MATHEMATICAL APPLICATIONS 1

WEEK 6 NOTES & EXERCISES

At various stages of our lives, we will be faced with making important and informed decisions regarding our finances. We may find ourselves asking:

• If I wish to purchase a car, should I borrow from the bank or accept a payment plan offered by the car dealer?

• If I wish to invest $5000, how much interest will I earn annually?

• If I wish to borrow a sum of $100 000 from a bank or finance company, how much money will I need to repay in total?

To answer these questions, we need to have a basic understanding of 'money matters'. This week we will look at simple interest.

Simple interest

When you lend money for a certain period of time (a term deposit) to a bank, building society, or other financial institution, you expect to be rewarded by eventually getting your money back, plus an extra amount commonly known as *interest* (*I*).

Similarly, if you borrow money from any institution by taking out a loan or mortgage, you must pay back the original sum plus interest.

The following examples deal with *simple interest*, that is, interest which is paid only on the original sum of money invested or borrowed.

The formula used to calculate simple interest is given by:

$$I = \frac{PRT}{100}$$

where:
I = interest, $
P = principal, $ — that is, the sum of money borrowed or invested
R = rate of interest p.a., % — that is, per annum, (per year)
T = term of interest, years — that is, the period of time for which the sum of money is to be borrowed or invested

The sum of the principal, *P*, and the interest, *I*, is called the *total amount* and is denoted by the symbol *A*.

The formula used to calculate the total amount is given by:

$$A = P + I$$

where:
A = total amount at the end of the term, $
P = principal, $
I = simple interest, $

Example 1

Calculate the amount of simple interest, I, earned and the total amount, A, at the end of the term, if:
a $12 000 is invested for 5 years at 9.5% p.a.
b $2500 is invested for 3 months at 4.5% p.a.

THINK	WRITE
a 1 Write down the formula for simple interest.	**a** $I = \frac{PRT}{100}$
2 Write down the known values of the variables.	$P = \$12\,000 \quad R = 9.5\%$ $T = 5$ years
3 Substitute the values into the given formula.	$I = \frac{12\,000 \times 9.5 \times 5}{100}$
4 Evaluate.	$= \frac{570\,000}{100}$ $= 5700$
5 Answer the question and include the appropriate unit.	The amount of interest earned is $5700.
6 Write down the formula for the total amount.	$A = P + I$
7 Substitute the values for P and I.	$= 12\,000 + 5700$
8 Evaluate.	$= 17\,700$
9 Answer the question and include the appropriate unit.	The total amount at the end of the term is $17 700.
b 1 Write down the formula for simple interest.	**b** $I = \frac{PRT}{100}$
2 Write down the known values of the variables. *Note*: T must be expressed in years, so divide 3 months by 12 months	$P = \$2500 \quad R = 4.5\%$ $T = 3$ months $= \frac{3}{12}$ or 0.25 years
3 Substitute the values into the given formula.	$I = \frac{2500 \times 4.5 \times 0.25}{100}$
4 Evaluate and round off the answer to 2 decimal places.	$= \frac{2812.5}{100}$ $= 28.13$
5 Answer the question and include the appropriate unit.	The amount of interest earned is $28.13.
6 Write down the formula for the total amount.	$A = P + I$
7 Substitute the values for P and I.	$= 2500 + 28.13$
8 Evaluate.	$= 2528.13$
9 Answer the question and include the appropriate unit.	The total amount at the end of the term is $2528.13.

remember

1. Simple interest is given by $I = \frac{PRT}{100}$.
2. The total amount is given by $A = P + I$.
3. When calculating simple interest, the interest earned is the same for each time period.

Exercise Set 1

Q1. Calculate the simple interest paid per year on the following investments:

a) $3690 at 11% p.a.

b) $22 400 at 6.85% p.a.

c) $620 at 14 % p.a.

d) $16 000 at 4.8% p.a.

Q2. For each of the following calculate:

i the amount of simple interest, *I*, earned

ii the total amount, *A*, at the end of the term.

a) $1200 for 1 year at 10.5% p.a.

b) $8320 for 3 years at 6.45% p.a.

c) $960 for 2 years at 9.20% p.a.

d) $126 000 for 6 months at 8.35% p.a.

e) $5000 for 3 months at 5.25% p.a.

f) $7920 for 120 days at 8.26% p.a.

Q3. Norman borrowed $3500 for 8 months at 11 % p.a. simple interest. Calculate the total amount Norman must repay at the end of the term of the loan.

Q4. Rodney borrowed $3500 from the student credit union for 2 years at 8% per annum simple interest with repayments made by equal instalments at the end of each calendar month. Calculate the amount Rodney has to pay at the end of each month to the Credit Union.

Q5. Sue and Harry invested $14 500 in State Government bonds at 8.65% p.a. The investment is for 10 years and the interest is paid semi-annually (that is, every six months). Calculate how much interest

a) they receive every payment

b) will be received in total.

Simple Interest Calculator

The link below takes you to a Simple Interest Calculator. There are four variables associated with simple interest. The principal, P, the rate, r, the time ,T and interest earned I. This calculator allows you to put any three of these variable in and it will calculate the fourth.

