Week 11&12
Term 2
2021

Learning Brief

Mathematical Applications

Year 11 Semester 1 - MAP1T

Goals

Oops, wrong matrix

Unit goals

- Understand the concepts and techniques introduced in consumer arithmetic, algebra and matrices, and shape and measurement.
- Apply reasoning skills and solve practical problems.

This week:

- What is a matrix
- Information displayed in a matrix.
- Scalar multiplication
- Matrix multiplication

Theoretical Components

Resources:

For this week the theory work is in the *PDF file:* Week 11&*12* Notes & Exercises

The following clip gives a good introduction to matrix multiplication.

https://www.khanacademy.org/math/precalculus/precalc-matrices/multiplying-matrices-by-matrices/v/matrix-multiplication-intro

Knowledge Checklist

- What is a matrix
- Rows and columns
- Elements of a matrix
- Order of a matrix
- Understanding the information given by the elements in a matrix.
- Drawing a diagram from a matrix.
- Addition and subtraction of matrices.
- Scalar multiplication
- Determining if matrix multiplication is possible
- Matrix multiplication
- Rules for matrix multiplication
- Applications of matrix multiplication

Practical Components

Order of things to do

1. Complete the questions in the *Booklet*
2. Complete the Investigation below.
3. Submit the questions for marking.
4. Complete the Mathspace exercise.

Investigation

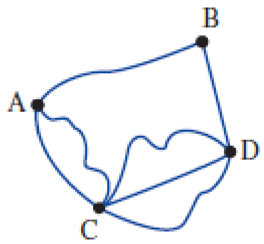
On HawkerMaths.com - attached to the Brief

Quiz

There is Mathspace on mathspace.

WEEK 11 NOTES & EXERCISES

Introduction to Matrices



Four towns are connected by roads as shown in the figure. There is one road connecting A and B, two roads connecting A and C and so on. This information may be represented as shown in the table.

		To			
		A	B	C	D
From	A	0	1	2	0
	B	1	0	0	1
	C	2	0	0	3
	D	0	1	3	0

If the headings at the top and side of this display are removed, an *array* of numbers only is left:

$$\begin{bmatrix} 0 & 1 & 2 & 0 \\ 1 & 0 & 0 & 1 \\ 2 & 0 & 0 & 3 \\ 0 & 1 & 3 & 0 \end{bmatrix}$$

This array of numbers is called a *matrix* (plural, *matrices*).
The arrangement of numbers in matrices is an extension of our number system and, as we will see, the rules that govern matrix calculations have many similarities with the arithmetic of numbers. Matrices can be used to store information, solve simultaneous equations, find optimal solutions in business, analyses networks, predict final standings in a football season, encode information and devise the best strategies in game theory.

A *matrix* is a rectangular array of numbers arranged in rows and columns.
The numbers in the matrix are called the *elements* of the matrix.

The matrix above is a 4 × 4 matrix as it has 4 rows and 4 columns. We say the *order* of the matrix is 4 by 4.

The matrix below is a 3 × 2 matrix because it has 3 rows and 2 columns.

$$\begin{bmatrix} 2 & 0 \\ 1 & -4 \\ -1 & 2 \end{bmatrix}$$

A matrix with *m* rows and *n* columns is called an *m* × *n* matrix.
We say the order of the matrix is *m* × *n.*
The dimensions of a matrix are always given as the number of rows multiplied by the number of columns.

The elements of the matrix are referred to by the row and then by the column position. In the 3 × 2 matrix above, the row 1, column 1 element is 2, the row 3, column 1 element is -1 and the row 1, column 2 element is 0.

We often use capital letters as symbols for matrices. Thus, we may write

$$A = \begin{bmatrix} 2 & 0 \\ 1 & -4 \\ -1 & 2 \end{bmatrix}$$

In general, the elements of a matrix *A* are referred to as a_{ij} where '**i**' refers to the row position and '***j***' refers to the column position.

