1 10% of \$1000 $I_1 = \$100$	Interest for Year 1 10% of \$1000 $I_1 = \$100$
Principal at the beginning of Year 2 $P_2 = \$1000$	Principal at the beginning of Year 2 $P_2 = \$1000 + \100 = \$1100
Interest for Year 2 10% of \$1000 $I_2 = \$100$	Interest for Year 2 10% of \$1100 $I_2 = \$110$
Principal at the beginning of Year 3 $P_3 = \$1000$	Principal at the beginning of Year 3 $P_3 = \$1100 + \110 = \$1210
Interest for Year 3 10% of \$1000 $I_3 = \$100$	Interest for Year 3 10% of \$1210 $I_3 = \$121$

Note that for simple interest, the interest earned each year is \$100, while in compound interest the amount of interest earned increases each year. So the amount of interest earned with simple interest is $\$100 + \$100 + \$100 = \300 while the amount of interest earned with compound interest is $\$100 + \$110 + \$121 = \331 .

Formula for compound interest

$$A = P \left(1 + \frac{r}{100} \right)^n$$

where:

A = the amount at the end of n compounding periods, \$

P = principal, \$

r = rate of interest per period

n = number of compounding periods

Note that in the formula for compound interest, r is the rate of interest *per period*, not *per annum* and n is the number of compounding *periods*, not *years*. It reflects the fact that compounding occurs not only on an annual basis but can be more frequent: that is, semi-annually (half-yearly), quarterly (every three months), monthly, weekly or daily.

Example

Tamara has \$15000 to invest for 3 years. She considers the following options:

a) a term deposit at 5.25% pa compounded annually.

Here we use the formula $A = P \left(1 + \frac{r}{100}\right)^n$ where $P = \$15000$, $r = 5.25$ and $n = 5$

Evaluating for A gives \$17488.70, thus the interest earned is $\$17488.70 - \$15000 = \$2488.70$

b) shares, paying a dividend rate of 5.08% pa, compounded quarterly.

Again, we use the compound interest formula, but now $r = \frac{5.08}{4} = 1.27\%$ per quarter and $n = 4 \times 3$

= 12 quarters in 3 years. Substituting in $A = P \left(1 + \frac{r}{100}\right)^n$ gives $A = P \left(1 + \frac{1.27}{100}\right)^{12} = \17452.63 .

Thus the interest earned is $\$17452.63 - \$15000 = \$2452.63$

c) a building society, paying a return of 5.4% pa compounded monthly.

Again, we use the compound interest formula, but now $r = \frac{5.4}{12} = 0.45\%$ per month and $n = 12 \times 3$

= 36 months in 3 years. Substituting in $A = P \left(1 + \frac{r}{100}\right)^n$ gives $A = P \left(1 + \frac{0.45}{100}\right)^{36} = \17631.50

Thus the interest earned is $\$17631.50 - \$15000 = \$2631.50$

d) a business venture with guaranteed return of 7.3% compounded daily.

Again, we use the compound interest formula, but now $r = \frac{7.3}{365} = 0.02\%$ per day and $n = 365 \times 3$

= 1095 days in 3 years. Substituting in $A = P \left(1 + \frac{r}{100}\right)^n$ gives $A = P \left(1 + \frac{0.02}{100}\right)^{1095} = \18672.06 .

Thus, the interest earned is $\$18672.08 - \$15000 = \$3672.06$.

The more often the interest is compounded the greater the return (assuming the interest rate is the same).

Exercise Set 2

Q1. Warren wishes to invest \$10 000. The following investment alternatives are suggested to him. All investments are for 7 years. Remember the simple interest formula gives you the interest earned while the compound interest formula gives the total amount.

a) simple interest at 9% p.a.

b) compound interest at 8% p.a.

c) compound interest at 7 % p.a. adjusted quarterly. Remember to adjust the interest rate and time period.

d) compound interest at 7% p.a. adjusted daily. Remember to adjust the interest rate and time period.

e) Which investment alternative will produce the greatest return on his money?

