

Learning Brief

Mathematical Applications

Year 11 Semester 1 - MAP1T

Goals



Unit goals

- Understand the concepts and techniques introduced in consumer arithmetic, algebra and matrices, and shape and measurement.
- Apply reasoning skills and solve practical problems.

This week the work is on

- Wages and Salaries
- Overtime and Extra Income
- Commission and Other Income
- Allowances
- Budgeting
- Simple Interest

Theoretical Components

Resources:

For this week the theory work is in the *PDF file: Week 5&6 Notes & Exercises*

Subtopics: → Wages and Salaries

→ Overtime and Extra Income

→ Commission and Other Income

→ Allowances

→ Budgeting

Knowledge Checklist

- What is a wage, what is a salary?
- Time-and-a-half, double time, and holiday loading
- Gross income, net income
- Commission
- Receiving an allowance
- What is a budget and why is it important?
- Terms: income, expenses, fixed income, variable income
- Words associated with simple interest: principal, interest, rate, term

Order

1. Look at the Investigation.
2. Work through the examples to develop the skills necessary to complete the worksheet questions.
3. Complete all sets of questions
4. Complete the Investigation.
5. Show your completed booklet to Aaron and submit the Investigation for marking.

Practical Components

There are five sets of questions for this week.

They are

- Wages & Salaries
- Overtime
- Commission
- Allowances
- Budgeting

Investigation

In Week 6 you are to sit an In-Class Task worth 20% (with your weekly investigations) in your double line. It is an "open book" task given under test conditions. You will be allowed to bring in any of your notes and worked exercises since Week 1 and, of course, your calculator.

Quiz

No quiz this week.

MATHEMATICAL APPLICATIONS 1

Week 5&6 Notes & Exercises

Allowances

Allowances

When we talk about receiving an allowance, we can either talk about receiving an allowance from our employer or from the government. You'll see more about government allowances in a later chapter.

Let's focus now on the types of allowances you commonly receive from your employer.

Holiday loading

A holiday loading is a payment employees receive when they take paid annual leave. In Australia, the award rate is 17.5% in addition to your regular rate of pay. So basically you're getting paid to take holidays- sounds awesome right?!

There are a couple of reasons why holiday loadings were introduced

Since you generally spend more money when you are on holidays than when you are at work, holiday loadings help people with lower incomes afford to take holidays (especially since they are unable to earn overtime when they aren't working).

It encourages employees to take holidays. Theoretically, a person could "save up" all their annual leave and then take a year off which wouldn't be very helpful to a business who would have to pay the person who is away + someone to fill their role.

Allowances

Just like you may receive a weekly allowance from your parents, allowances in terms of employment are additional amounts of money employers pay their employees for various work-related expenses. This may include a car or travel allowance, a tool allowance or a uniform allowance depending on your job.

Depending on your job, there are also allowances that you might be paid to do dangerous or unpleasant work. For example, you might be paid an extra allowance, on top of your salary, to work high up on a tall building, or for doing work that requires deep sea diving.

Worked Example 1.

Neil, a miner, receives allowances of \$1.40 per hour for working in confined spaces, 70 cents per hour for being exposed to potentially toxic substances during work, and 80 cents per hour for the noise caused by general mining operations. Calculate the total value of Neil's allowances when he works 38 hours.

Hide Solution

▶ Watch video

Try this problem!

Sum the three hourly allowances. Then multiply this by the total number of hours worked.

$$= (1.4 + 0.7 + 0.8) \times 38 \text{ dollars}$$

Evaluate the sum.

$$= 2.9 \times 38 \text{ dollars}$$

Use your calculator to evaluate.

$$= 110.20 \text{ dollars}$$

Exercise Set 1.

Q1. A worker receives an allowance of \$0.50 per hour for performing unpleasant work. What is the total value of this allowance if he works 37 hours?

Q2. Maria, a window cleaner, receives a height allowance of \$0.90 per hour. If her normal hourly rate is \$24.80, find:

- Her normal weekly wage without allowances (assuming a 35 –hour working week).
- Her gross weekly income (including allowances).
- Her holiday loading at 17.5% of 4 weeks' income.
- The amount she will receive when she takes her 4 weeks' holiday.
- Her annual gross income.

Q3. James, a pest inspector, is paid \$17.00 per hour. In addition, he is paid allowances of \$1.50 per hour for being exposed to potentially toxic substances, 50 cents for exposure to potentially dangerous insect bites and 70 cents for wet work. Assuming a 39 –hour working week, find:

- a) His weekly wage before allowances.