For example, how long will it take to earn $500 (the interest) if $5000 is invested (the principal) at 3% pa (the rate).

https://www.easycalculation.com/simple-interest.php

Set the input *I want to calculate* to Time Period (T). Then enter Principal or Sum (P) as $5000, set Rate per Annum (R) as 3, set Simple Interest (SI) $500. Note that Time (T) is highlighted in red – as that is what we want to calculate. Click on *Calculate* and the result is given as 3 years.

Exercise Set 2

Use this calculator to solve the following.

Q1. How long, to the nearest month, will it take to earn $1950 simple interest if $16 325 is invested at 9.75% p.a.? Note: Make sure Time (T) is in months.

Q2. What sum, to the nearest dollar, must be invested for one year at 6% per annum simple interest in order to earn $1200 interest?

Q3. Find the total value of an investment of $3500 after 2 years and 6 months if simple interest is paid at the rate of 5% per annum. Remember to use $A = P + I$.

Q4. For how long must a principal of $15 750 be invested at 9.8% p.a. for it to earn $3087 in simple interest?

Q5. What principal would earn $3729.60 in interest if invested for 3 years at 16.8% p.a.?

Q6. An amount of $9020 was invested for 2 years and earned $1731.84 in simple interest. Calculate the monthly interest rate.

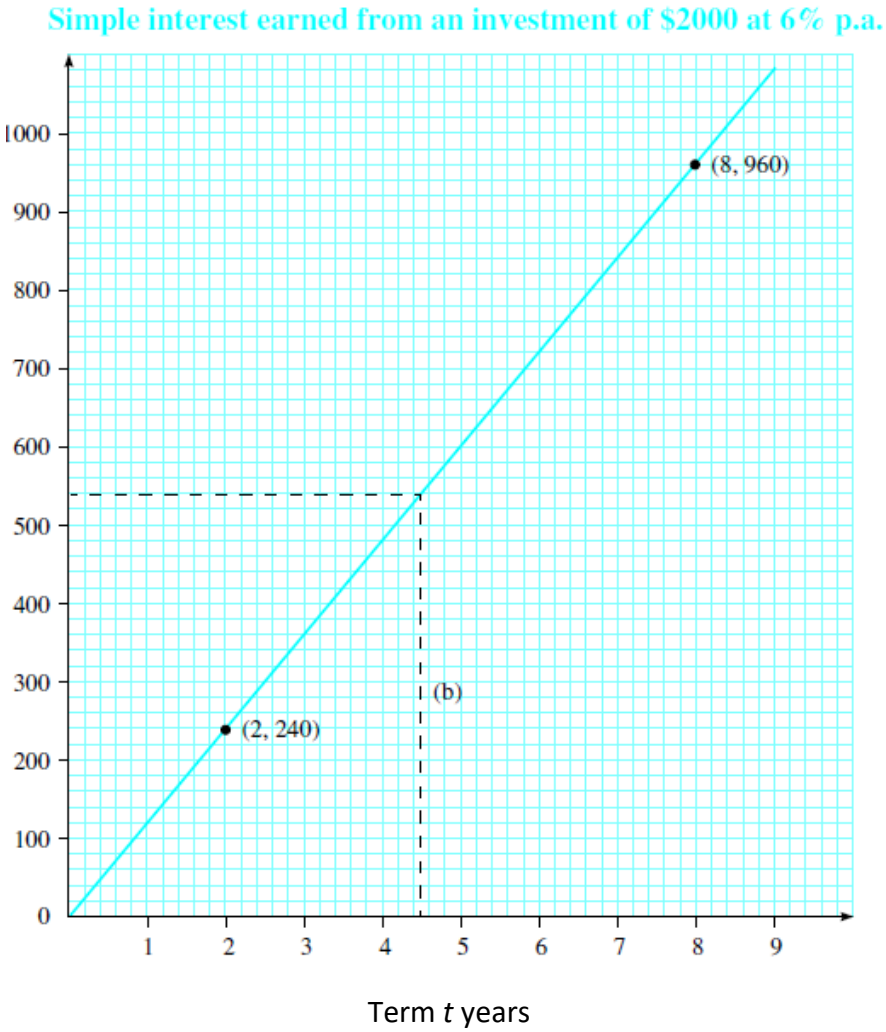
Graphing Simple Interest

Example: Rachel invests $2000 in an account that earns 6% p.a. simple interest. Construct a graph that shows the simple interest *I* earned in dollars over *n* years, for values of *n* from 0 to 8.

We use the formula $I = \frac{Prt}{100}$ and substitute *P* = 2000, *r* = r, which gives $I = \frac{2000 \times 6 \times t}{100}$ which simplifies to *I = 120 t.* This is a simple equation and we can construct a simple table to help graph it.

t (years)	**0**	**2**	**8**
I ($)	**0**	**240**	**960**

We can use this table to produce the graph below.



The dotted line shows that after 4.5 years the money invested would earn $640.

Exercise Set 3

Q1. Use the simple interest graph above to estimate the interest earned when $2000 is invested at 6% p.a. for:

a) 9 years

b) 2.5 years

Q2. What would the total value of the investment be after:

a) 3 years

b) 6 years?

Q3. Graph on the same axes the simple interest, *I,* earned when $4000 is invested in an account earning:

a) 4% p.a.

b) 12% p.a.

over *t* years, for values of *n* from 0 to 10. Draw up a table for each. Use graph paper.