MATHEMATICAL APPLICATIONS 1

WEEK 7 NOTES & EXERCISES

WHAT DO YOU GET TAXED ON?

We've already learnt about people earning wages and salaries. However, most people don't get to keep all of the money they earn. They have to pay tax, which is a percentage of their income paid to the government based on their taxable income. Taxes are used to fund public projects like schools, hospitals and roads. Depending on where you work, each year you or your employer will have to lodge a tax return, so knowing how to calculate your taxable income is important.

You'll need to know the difference between your GROSS income and your NET income. Your gross income is all the money you earn. Your NET income is the amount you actually get paid AFTER tax is taken out. Think of this fishing analogy to help you remember: if you lifted your net out of the gross ocean, all the tax and deductions would drain away and all that would be left in the net is the big, juicy amount of money you get paid.

Terminology

Gross income: the amount of income you receive before tax and deductions.

Net income: the amount of income you receive after tax and deductions.

Taxable income: the income you are required to pay tax on.

Tax rates: The different tax amounts people are required to pay. This varies depending on your taxable income.

Deductions: expenses that are directly related to earning your income which you are allowed to subtract from your gross income. These include uniform expenses, vehicle and travel expenses, self-education expenses, as well as charitable donations.

Medicare levy: a tax of 1.5% of one's taxable income paid by most Australian taxpayers that is used to provide Australian residents with access to public health care.

CALCULATING YOUR INCOME

Each pay period (whether it's weekly, fortnightly or monthly), your employer will take out a percentage of your income as tax, which is based on your gross income. However, you may have deductions that you can subtract from your gross income, which in turn will reduce your taxable income. If this is the case, and you have paid more tax than you needed to, you may be eligible for a tax return.

However, prior to lodging tax returns, it's also helpful to be able to calculate your net income, so you know how much money you will ACTUALLY receive each pay period. To do this, you need to subtract the amount of tax you are required to pay from your gross income. Then you can divide this net amount to find your weekly net wage, monthly net wage etc.

EXAMPLES

1. Calculate the net **weekly** wage of a carpenter who must pay 16% tax on an annual salary of $38080.

Give your answer correct to two decimal places.

Think: What would his annual net salary be? Remember that there are $52 weeks in a year.

Do: His annual net salary would be 100−16, or 84% of his gross income:

$0.84 \times 38080 = \$31987$

Then, to calculate his net weekly income:

$\$31987.20 \div 52 = \$615.14$

2. Sharon is a project manager with a taxable income of $99192. Throughout the year, her employer has deducted $512 per week in tax instalments.

The taxable income rates for individuals is provided in the table.

Income Bracket	Tax Payable
$1 - $18 200	Nil
$18 201 - $37 000	19 cents for each $1 over $18 200
$37 001 - $87 000	$3 572 + 32.5 cents for each $1 over $37 000
$87 001 - $180 000	$19 822 + 37 cents for each $1 over $87 000
$180 001 and over	$54 232 + 45 cents for each $1 over $180 000

a) Calculate the tax payable to the nearest cent.

Sharon's taxable income lies in the range of $87001 - $180000.

This means she needs to pay $19822 plus an additional amount, found as follows.

$99192 exceeds $87000 by $12192. This amount is taxed at 37 cents in the dollar.

$12192 × 0.37 = $4511.04

Thus the total tax payable is $19822 + $4511.04 = $24333.04

b) Calculate the Medicare Levy, which is 2% of taxable income.

The taxable income is $99192. We can find 2% of this amount.

$\frac{2}{100}$ × 99192 = $1983.84

Medicare Levy is $1983.84

c) Is Sharon entitled to a refund? Sharon needs to pay the total of her tax and the Medicare Levy but she has already paid $512 each week.

Thus she has paid $512 × 52 = $26624 already.

Her tax and Medicare Levy come to $24333.04 + $1983.84 = $26316.88

Now $26624 - $26316.88 = $307.12

Thus she is entitled to a refund of $307.12

Exercise Set 1

Q1. For a gross income of $52000, calculate the net income when tax is $11100 and superannuation is $2500.

Q2. If tax is stated as being 15.9 cents in the dollar, what percentage tax is this?

Q3. Medicare Levy is charged at 1.5% of taxable income. Calculate the Medicare levy on a taxable income of $60000.

Q4. Calculate the net weekly wage of a carpenter who must pay 16% tax on an annual salary of $38080 Give your answer correct to two decimal places.

Q5. Calculate the percentage of gross income going to taxes , correct to 2 decimal places, when:
(Note: Don't forget to include the percentage symbol where required)
a) gross income is $56000 and income tax is $15000

b) gross income is $49000, income tax is $13000.

Q6. A self-employed tradesman must put aside 9% of each fortnightly wage towards superannuation, and pay 10% GST to the government. If he earns $1067 in a fortnight, what will his net income be after these
deductions? Give your answer correct to two decimal places. Note: Do not use repeated percentage change.

Q7. Amelia's employer deducts a percentage of her income in tax. She sees the fortnightly amount deposited into her account is $1964 and knows that she earns an annual gross income of $63830.00 .
a) Use the amount deposited in her account to calculate her net annual income.

b) Hence, calculate the rate at which she is being taxed. Give your answer to the nearest whole percentage.

For the following questions use the tax table provided below:

Income thresholds	Rate	Tax payable on this income
$0 – $18,200	0%	Nil
$18,201 – $37,000	19%	19c for each $1 over $18,200
$37,001 – $90,000	32.5%	$3,572 plus 32.5% of amounts over $37,000
$90,001 – $180,000	37%	$20,797 plus 37% of amounts over $90,000
$180,000 and over	45%	$54,096 plus 45% of amounts over $180,000

Source: ATO

Q8. Calculate the tax payable, to the nearest cent, on a taxable income of:

a) $5347

b) $22968

c) $78671

Q9. Dave is a teacher who earns an annual salary of $78600. He earns an additional $4660 per year from private tutoring. He is allowed various tax deductions (for resource books etc.) amounting to $852 and he has paid tax instalments of $8430 during the year. Find:

a) His total gross income.

b) His taxable income.

c) Tax payable using the table.

d) the tax still owing.

Q10. Sally earns an annual income of $50000. With tax deductions of $1300 and having already paid $4900 in
tax instalments throughout the year, calculate her:

a) taxable income

b) tax payable to the nearest cent.

c) Medicare levy (1.5% of taxable income).

d) Amount due after levies and tax have been already paid.

Compound interest

In reality, calculating interest is not so 'simple' and straightforward. Simple interest is used only when the interest earned is collected by the investor and not added to the investment, such as in a term deposit account. With most accounts, however, the balance plus the interest becomes the new balance on which the interest is calculated next time. In other words, the interest will increase because you also earn 'interest on your interest'. This is called **compound interest**. Compound means 'to combine'.

The effect of compounding (which oil billionaire J. P. Getty called the 'eighth wonder in the world' and theoretical physicist Albert Einstein described as 'the driving force of the Universe') is a secret of financial wealth creation.

As mentioned previously, when we are dealing with simple interest, the interest is the same for each time period. The difference compounding makes can be seen in the following illustration.

Let us consider an amount of $1000, to be invested for a period of 3 years at an interest rate of 10% p.a. We will compare the interest earned using (i) simple interest, and (ii) compound interest.

Simple interest	Compound interest
Initial principal, $P = \$1000$	Initial principal, $P = \$1000$
Rate of interest, $R = 10\%$	Rate of interest, $R = 10\%$
Interest for Year 1 10% of \$1000 $I_1 = \$100$	Interest for Year 1 10% of \$1000 $I_1 = \$100$
Principal at the beginning of Year 2 $P_2 = \$1000$	Principal at the beginning of Year 2 $P_2 = \$1000 + \$100$ $= \$1100$
Interest for Year 2 10% of \$1000 $I_2 = \$100$	Interest for Year 2 10% of \$1100 $I_2 = \$110$
Principal at the beginning of Year 3 $P_3 = \$1000$	Principal at the beginning of Year 3 $P_3 = \$1100 + \$110$ $= \$1210$
Interest for Year 3 10% of \$1000 $I_3 = \$100$	Interest for Year 3 10% of \$1210 $I_3 = \$121$

Note that for simple interest, the interest earned each year is $100, while in compound interest the amount of interest earned increases each year. So the amount of interest earned with simple interest is $100 + $100 + $100 = $300 while the amount of interest earned with compound interest is $100 + $110 + $121 = $331.

Formula for compound interest

$$A = P\left(1 + \frac{r}{100}\right)^n$$

where:

A = the amount at the end of n compounding periods, \$
P = principal, \$
r = rate of interest per period
n = number of compounding periods

Note that in the formula for compound interest, *r* is the rate of interest *per period, not per annum* and *n* is the number of compounding *periods, not years*. It reflects the fact that compounding occurs not only on an annual basis but can be more frequent: that is, semi-annually (half-yearly), quarterly (every three months), monthly, weekly or daily.

Example

Tamara has $15000 to invest for 3 years. She considers the following options:

a) a term deposit at 5.25% pa compounded annually.

Here we use the formula $A = P\left(1 + \frac{r}{100}\right)^n$ where *P* =$15000, *r* = 5.25 and *n* = 5

Evaluating for *A* gives $17488.70, thus the interest earned is $17488.70 - $15000 = $2488.70

b) shares, paying a dividend rate of 5.08% pa, compounded quarterly.

Again, we use the compound interest formula, but now *r* = $\frac{5.08}{4}$ = 1.27% per quarter and *n* = 4 × 3

= 12 quarters in 3 years. Substituting in $A = P\left(1 + \frac{r}{100}\right)^n$ gives $A = P\left(1 + \frac{1.27}{100}\right)^{12}$ = $17452.63.

Thus the interest earned is $17452.63 - $15000 = $2452.63

c) a building society, paying a return of 5.4% pa compounded monthly.

Again, we use the compound interest formula, but now *r* = $\frac{5.4}{12}$ = 0.45% per month and *n* = 12 × 3

= 36 months in 3 years. Substituting in $A = P\left(1 + \frac{r}{100}\right)^n$ gives $A = P\left(1 + \frac{0.45}{100}\right)^{36}$ = $17631.50

Thus the interest earned is $17631.50 - $15000 = $2631.50

d) a business venture with guaranteed return of 7.3% compounded daily.

Again, we use the compound interest formula, but now *r* = $\frac{7.3}{365}$ = 0.02% per day and *n* = 4365 × 3

= 1095 days in 3 years. Substituting in $A = P\left(1 + \frac{r}{100}\right)^n$ gives $A = P\left(1 + \frac{0.02}{100}\right)^{1095}$ = $18672.06.

Thus the interest earned is $18672.08 - $15000 = $3672.06.

The more often the interest is compounded the greater the return (assuming the interest rate is the same).

Exercise Set 2

Q1. Warren wishes to invest $10 000. The following investment alternatives are suggested to him. All investments are for 7 years. Remember the simple interest formula gives you the interest earned while the compound interest formula gives the total amount.

a) simple interest at 9% p.a.

b) compound interest at 8% p.a.

c) compound interest at 7 % p.a. adjusted quarterly. Remember to adjust the interest rate and time period.

d) compound interest at 7% p.a. adjusted daily. Remember to adjust the interest rate and time period.

e) Which investment alternative will produce the greatest return on his money?

Q2. Rosemary has $25 000 to invest for 5 years. She considers the following options:

a) a term deposit at 6.75% p.a. compounded annually

b) shares, paying a dividend rate of 5.15% p.a. compounded quarterly

c) a building society, paying a return of 5.3% p.a. compounded monthly (change interest rate and time period)

d) a business venture with guaranteed return of 6.4% p.a. compounded daily.

(Assume there is only one leap year in the given 5 year period.)

e) All the investments are equally secure. Advise Rosemary which option to take and why?

Q3. Over the last 3 years a comprehensive hospital cover from 'TakeCare' private medical insurance rose at an average of 9.5% and currently costs $1980 per year. With this rate of increase continued, what would be the insurance premium after another 3 years? (*Hint*: The increase in premium compounds each year.)

Appreciation

Items which represent scarce or valuable resources such as land, collectables, paintings and antiques normally increase or appreciate in value over time. They become more valuable as time passes because they become more rare or scarce. This is called **appreciation**. Some people like to invest their money by buying and selling such items.

Q4. A painting that Elizabeth bought for $240 from an art exhibition appreciates (increases in value) by 14% p.a. If this rate of appreciation continued, what would be the value of the painting after 25 years?

Compound Interest Calculator

The link below is for a compound interest calculator. This calculator allows you to calculate A, P, r or t.

https://www.calculatorsoup.com/calculators/financial/compound-interest-calculator.php

Use the calculator to solve the following.

Q5. An original Elio Sanciolo painting is currently valued at $4500 and is known to appreciate an average 8% per year. After how many years would the painting have a value of at least $25 000?

In this question P = $4500, r = 8%, A = $25000 and we want to find t. so set 'Calculate' to Time (t).

Q6. According to the last census, Whitehorse Marsh has a population of 23 600. The population increases at will it take for the population of Whitehorse Marsh to reach the 40 000 mark?

Q7. What is the amount, rounded to the nearest $100, to be invested for 6 years and compounded semi-annually at 8% p.a.? The value of the investment at the end of the term is $15 000. Set the 'Compound (n)' to Semi-annually.

Q8. Gaetano bought a house for $175 000 in an area where house prices appreciate an average 3% per year. He decided to hold on to his house until its value is at least $250 000. How many years should he wait until he sells his current house?

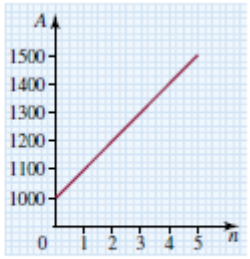
Graphing Compound Interest

We can use the data from the example at the beginning of this booklet to compare the graphs of simple and compound interest.

If we were to place the set of data obtained in two separate tables and represent each set graphically — as the total amount of the investment, *A*, versus the year of the investment, *n* — we would find that:

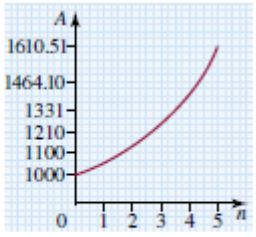
1. The simple interest investment is represented by a straight line as shown below.

n	*A*
0	1000
1	1100
2	1200
3	1300
4	1400
5	1500



2. The compound interest investment is represented by an exponential graph as shown below.

n	*A*
0	1000
1	1100
2	1210
3	1331
4	1464.10
5	1610.51



The graph of any simple interest scenario is always a straight line (linear), while the graph of compound interest is always represented by an exponential curve.

The total amount, A, in compound interest always grows at a much faster rate than in simple interest.