- b) His gross weekly income.

- c) His holiday loading at 17.5% of 4 weeks' income.

- d) The total amount he will receive when he takes his 4 weeks' holiday.

- e) His annual gross income.

Commission

What is Commission?

A commission is a fee paid to an agent as compensation for executing a transaction. It is calculated either as a percentage of the transaction value or as a flat fee.

How Does Commission Work?

Let's assume you would like to purchase 100 shares of Company XYZ at \$35 per share, and your broker charges a 2% commission to make the trade.

The shares themselves would cost \$3,500 ($\35×100 shares), but the broker would also need to be paid for finding someone to sell the shares to you. For their services, they would charge \$70 ($\$3,500 \times 2\%$). The total cost of the transaction would be $\$3,500 + \$70 = \$3,570$.

Four months later, you decide to sell your XYZ shares. Now selling at \$50, you would receive \$5,000 ($\$50 \times 100$ shares) from the sale. But once again, the brokerage would take 2% (\$100), so the actual proceeds from the transaction would total $\$4,900$ ($\$5,000 - \100).

Exercise Set 2.

Q1. Calculate the income earned for the following amounts of piecework:

a) washing 24 cars at \$10 per car

b) baking 26 croissants at \$7 per croissant

c) sewing 84 shirts at \$8 per shirt

Q2. A process worker earns \$3 for each article assembled. Calculate the number of articles he has to assemble to earn \$2400.

Q3. Sally is paid 7% on the first \$2600 of goods sold and 1% on any value thereafter. Goods to the value of \$15400 are sold.

a) Calculate the commission earned on the first \$2600 of goods sold.

b) Calculate the commission earned on the amount of goods sold in excess of \$2600.

c) What is the total commission earned?

Q4. Dave sold motorbikes to the value of \$66000 in one week. His pay for the week was \$2434, which comprised a retainer plus a commission of 2.9% on his sales. What retainer was Dave paid?

Q5. Sally is paid a retainer of \$210 per week plus a commission based on her weekly sales. In one week, she sells \$9800 worth of lighting. Her total pay for the week was \$504.

a) How much commission did she earn?

b) What is Sally's commission rate? Give your answer as a percentage.

Q6. Amelia earns a weekly retainer of \$300 plus a commission of 4.6% based on sales of airline tickets. If she is aiming to earn \$1183.20 each week, what value of ticket sales must she achieve?

Overtime

Overtime refers to any hours worked by an employee that exceed their normally scheduled working hours. While a generalized overtime definition refers simply to those hours worked outside of the standard working schedule, overtime commonly refers concurrently to the employee's remunerations of such work.

Common overtime rates

Time-and-a-half: payment to an employee at 1.5 times their usual hourly rate.

Double time: payment to an employee at 2 times their usual hourly rate.

Worked Example 3.

If the normal rate is \$13.40 per hour:

A) What is the time-and-a-half rate? Give your answer correct to two decimal places.

Think: We need to work out what 1.5 times the normal rate is.

Do:

$$\begin{aligned} 13.40 \times 1.5 &= 20.1 \\ &= \$20.10 \text{ (to 2 d.p.)} \end{aligned}$$

B) What is the double rate? Give your answer correct to two decimal places.

Think: We need to work out what two times the normal rate is.

Do:

$$\begin{aligned} 13.40 \times 2 &= 26.8 \\ &= \$26.80 \text{ (to 2 d.p.)} \end{aligned}$$

Exercise Set 3.

Q1. If the normal rate is \$13.40 per hour:

a) What is the time-and-a-half rate? Give your answer correct to two decimal places.

b) What is the double time rate? Give your answer correct to two decimal places.

Q2. A job advertisement states that the hourly wage based on a 35 -hour working week is \$38 and that there is a 17% holiday loading on 4 weeks wages. How much holiday loading would the employee receive? Give your answer correct to two decimal places.

Q3. Calculate the holiday loading for a worker who is given 20% of 4 weeks normal pay, and earns:

a) \$3760 in 4 weeks. Round your answer to 2 decimal places if necessary.

b) \$2150 in a fortnight. Round your answer to 2 decimal places if necessary.

c) \$650 a week. Round your answer to 2 decimal places if necessary.

d) \$15 per hour (assuming a 37-hour working week). Round your answer to 2 decimal places if necessary.