Q2. Rosemary has \$25 000 to invest for 5 years. She considers the following options:

a) a term deposit at 6.75% p.a. compounded annually

b) shares, paying a dividend rate of 5.15% p.a. compounded quarterly

c) a building society, paying a return of 5.3% p.a. compounded monthly (change interest rate and time period)

d) a business venture with guaranteed return of 6.4% p.a. compounded daily.
(Assume there is only one leap year in the given 5 year period.)

e) All the investments are equally secure. Advise Rosemary which option to take and why?

Q3. Over the last 3 years a comprehensive hospital cover from 'TakeCare' private medical insurance rose at an average of 9.5% and currently costs \$1980 per year. With this rate of increase continued, what would be the insurance premium after another 3 years? (*Hint*: The increase in premium compounds each year.)

Appreciation

Items which represent scarce or valuable resources such as land, collectables, paintings and antiques normally increase or appreciate in value over time. They become more valuable as time passes because they become more rare or scarce. This is called **appreciation**. Some people like to invest their money by buying and selling such items.

Q4. A painting that Elizabeth bought for \$240 from an art exhibition appreciates (increases in value) by 14% p.a. If this rate of appreciation continued, what would be the value of the painting after 25 years?

Compound Interest Calculator

The link below is for a compound interest calculator. This calculator allows you to calculate A, P, r or t.

<https://www.calculatorsoup.com/calculators/financial/compound-interest-calculator.php>

Use the calculator to solve the following. (you can find this link at www.hawkermaths.com)

Q5. An original Elio Sanciolo painting is currently valued at \$4500 and is known to appreciate an average 8% per year. After how many years would the painting have a value of at least \$25 000?

In this question $P = \$4500$, $r = 8\%$, $A = \$25000$ and we want to find t . so set 'Calculate' to Time (t).

Q6. According to the last census, Whitehorse Marsh has a population of 23 600. The population increases at will it take for the population of Whitehorse Marsh to reach the 40 000 mark?

Q7. What is the amount, rounded to the nearest \$100, to be invested for 6 years and compounded semi-annually at 8% p.a.? The value of the investment at the end of the term is \$15 000. Set the 'Compound (n)' to Semi-annually.

Q8. Gaetano bought a house for \$175 000 in an area where house prices appreciate an average 3% per year. He decided to hold on to his house until its value is at least \$250 000. How many years should he wait until he sells his current house?

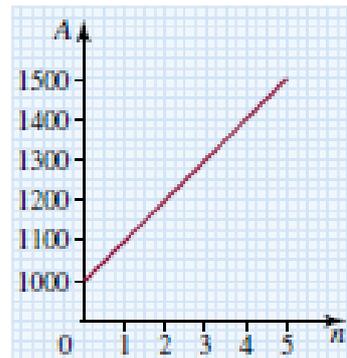
Graphing Compound Interest

We can use the data from the example at the beginning of this booklet to compare the graphs of simple and compound interest.

If we were to place the set of data obtained in two separate tables and represent each set graphically — as the total amount of the investment, A , versus the year of the investment, n — we would find that:

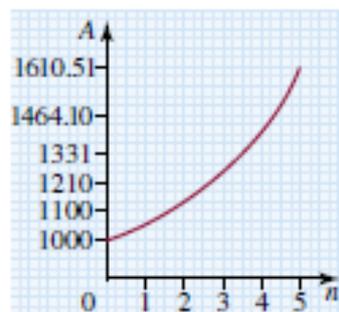
1. The simple interest investment is represented by a straight line as shown below.

n	A
0	1000
1	1100
2	1200
3	1300
4	1400
5	1500



2. The compound interest investment is represented by an exponential graph as shown below.

n	A
0	1000
1	1100
2	1210
3	1331
4	1464.10
5	1610.51



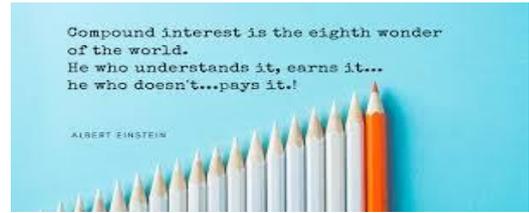
The graph of any simple interest scenario is always a straight line (linear), while the graph of compound interest is always represented by an exponential curve.