For example, in the *2 × 3* matrix below

$$A = \begin{bmatrix} 6 & 8 & 4 \\ 3 & 7 & 1 \end{bmatrix}$$

- element a_{13} is in row 1, column 3 and its value is 4
- element a_{22} is in row 2, column 3 and its value is 7

Example 1: Interpreting the elements in a matrix.

Matrix B shows the number of boys and girls in Years 10 to 12 in a particular school.

$$B = \begin{matrix} \\ \text{Year 10} \\ \text{Year 11} \\ \text{Year 12} \end{matrix} \begin{matrix} \text{Boys} & \text{Girls} \\ \begin{bmatrix} 57 & 63 \\ 48 & 54 \\ 39 & 45 \end{bmatrix} \end{matrix}$$

- the order of the matrix is 3 × 2 as there 3 rows and two columns.
- element b_{12} is in row 1 column 2 and its value is 63. There are 63 girls in Year 10.
- the number of girls in Year 12 is 45. This is element a_{32}
- the sum of the boy's column gives the total number of boys, i.e. 57 + 48 + 39 = 144
- the sum of the Year 11 row gives the number of students in Year 11, i.e. 48 + 54 = 102

Example 2: A market stall operates on Friday and Saturday. Sales could be recorded by using matrix *A.*

Matrix A :

$$A = \begin{array}{c} \\ \textit{Friday} \\ \textit{Saturday} \\ \\ \end{array} \begin{array}{c} \begin{array}{ccc} \textit{Shirts} & \textit{Jeans} & \textit{Belts} \end{array} \\ \left[\begin{array}{ccc} 6 & 8 & 4 \\ 3 & 7 & 1 \end{array}\right] \\ \begin{array}{ccc} \textit{column 1} & \textit{column 2} & \textit{column 3} \end{array} \end{array} \begin{array}{c} \\ \textit{row 1} \\ \textit{row 2} \\ \\ \end{array}$$

Rows

Friday sales are listed in row 1

Saturday sales are listed in row 2

Columns

The number of shirts is shown in column 1

The number of pairs of jeans sold is listed in column 2

The number of belts sold is listed in column 3.

Row matrices

A **row matrix** has a *single row* of elements.

In matrix A, the Friday sales from the market stall can be represented by a 1×3 *row* matrix.

$$A = \begin{array}{c} \\ \textit{Friday} \\ \textit{Saturday} \end{array} \begin{array}{c} \begin{array}{ccc} \textit{Shirts} & \textit{Jeans} & \textit{Belts} \end{array} \\ \left[\begin{array}{ccc} 6 & 8 & 4 \\ 3 & 7 & 1 \end{array}\right] \end{array} \qquad \begin{array}{c} \\ \textit{Friday} \end{array} \begin{array}{c} \begin{array}{ccc} \textit{Shirts} & \textit{Jeans} & \textit{Belts} \end{array} \\ \left[\begin{array}{ccc} 6 & 8 & 4 \end{array}\right] \end{array}$$

Column matrices

A **column matrix** has a *single column* of elements.

In matrix A, the sales of jeans from the market stall can be represented by a 2×1 *column* matrix.

$$\begin{array}{c} \\ \textit{Friday} \\ \textit{Saturday} \end{array} \begin{array}{c} \textit{Jeans} \\ \left[\begin{array}{c} 8 \\ 7 \end{array}\right] \end{array}$$

Square matrices

In **square matrices** the number of *rows* equals the number of *columns*.

Here are three examples.

$$\underset{1 \times 1}{[9]} \qquad \underset{2 \times 2}{\begin{bmatrix} 5 & 4 \\ 4 & 2 \end{bmatrix}} \qquad \underset{3 \times 3}{\begin{bmatrix} 0 & 4 & 3 \\ 8 & 1 & 6 \\ 2 & 0 & 7 \end{bmatrix}}$$

Exercise Set 1

Q1. Matrix *C* is shown below.

$$C = \begin{bmatrix} 2 & 4 & 16 & 7 \\ 6 & 8 & 9 & 3 \\ 5 & 6 & 10 & 1 \end{bmatrix}$$

a) What is the order of matrix *C*

b) State the value of

(i) c_{13} (ii) c_{24} (iii) c_{31}

c) Find the sum of the elements n row 3.

d) Find the sum of the elements in column 2.

Q2. For each of the following matrices.

(i) state the order (ii) find the value of the required elements

a) Find a_{12} and a_{22}

$$A = \begin{bmatrix} 5 & 6 & 8 \\ 4 & 7 & 9 \end{bmatrix}$$

b) Find c_{32} and c_{12}

$$C = \begin{bmatrix} 4 & 5 \\ 3 & 1 \\ 8 & -4 \end{bmatrix}$$

c) Find f_{34} and f_{23}

$$F = \begin{bmatrix} 8 & 11 & 2 & 6 \\ 4 & 1 & 5 & 7 \\ 6 & 14 & 17 & 20 \end{bmatrix}$$

Q3. Some students were asked which of four sports they preferred to play and the results were entered in the following matrix.

$$S = \begin{array}{r} \\ Year\ 10 \\ Year\ 11 \\ Year\ 12 \end{array} \begin{array}{c} \begin{array}{cccc} Tennis & Basketball & Football & Hockey \end{array} \\ \begin{bmatrix} 19 & 18 & 31 & 14 \\ 16 & 32 & 22 & 12 \\ 21 & 25 & 5 & 7 \end{bmatrix} \end{array}$$

a) How many Year 11 students preferred basketball?

b) Write down the order of matrix X

c) What information is given by s_{23}?

d) Which sport was most popular. What column is this sport in?

Q4. Matrix F shows the number of hectares of land used for different purposes on two farms, *X* and *Y*. Row 1 represents Farm *X* and row 2 represents Farm *Y*.
Columns 1, 2 and 3 show the amount of land used for wheat, cattle and sheep (W, C, S) respectively in hectares.

$$\begin{array}{cc} & \begin{array}{ccc} W & C & S \end{array} \\ F = & \begin{bmatrix} 150 & 300 & 75 \\ 200 & 0 & 350 \end{bmatrix} \begin{array}{c} X \\ Y \end{array} \end{array}$$

a) How many hectares are used on:

(i) Farm *X* for sheep? (ii) Farm *X* for cattle? (iii) Farm *Y* for wheat?

b) Calculate the total number of hectares used on both farms for wheat.

c) Write down the information that is given by:

(i) f_{22} (ii) f_{13} (iii) f_{11}

d) Which elements of matrix *F* gives the number of hectares used:

(i) in Farm *Y* for sheep

(ii) on Farm *X* for cattle

(iii) on Farm *Y* for wheat?

e) What is the order of matrix *F*?

Using Matrices

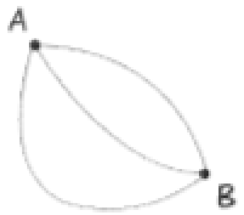
As seen at the beginning of these notes, matrices can be used to model road networks showing the number of connections between each of the towns in the network.

Exercise Set 2

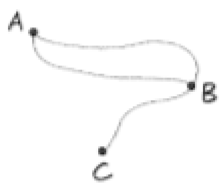
Q1. The road network shows roads connecting towns.

a) In each case use a matrix to record the number of ways of travelling directly from one town to another.

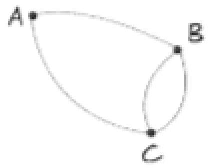
(i)



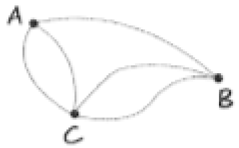
(ii)



(ii)



(iv)



b) What does the sum of the second column of each matrix represent?

(i) (ii)

(iii) (iv)

Q2. a) In each case draw graphs to show the direct connections between towns *A, B* and *C*.

(i)

$$\begin{array}{c} \begin{matrix} A & B & C \end{matrix} \\ \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{matrix} A \\ B \\ C \end{matrix} \end{array}$$

(ii)

$$\begin{array}{c} \begin{matrix} A & B & C \end{matrix} \\ \begin{bmatrix} 0 & 2 & 2 \\ 2 & 0 & 0 \\ 2 & 0 & 0 \end{bmatrix} \begin{matrix} A \\ B \\ C \end{matrix} \end{array}$$

b) State the information that is given by the sum of the first column in the matrices in Part a)

(i)

(ii)

Addition and Subtraction of Matrices

Matrices can be added and subtracted.

The rules for doing this are:

1. Matrices are added by adding the elements in the same position.
2. Matrices are subtracted by subtracting the elements in the same position.
3. Matrix addition and subtraction can only be done if the two matrices have the same order.

Example

a)

$$\begin{bmatrix} 2 & 4 \\ 5 & 1 \end{bmatrix} + \begin{bmatrix} 9 & 8 \\ 9 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 5 & 1 \end{bmatrix} + \begin{bmatrix} 9 & 8 \\ 9 & -1 \end{bmatrix} = \begin{bmatrix} 11 & 12 \\ 14 & 0 \end{bmatrix}$$

b)

$$\begin{bmatrix} 7 & 3 \\ 2 & 8 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 4 & 2 \\ -1 & 9 \\ 3 & 7 \end{bmatrix} = \begin{bmatrix} 7-4 & 3-2 \\ 2-(-1) & 8-9 \\ 1-3 & 0-7 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 3 & -1 \\ -2 & -7 \end{bmatrix}$$

Exercise Set 3

Q1. Using the matrices given:

$$A=\begin{bmatrix}3 & -2\\ 2 & 4\end{bmatrix} \quad B=\begin{bmatrix}5 & 7\\ 1 & 3\end{bmatrix} \quad C=\begin{bmatrix}6 & 2\\ 1 & 0\\ 3 & -8\end{bmatrix} \quad D=\begin{bmatrix}-3 & 5\\ 4 & -2\\ 1 & 7\end{bmatrix} \quad E=\begin{bmatrix}8\\ 3\end{bmatrix}$$

Find, where possible: (write 'impossible' for the one that cannot be done)

a) *A + B*

b) *B + A*

c) *A – B*

d) *B + E*

e) *C + D*

f) *B + C*

Q2. The weights and heights of four people were recorded and the checked again one year later.

2004 results:

	Aida	*Bianca*	*Chloe*	*Donna*
Weight (kg)	32	44	59	56
Height (cm)	145	155	160	164

2005 results:

	Aida	*Bianca*	*Chloe*	*Donna*
Weight (kg)	38	52	57	63
Height (cm)	150	163	167	170

a) Write a matrix that gives the changes in each person's weight and height after one year.

b) Who gained the most weight?

c) Which person had the greatest height increase?

Multiplication by a scalar

A *scalar* is a number. Multiplying a matrix by a number is called scalar multiplication.

Consider the matrix $B = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$

To find $3B$ we could use repeated addition: $3B = B + B + B$

$$= \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 12 \\ 6 & 9 \end{bmatrix}$$

$3B$ could have been calculated more efficiently by multiplying each element of B by 3.

Thus $3B = 3\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 3\times 1 & 3\times 4 \\ 3\times 2 & 3\times 3 \end{bmatrix} = \begin{bmatrix} 3 & 12 \\ 6 & 9 \end{bmatrix}$

The number 3 in the term 3B is called a scalar because it is a constant. Terms such as 3B refer to scalar multiplication of matrices.

When a matrix is multiplied by a scalar, each element of the matrix is multiplied by the scalar.

Example

If $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, find the matrix equal to $2A - 3B$.

Solution

1 Write $2A - 3B$ in expanded matrix form.

$$2A - 3B = 2 \times \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} - 3 \times \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

2 Multiply the elements in A by 2 and the elements in B by 3.

$$= \begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 0 & 3 \\ 3 & 0 \end{bmatrix}$$

3 Subtract the elements in corresponding positions.

$$= \begin{bmatrix} 2-0 & 2-3 \\ 0-3 & 2-0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$$

Scalar multiplication has many practical applications. It can be used to scale up elements in a matrix e.g. to add GST to the cost of items multiply the elements in the matrix by 1.1.

Example

A gymnasium has the enrolments in courses shown in this matrix.

$$\begin{array}{c} \\ Men \\ Women \end{array} \begin{array}{c} \begin{array}{ccc} \textit{Body building} & \textit{Aerobics} & \textit{Fitness} \end{array} \\ \begin{bmatrix} 70 & 20 & 80 \\ 10 & 50 & 60 \end{bmatrix} \end{array}$$

The manager wishes to double the enrolments in each course. Show this in a matrix.

Solution

1 Each element in the matrix is multiplied by 2.

$$2 \times \begin{bmatrix} 70 & 20 & 80 \\ 10 & 50 & 60 \end{bmatrix} = \begin{bmatrix} 2 \times 70 & 2 \times 20 & 2 \times 80 \\ 2 \times 10 & 2 \times 50 & 2 \times 60 \end{bmatrix}$$

2 Evaluate each element.

$$= \begin{array}{c} \\ Men \\ Women \end{array} \begin{array}{c} \begin{array}{ccc} \text{Body building} & \text{Aerobics} & \text{Fitness} \end{array} \\ \begin{bmatrix} 140 & 40 & 160 \\ 20 & 100 & 120 \end{bmatrix} \end{array}$$

Exercise Set 1

Q1.

If $A = \begin{bmatrix} -2 & 3 \\ 4 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 0 \\ -4 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & -3 \\ 2 & 6 \end{bmatrix}$, calculate:

a) 2*A*

b)2*A* + 3*B*

c) 3(*A + B)*

d) 2*A* + 3*B* – 4*C*

Q2. The expenses arising from costs and wages for each of three stores, *A, B* and *C,* are shown in the Costs matrix. The Sales matrix shows the money from the sale of goods in each section of the three stores. Figures represent the nearest million dollars.

Costs:

$$\begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} \textit{Clothing} & \textit{Furniture} & \textit{Electronics} \end{array} \\ \begin{bmatrix} 12 & 10 & 15 \\ 11 & 8 & 17 \\ 15 & 14 & 7 \end{bmatrix} \end{array}$$

Sales:

$$\begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} \textit{Clothing} & \textit{Furniture} & \textit{Electronics} \end{array} \\ \begin{bmatrix} 18 & 12 & 24 \\ 16 & 9 & 26 \\ 19 & 13 & 12 \end{bmatrix} \end{array}$$

a) Write a matrix showing the profit in each section of the store. Remember that the profit is Sales – Costs.

b) If 30% tax must be paid on profits, show the amount of tax that must be paid by each section of the store. No tax needs to be paid for a section that has made a loss.

Q3. Zoe competed in gymnastics rings and parallel bars events at a three-day gymnastics tournament. A win was recorded as a 1 and a loss as a zero. The three column matrices show the results for Saturday, Sunday and Monday.

$$\begin{array}{r} \\ \textit{Gymnastics rings} \\ \textit{Parallel bars} \end{array} \quad \begin{array}{ccc} \textit{Sat} & \textit{Sun} & \textit{Mon} \\ \begin{bmatrix} 1 \\ 0 \end{bmatrix} & \begin{bmatrix} 1 \\ 1 \end{bmatrix} & \begin{bmatrix} 1 \\ 1 \end{bmatrix} \end{array}$$

a) Give a 2×1 column matrix which records her total wins for the two types of events.

b) Zoe received $50 for a win. Give a 2×1 matrix which records her total prize money for each of the two types of events.

Q4. Find the values of *x, y* and *z* if 2**K** +3**L** = **M**

$$\mathbf{K}=\begin{bmatrix}5 & 2\\ 3 & 5\\ 7 & 8\\ 5 & 0\end{bmatrix}, \mathbf{L}=\begin{bmatrix}x & 3\\ 12 & 32\\ 14 & y\\ z & 57\end{bmatrix} \text{ and } \mathbf{M}=\begin{bmatrix}16 & 13\\ 42 & 102\\ 56 & 31\\ 55 & 171\end{bmatrix}.$$

Matrix Multiplication

Matrix multiplication is the multiplication of one matrix by another. It is not the same as scalar multiplication which is the multiplication of a matrix by a number.

The matrix multiplication of two matrices *A* and *B* can be written as *A × B* or simply *AB*.

The method of multiplying matrices is shown in the example below.

Example

The number of CD's and DVD's sold by Fatima and Gaia are recorded in matrix *N*. The selling prices of the CD's and DVD's are shown in matrix *P*.

$$N = \begin{matrix} & CDs & DVDs \\ Fatima & 7 & 4 \\ Gaia & 5 & 6 \end{matrix} \qquad P = \begin{matrix} & \$ \\ CDs & 20 \\ DVDs & 30 \end{matrix}$$

We need to make a matrix, *S*, that shows the value of the sales made by each person.

Fatima sold: 7 CD's at \$20 + 4 DVD's at \$30

Gaia sold: 5 CD's at \$20 + 6 DVD's at \$30

$$S = \begin{matrix} & \$ \\ Fatima & 7\times 20+4\times 30 \\ Gaia & 5\times 20+4\times 30 \end{matrix}$$

The steps used in this multiplication follow the routine for the matrix multiplication of *N × P*.

As we move across the first row of matrix *N* we move down the column matrix *P*, adding the products of the pairs as we go.

Then we move across the second row of the matrix *N* and down the column of matrix *P* adding the products of the pairs of numbers as we go.

$$N \times P$$

$$\begin{bmatrix}7 & 4\\ 5 & 6\end{bmatrix}\begin{bmatrix}20\\ 30\end{bmatrix}=\begin{bmatrix}7\times 20+4\times 30\\ \square \quad \square\end{bmatrix}$$

$$\begin{bmatrix}7 & 4\\ 5 & 6\end{bmatrix}\begin{bmatrix}20\\ 30\end{bmatrix}=\begin{bmatrix}7\times 20+4\times 30\\ 5\times 20+6\times 30\end{bmatrix}$$

$$=\begin{bmatrix}140+120\\ 100+180\end{bmatrix}$$

$$=\begin{matrix} \$ & \\ 260 & Fatima \\ 280 & Gaia \end{matrix}$$

Rules for matrix multiplication

The number of columns in the first matrix must equal the number of rows in the second matrix.
Otherwise, matrix multiplication cannot occur, meaning not possible (undefined).
In our example of the CD and DVD sales: order of 1st matrix order of 2nd matrix

$m \times n$ $\quad$ $n \times m$

↑ must be the same ↑

The outside numbers give the order of the product matrix: the matrix made by multiplying the two matrices. In our case, the answer is a 2 × 1 matrix.

Example

For the following matrices: $A = \begin{bmatrix} 5 & 2 \\ 4 & 6 \\ 1 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 8 \\ 9 \end{bmatrix}$ $C = \begin{bmatrix} 2 & 4 & 7 \end{bmatrix}$ $D = \begin{bmatrix} 8 \\ 6 \\ 5 \end{bmatrix}$

1. Decide if matrix multiplication can occur ie defined
2. If it can occur do the matrix multiplication.

a) *AB* $\quad$ *A* $\quad$ *B*

Write the order of each matrix. $\quad$ 3 × 2 $\quad$ 2 × 1

The inside numbers are the same $\quad$ thus matrix multiplication is defined

The outside numbers give the order $\quad$ the order of the product of *AB* is 3 × 1

Step 1

$$\begin{bmatrix} 5 & 2 \\ 4 & 6 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 8 \\ 9 \end{bmatrix} = \begin{bmatrix} 5 \times 8 + 2 \times 9 \\ \\ \\ \end{bmatrix}$$

Step 2

$$\begin{bmatrix} 5 & 2 \\ 4 & 6 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 8 \\ 9 \end{bmatrix} = \begin{bmatrix} 5 \times 8 + 2 \times 9 \\ 4 \times 8 + 6 \times 9 \end{bmatrix}$$

Step 3

$$\begin{bmatrix} 5 & 2 \\ 4 & 6 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 8 \\ 9 \end{bmatrix} = \begin{bmatrix} 5 \times 8 + 2 \times 9 \\ 4 \times 8 + 6 \times 9 \\ 1 \times 8 + 3 \times 9 \end{bmatrix}$$

Which gives

$$= \begin{bmatrix} 40 + 18 \\ 32 + 54 \\ 8 + 27 \end{bmatrix}$$

So $A \times B$

$$= \begin{bmatrix} 58 \\ 86 \\ 35 \end{bmatrix}$$

b) *BA* *B* *A*

Write the order of each matrix 2×1 3×2

The inside numbers are not the same thus multiplication is not defined for $B \times A$

c) *CD* *C* *D*

Write the order of each matrix 1×3 3×1

Multiplication is defined

The order of *CD* will be 1×1

Move across the row of *C* and down
The column of *D*, adding the products
of the pairs

$$\begin{bmatrix} 2 & 4 & 7 \end{bmatrix} \begin{bmatrix} 8 \\ 6 \\ 5 \end{bmatrix} = \begin{bmatrix} 2 \times 8 + 4 \times 6 + 7 \times 5 \end{bmatrix}$$

So $C \times D$ $= |75|$

Exercise Set 2

Q1. For the following matrices: $A = \begin{bmatrix} 6 & 2 \\ 3 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 3 \\ 0 & 8 \\ 2 & -5 \end{bmatrix}$

(i) Decide if matrix multiplication is possible

(ii) If matrix multiplication is defined, give the order of the matrix multiplication and carry out the multiplication

a) *AB=*

b) *BA=*

c) *CB=*

d) BC =

e) *AC=*

Q2. Carry out the matrix multiplication for the following pair of matrices.

$$\begin{bmatrix} 1 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

Applications of matrices

Data represented in matrix form can be multiplied to produce new useful information.

Example

The sports results at Mathglen High School were:

	Position		
House	**1st**	**2nd**	**3rd**
Hamilton	60	63	51
Leslie	71	64	74
Barnes	64	69	71
Cunningham	69	72	68

Position	Points
1st	5
2nd	3
3rd	1

To calculate the total points for each house, this matrix is multiplied by $\begin{bmatrix} 5 \\ 3 \\ 1 \end{bmatrix}$ since 5 points are awarded for first, 3 for second and 1 for third.

The result can be obtained using the following operations.

Hamilton: $60 \times 5 + 63 \times 3 + 51 \times 1 = 540$
Leslie: $71 \times 5 + 64 \times 3 + 74 \times 1 = 621$
Barnes: $64 \times 5 + 69 \times 3 + 71 \times 1 = 598$
Cunningham: $69 \times 5 + 72 \times 3 + 68 \times 1 = 629$

We can also write A × B = C, where

$$A = \begin{bmatrix} 60 & 63 & 51 \\ 71 & 64 & 74 \\ 64 & 69 & 71 \\ 69 & 72 & 68 \end{bmatrix}, \; B = \begin{bmatrix} 5 \\ 3 \\ 1 \end{bmatrix} \text{ and } C = \begin{bmatrix} 540 \\ 621 \\ 598 \\ 629 \end{bmatrix}$$

The order of A is 4 × 3, B is 3 × 1 and C is 4 × 1.

Exercise Set 2

Q1. The matrix below shows the number of wins, draws and losses for two soccer teams, the Sharks and the Dolphins.

$$\begin{bmatrix} 10 & 2 & 5 \\ 8 & 7 & 2 \end{bmatrix}$$

Thus the Sharks have 10 wins, 2 draws and 5 losses. If 3 points are awarded for a win, 1 for a draw and 0 for a loss:

a) write down a 3 × 1 matrix for the points awarded

b) use matrix multiplication to find the total points for the two teams.

Q2. Two shops, A and B, are supplied with boxes of different brands of chocolates — Yummy, Scrummy and Creamy — as shown in this table:

	Yummy	Scrummy	Creamy
Shop A	20	20	10
Shop B	10	5	10

The cost of the boxes are Yummy $10, Scrummy $25 and Creamy $12.

a) Write down the costs in a 3 × 1 matrix.

b) Use matrix multiplication to find the total cost for each shop.

Q3. The first matrix shows the number of cars and bicycles owned by two families. The second matrix records the wheels and seats for cars and bicycles.

$$\begin{array}{c} \\ Smith \\ Jones \end{array}\begin{array}{c} \begin{array}{cc} Cars & Bicycles \end{array} \\ \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \end{array} \qquad \begin{array}{c} \\ Car \\ Bicycle \end{array}\begin{array}{c} \begin{array}{cc} Wheels & Seats \end{array} \\ \begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix} \end{array}$$

Use a matrix product to find a matrix that gives the number of wheels and seats owned by each family.

Q4. Fast-food chain McDonuts has outlets in the centre of the city. An overall view of sales is maintained to monitor demand. Hamburgers, drinks and chips were surveyed. In January, Store A sold 1200 hamburgers, 2367 drinks and 4219 serves of chips.

a) Write this as a (1 × 3) matrix.

b) The selling price of each product is \$4.50, \$1.95 and \$2.45 respectively. Write this as a (3 × 1) matrix. C

c) Multiply these matrices to calculate the total income from these three items.

2021 MA1 Week 12 Investigation

The graph below shows the friendships within a group of people.

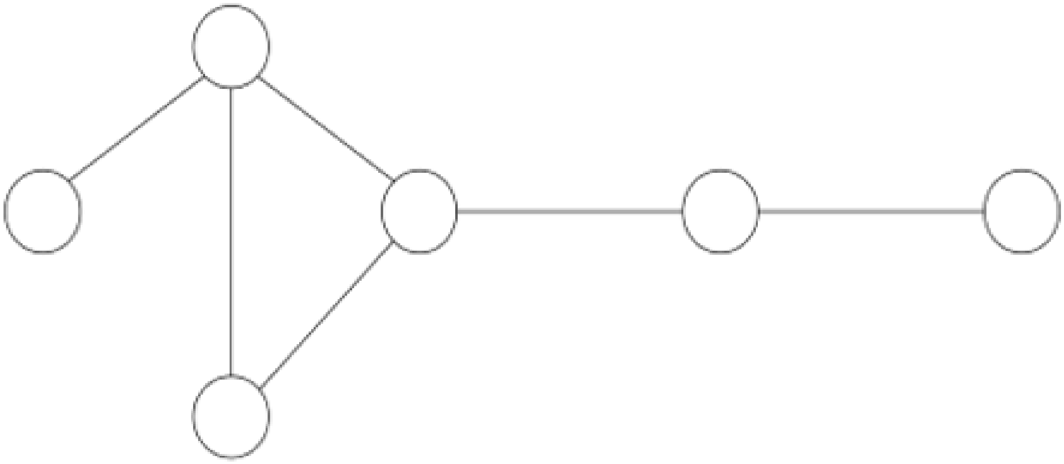
Part A

You need to work out who's who using the clues provided.

Clues: Alan has 3 friends; Barney, Charlie and Daniel

Barney and Ed are both friends with Charlie.

Ed is Franks only friend.



Part B

Write your solution in the form of a matrix.