Wages & Salaries

People earn money in different ways. However, the most common sources of income people receive are wages and salaries. Let's start by defining these terms.

A **wage** is an amount paid to an employee at a certain rate. It may be calculated as a fixed task-based amount, or at an hourly rate, or based on the quantity of work done (e.g. piecemeal work). So, if you are paid per hour, your wage will increase the more hours you work.

In contrast, a **salary** is a fixed income that is based on a fixed number of working hours. People normally sign a contract with an agreed salary amount before they start working with a company. An employee may receive a weekly, fortnightly, or monthly salary. The employee will receive the same amount of income each time period, regardless of whether they work more or less hours, as it is thought this will be averaged out. A salary is normally written as an annual amount. However, it can be helpful to calculate how much money you'll receive each week, to help you set a budget for your spending. Similarly, you may want to work out how much money you will earn in a year based on the amount you get paid fortnightly. So, let's look at some examples of how to solve questions involving salaries and wages.

Remember

- There are 52 weeks in a year.
- There are 2 weeks in a fortnight.
- There are 26 fortnights in a year.

Worked Example 4.

Beth worked for 281 days in the year, with an average of 7 hours a day. For 62% of those hours, she was paid \$15 per hour, and for the rest she was paid \$25 per hour.

A) For how many hours was she paid \$15 per hour? Give your answer to 1 decimal place.

Think: How many hours does she work in a year?

Do:

$$281 \times 7 = 1\,967 \text{ hours that she works in a year}$$

$$1\,967 \times 62\% = 1\,219.5 \text{ hours}$$

B) For how many hours was she paid \$25 per hour? Give your answer to 1 decimal place.

Think: She was earning \$15 for 1219.5 hours, and \$25 for the rest of the year.

Do: $1\,967 - 1\,219.5 = 747.5 \text{ hours}$

C) What was her total income for the year (to the nearest cent)?

$$1\,219.5 \times 15 = \$18\,292.50$$

$$747.5 \times 25 = \$18\,687.50$$

Total

$$18\,292.50 + 18\,687.50 = \$36\,980$$

D) What was her average weekly income for the year (to the nearest cent)?

Think: There are 52 weeks in a year.

Do: $36\,980 \div 52 = \$711.15$

Exercise Set 4.

Q1. Christa earns \$21 per hour working as a receptionist. If she works 19 hours per week, how much is her weekly wage?

Q2. Find the annual salary of a worker whose weekly pay is \$307. Assume that there are 52 weeks in the year.

Q3. Noah's annual salary is \$90282. If Noah gets paid monthly, calculate his monthly income, giving your answer correct to the nearest cent.

Q4. Dave has a job that pays him \$21 per hour. Assume a working week of 38 hours.

a) Calculate his weekly income.

b) Calculate his fortnightly income

c) Calculate his annual income.

d) Calculate his monthly income (to the nearest cent).

Q5. Han earns \$18.00 per hour from 9am-5pm and \$27.00 per hour outside business hours (assume Han has paid breaks). Calculate his income, to the nearest cent, for the shifts that he worked between the following times:

a) 9am to 5pm

b) 11am to 4pm

c) 6am to 4pm

d) 10am to 11pm

e) 5am to 10pm

Q6. Neil's contract is based on a 30-hour working week and an hourly rate of \$20. Amelia's contract is based on an annual salary of \$33040.

a) What is Neil's weekly income?

b) What is Amelia's weekly income (to the nearest cent)?

c) Who has the higher weekly income?

A Amelia B Neil

d) How much higher is Amelia's weekly income compared to Neil's?

Interest

At various stages of our lives, we will be faced with making important and informed decisions regarding our finances. We may find ourselves asking:

- If I wish to purchase a car, should I borrow from the bank or accept a payment plan offered by the car dealer?
- If I wish to invest \$5000, how much interest will I earn annually?
- If I wish to borrow a sum of \$100 000 from a bank or finance company, how much money will I need to repay in total?

To answer these questions, we need to have a basic understanding of 'money matters'. This week we will look at simple interest.

Simple interest

When you lend money for a certain period of time (a term deposit) to a bank, building society, or other financial institution, you expect to be rewarded by eventually getting your money back, plus an extra amount commonly known as *interest* (I).

Similarly, if you borrow money from any institution by taking out a loan or mortgage, you must pay back the original sum plus interest.

The following examples deal with *simple interest*, that is, interest which is paid only on the original sum of money invested or borrowed.

The formula used to calculate simple interest is given by:

$$I = \frac{PRT}{100}$$

where:

I = interest, \$

P = principal, \$ — that is, the sum of money borrowed or invested

R = rate of interest p.a., % — that is, per annum, (per year)

T = term of interest, years — that is, the period of time for which the sum of money is to be borrowed or invested

The sum of the principal, P , and the interest, I , is called the *total amount* and is denoted by the symbol A .

The formula used to calculate the total amount is given by:

$$A = P + I$$

where:

A = total amount at the end of the term, \$

P = principal, \$

I = simple interest, \$

Worked Example 5.

Calculate the amount of simple interest, I , earned and the total amount, A , at the end of the term, if:

- a \$12 000 is invested for 5 years at 9.5% p.a.
- b \$2500 is invested for 3 months at 4.5% p.a.

THINK

- a
 - 1 Write down the formula for simple interest.
 - 2 Write down the known values of the variables.
 - 3 Substitute the values into the given formula.
 - 4 Evaluate.
 - 5 Answer the question and include the appropriate unit.
 - 6 Write down the formula for the total amount.
 - 7 Substitute the values for P and I .
 - 8 Evaluate.
 - 9 Answer the question and include the appropriate unit.

- b
 - 1 Write down the formula for simple interest.
 - 2 Write down the known values of the variables.
Note: T must be expressed in years, so divide 3 months by 12 months
 - 3 Substitute the values into the given formula.
 - 4 Evaluate and round off the answer to 2 decimal places.
 - 5 Answer the question and include the appropriate unit.
 - 6 Write down the formula for the total amount.
 - 7 Substitute the values for P and I .
 - 8 Evaluate.
 - 9 Answer the question and include the appropriate unit.

WRITE

$$\begin{aligned} \text{a } I &= \frac{PRT}{100} \\ P &= \$12\,000 \quad R = 9.5\% \\ T &= 5 \text{ years} \\ I &= \frac{12\,000 \times 9.5 \times 5}{100} \\ &= \frac{570\,000}{100} \\ &= 5700 \end{aligned}$$

The amount of interest earned is \$5700.

$$A = P + I$$

$$\begin{aligned} &= 12\,000 + 5700 \\ &= 17\,700 \end{aligned}$$

The total amount at the end of the term is \$17 700.

$$\begin{aligned} \text{b } I &= \frac{PRT}{100} \\ P &= \$2500 \quad R = 4.5\% \\ T &= 3 \text{ months} \\ &= \frac{3}{12} \text{ or } 0.25 \text{ years} \end{aligned}$$

$$\begin{aligned} I &= \frac{2500 \times 4.5 \times 0.25}{100} \\ &= \frac{2812.5}{100} \\ &= 28.13 \end{aligned}$$

The amount of interest earned is \$28.13.

$$A = P + I$$

$$\begin{aligned} &= 2500 + 28.13 \\ &= 2528.13 \end{aligned}$$

The total amount at the end of the term is \$2528.13.

remember

1. Simple interest is given by $I = \frac{PRT}{100}$.
2. The total amount is given by $A = P + I$.
3. When calculating simple interest, the interest earned is the same for each time period.

Q4. Rodney borrowed \$3500 from the student credit union for 2 years at 8% per annum simple interest with repayments made by equal instalments at the end of each calendar month. Calculate the amount Rodney has to pay at the end of each month to the Credit Union.

Q5. Sue and Harry invested \$14 500 in State Government bonds at 8.65% p.a. The investment is for 10 years and the interest is paid semi-annually (that is, every six months). Calculate how much interest

a) they receive every payment

b) will be received in total.

Simple Interest Calculator

The link below takes you to a Simple Interest Calculator. There are four variables associated with simple interest. The principal, P , the rate, r , the time, T and interest earned I . This calculator allows you to put any three of these variables in and it will calculate the fourth.

For example, how long will it take to earn \$500 (the interest) if \$5000 is invested (the principal) at 3% pa (the rate).

<https://www.easycalculation.com/simple-interest.php>

Set the input *I want to calculate* to Time Period (T). Then enter Principal or Sum (P) as \$5000, set Rate per Annum (R) as 3, set Simple Interest (SI) \$500. Note that Time (T) is highlighted in red – as that is what we want to calculate. Click on *Calculate* and the result is given as 3 years.

Exercise Set 6. Use this calculator to solve the following.

Q1. How long, to the nearest month, will it take to earn \$1950 simple interest if \$16 325 is invested at 9.75% p.a.? Note: Make sure Time (T) is in months.

Q2. What sum, to the nearest dollar, must be invested for one year at 6% per annum simple interest in order to earn \$1200 interest?

Q3. Find the total value of an investment of \$3500 after 2 years and 6 months if simple interest is paid at the rate of 5% per annum. Remember to use $A = P + I$.

Q4. For how long must a principal of \$15 750 be invested at 9.8% p.a. for it to earn \$3087 in simple interest?

Q5. What principal would earn \$3729.60 in interest if invested for 3 years at 16.8% p.a.?

Q6. An amount of \$9020 was invested for 2 years and earned \$1731.84 in simple interest. Calculate the monthly interest rate.

Graphing Simple Interest

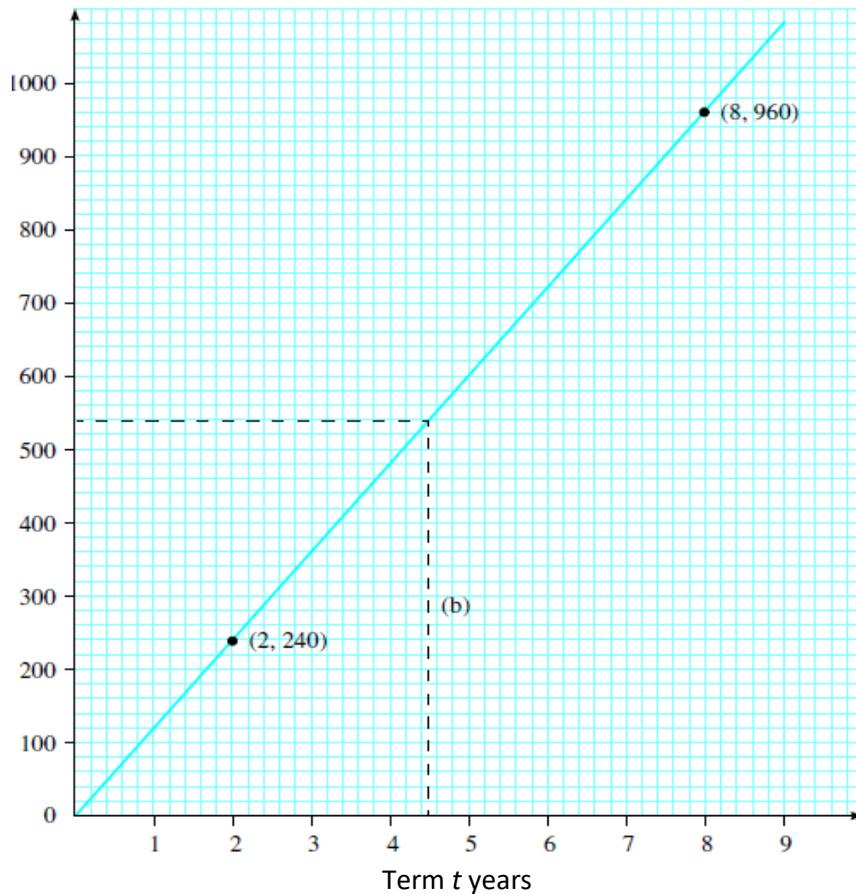
Example: Rachel invests \$2000 in an account that earns 6% p.a. simple interest. Construct a graph that shows the simple interest I earned in dollars over n years, for values of n from 0 to 8.

We use the formula $I = \frac{Prt}{100}$ and substitute $P = 2000$, $r = 6$, which gives $I = \frac{2000 \times 6 \times t}{100}$ which simplifies to $I = 120t$. This is a simple equation and we can construct a simple table to help graph it.

t (years)	0	2	8
I (\$)	0	240	960

We can use this table to produce the graph below.

Simple interest earned from an investment of \$2000 at 6% p.a.



The dotted line shows that after 4.5 years the money invested would earn \$540.

Exercise Set 7.

Q1. Use the simple interest graph above to estimate the interest earned when \$2000 is invested at 6% p.a. for:

- a) 9 years b) 2.5 years

Q2. What would the total value of the investment be after:

- a) 3 years b) 6 years?