The total amount, A , in compound interest always grows at a much faster rate than in simple interest.

Week 9 Investigation

Finance Investigation

Compound Interest Comparison



Investigation - As a real-life example, let us compare the price of a car at different dealership with different financing plans.

Below is a list of prices and financing terms for the purchase of a new Toyota Camry.

Dealer	Price	Interest Rate	Payment Frequency	Term of Loan
Boch Toyota	\$24,000	5%	Monthly	5 Years
Herb Chambers	\$23,000	5.5%	Monthly	5 Years
Ira Toyota	\$25,000	5%	Monthly	3 Years
Clair Toyota	\$23,000	5%	Semi Annual	5 Years



Question 1. Using what you know about compounding interest, which is the best buy? Show all necessary calculations.

Question 2. You finally decide to buy this car from Ira Toyota as the term of the loan is shorter, but base on your income, the dealership can only finance half of the car price for you, which means you will have to pay half of the car price first. Your hourly net income is \$28.50, and you work 35 hours per week. If you are willing to put away 20% of you weekly income to a saving account for the car. How long it takes you to save enough money to start your finance?

SCORING NOTES

Formatting **if necessary** for all typed/written assessments should be as follows:

Google Doc	11-12 Pt	1.15-1.5 Line Spacing	1 Space between paragraphs	Spelling and Grammar "Soft Limit"	In-Text Citations with footnotes	Title Page/Slide: <ul style="list-style-type: none"> Name Date Class Aim Assessment title
Slides	10-12 pt font text 14-24 pt font titles	1.0 1.15 Line Spacing	Bullet Points Preferred	Word Count per slide >100-110 "Soft Limit"	Approved Templates and Themes	

If there is a **Timeliness** Criterion - Student will receive full marks for a submission on time, half marks for a submission within one week of the deadline and no marks for a submission after this without prior arrangement with the teacher. Deadline exceptions can be discussed with your teacher for special considerations.

"Soft Limits" are not rigidly defined limits and will be assessed on a case-by-case basis. Ask for clarification for specific tasks.

Possible Scoring Groups are out of 2 or 4 Points.

2-Point Criteria

	0 Points	1 Point	2 Points
2 Point Criteria	Not present or able to be assessed as the required criteria	Item is presented and does not meet expectations for quality, rigour, or detail	Item is presented and does meet expectations for quality, rigour, or detail

4-Point Criteria

	0 Points	1 Point	2 Points	4 Points
4 Point Criteria	Not present or able to be assessed as the required criteria	Item is presented, but does not meet expectations for quality, rigour, or detail	Item is presented and does meet expectations for quality, rigour, or detail	Item is presented and does meet expectations for quality, rigour, or detail. Response presented is consistently accurate and demonstrates very high level of proficiency.

Multiplier

Criteria will be combined with a **Multiplier**. While each criterion will be scored on the 0-1-2-4 scale, the multiplier will attach relevant worth to each criterion. Be aware of these multipliers and dedicate appropriate time to ensure you achieve your best result.

Achievement Standards:

Achievement standards can be found on the BSSS course description.

MARKING RUBRIC

CRITERIA	EXPECTATIONS	POSS	MULT	GIVEN	TOTAL
Practical	Student completes practical work, including exercises and Mathspace task, of the brief to an acceptable standard set by the teacher.	2	3		/6
Investigation Task	Student completes the investigation task of the week to an acceptable standard set by the teacher.	2	2		/4
Communication and Reasoning	Student responses are accurate and appropriate in presentation of mathematical ideas in different contexts, with clear and logical working out shown.	4	-		/4
Knowledge and Application	Student submitted work selects and applies appropriate mathematical modelling and problem solving techniques to solve practical problems and demonstrates proficiency in the use of mathematical facts, techniques and formulae.	4	-		/4
Submission Guidelines					
Timeliness	Student submits the exercises, Mathspace task and investigation by the set deadline. See scoring guidelines for specific details.	2	-		/2
				FINAL	/20

Student Reflection: How did you go with this week's work? What was interesting? What did you find easy? What do you need to work on?

Mathspace task title:
Mathspace score: