Week 14
Term 2
2024

HAWKER COLLEGE

Engage | Inspire | Achieve

MA1
Shape and measurement

Goals

"Today's lesson is – hey, who wrote junk on the Maths board?"

This fortnight we are going to:

- review Pythagoras' Theorem and use it to solve practical problems in two dimensions and for simple applications in three dimensions

Theoretical Components

Resources:
PDF file: Week 14 Notes and Exercises
https://www.hawkermaths.com/uploads/1/1/7/0/11707964/4._geometry_shapes_and_solids.pdf

Knowledge Checklist

- Naming sides in a right-angled triangle
- Pythagoras' theorem
- Calculating the hypotenuse and shorter sides
- Applications of Pythagoras' theorem
- Using Pythagoras' theorem in 3-dimensions
- Common units in the metric system
- Perimeter
- Composite
- Area

Order

1. Read through the notes and examples
2. Work through the exercises
3. Complete the investigation at the end of the booklet.
4. Complete the reflection at the end of the booklet
5. Come and see your teacher and make sure you are up to date.

Practical Components

Work through the exercises and show the completed tasks to your teacher.

Be sure to ask for help as needed for the successful completion of all tasks.

Remember to regularly check Google Classroom for messages.

Investigation

Complete the task at the end of the booklet and submit your work for checking. 😊

QFO

Quiz/Forum/Other

Remember to check hawkermaths.com **for each week's learning brief.**

Make sure you have joined Google Classroom. If you have not, see your teacher.

MATHEMATICAL APPLICATIONS 1

WEEK 14 NOTES AND EXERCISES

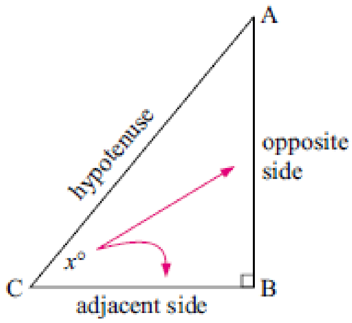
NAMING THE SIDES OF RIGHT-ANGLED TRIANGLES

A right-angled triangle is a triangle with one angle of 90° i.e. a right angle.

In a right-angled triangle, the side opposite the right angle is called the *hypotenuse.*

The other two sides are named in relation to the angle in question, x° (other letters can be used).

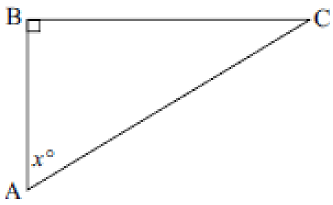
The side furthest away from the angle is called the *opposite* side. The remaining side, which is next to the angle, is called the *adjacent* side.



EXERCISE 1

1. Label the sides of the following triangles (hypotenuse, opposite, adjacent).

a)



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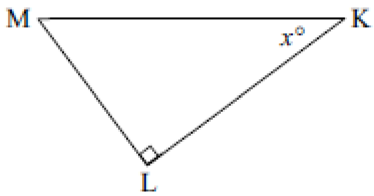

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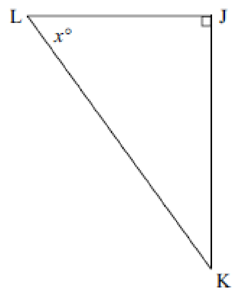


PYTHAGORAS' THEOREM

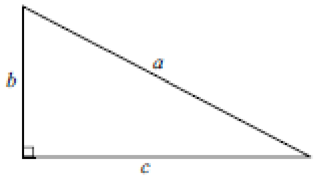
Pythagoras was a famous Greek mathematician and mystic but is now best known for his theorem about the sides of a triangle. He was born on Samos Island. It is believed that he was born about 580 BC and died about 500 BC.

When Pythagoras was a young man, he travelled to Egypt and Babylonia (Mesopotamia) where he learned much of his mathematics and developed an interest in investigating it further.

He founded a cult with the idea that 'the essence of all things is a number'. This group believed that all nature could be expressed in terms of numbers.

He is credited with the discovery now known as 'Pythagoras' theorem' which states that in a right-angled triangle, the sum of the squares of the the two shorter sides is equal to the square of the hypotenuse.

Remember: A right-angle is an angle of 90°. The hypotenuse is the side opposite the right-angle and is always the longest side.

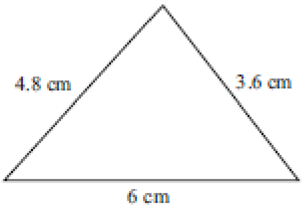


$$a^2 = b^2 + c^2$$

Thus, if we know the lengths of two sides we can calculate the length of the third side. We can also determine if a triangle is a right-angled triangle.

Example

Determine if the following triangle is right-angled and if so sketch and mark the right angle.



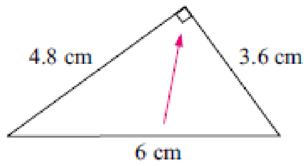
$$\text{Longest side, squared} = 6^2 = 36$$

$$\begin{aligned}\text{Sum the squares of the other two sides} &= 4.8^2 + 3.6^2 \\ &= 23.04 + 12.96 \\ &= 36\end{aligned}$$

$$\text{i.e.,} \quad 6^2 = 4.8^2 + 3.6^2$$

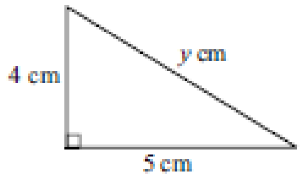
i.e., Pythagoras' theorem is true for this triangle, hence it is right-angled.

The right-angle is opposite the longest side, hence



EXAMPLE 1

Find the length of the hypotenuse in the given triangle.

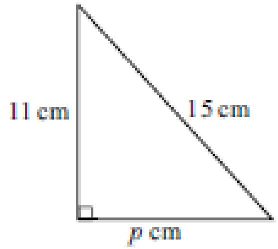


$$y^2 = 4^2 + 5^2$$
$$\therefore \quad y^2 = 16 + 25$$
$$\therefore \quad y^2 = 41$$
$$\therefore \quad y = \sqrt{41} = 6.4 \quad \text{(to 1 d.p.)}$$

i.e., the length of the hypotenuse is 6.4 cm.

EXAMPLE 2

Find the length of the unknown side in the given triangle.



Using Pythagoras' theorem in this triangle

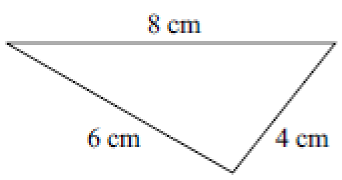
$$p^2 + 11^2 = 15^2$$
$$\text{i.e.,} \quad p^2 + 121 = 225$$
$$\therefore \quad p^2 = 225 - 121$$
$$\therefore \quad p^2 = 104$$
$$\therefore \quad p = \sqrt{104} = 10.2 \quad \text{(to 1 d.p.)}$$

i.e., the length of the unknown side is 10.2 cm.

EXERCISE 2

1. Determine which of these triangles are right-angled and if so, mark the angle accordingly.

a)



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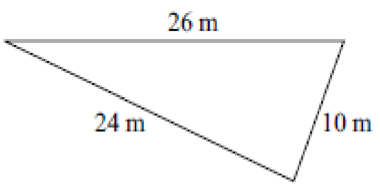

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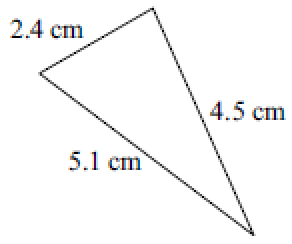
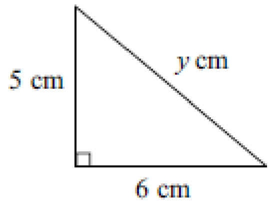


2. Find the length of the hypotenuse, correct to one decimal place.

a)



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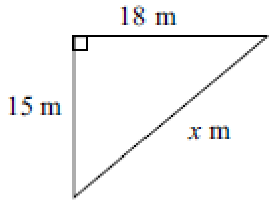

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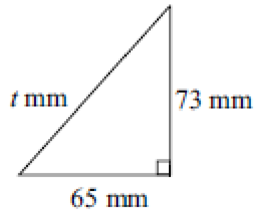
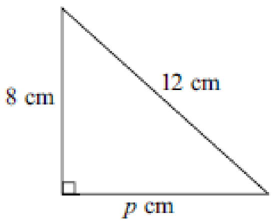


3. Find the length of the unknown side.

a)



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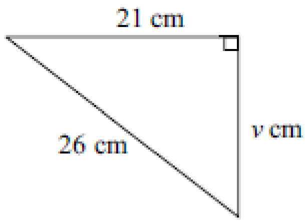

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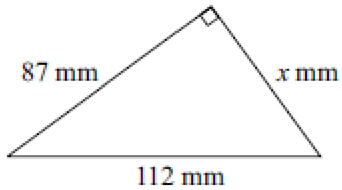
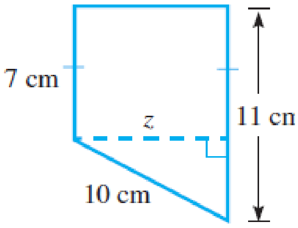


4. Find the value of the pronumeral in each of these figures, use 2 decimal places.

a)



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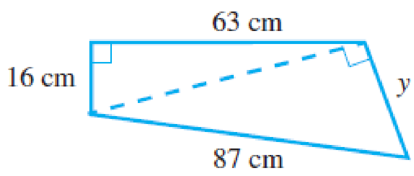


PYTHAGOREAN TRIADS

Pythagorean triads (or **Pythagorean triples**) are sets of 3 numbers which satisfy Pythagoras' theorem.

A right-angled triangle with side lengths of 3 cm, 4 cm and 5 cm satisfies Pythagoras' theorem, so the numbers 3, 4 and 5 form a Pythagorean triad or triple.

In fact, any multiple of these numbers, for example, 6, 8 and 10, forms a Pythagorean triad.

Some other triads are: 5, 12, 13 and 8, 15, 17 and 9, 40, 41 as the first two numbers squared and added equals the third number squared.

EXERCISE 3

1. Which of the following are Pythagorean triads?

a) 9, 12, 15 b) 4, 5, 6 c) 30, 40, 50

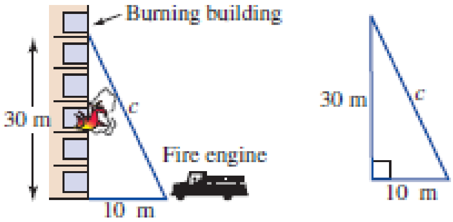
d) 14, 20, 30 e) 10, 24, 26 f) 12, 16, 20

2. Complete the following triads. Assume the numbers are in ascending order.

a) 9, ______, 15 b) _____, 24, 25 c) 11, 60, _____

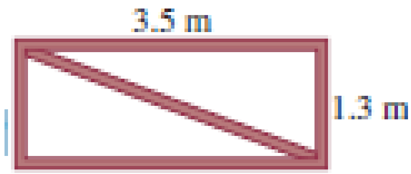
USING PYTHAGORAS

The fire brigade attends a blaze in a tall building. They need to rescue a person from the 6th floor of the building, which is 30 metres above ground level. Their ladder is 32 metres long and must be at least 10 metres from the foot of the building. Can the ladder be used to reach the people needing rescue?

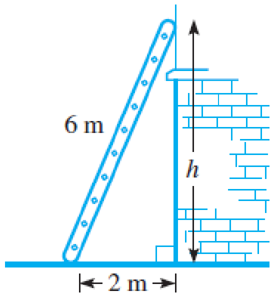
THINK	WRITE
1 Draw a diagram and show all given information.	
2 Write the formula after deciding if you are finding the hypotenuse or a shorter side.	$\text{Hypotenuse}^2 = \text{base}^2 + \text{height}^2$
3 Substitute the lengths of the known sides.	$c^2 = 10^2 + 30^2$
4 Evaluate the expression.	$= 100 + 900$ $= 1000$
5 Find the answer by taking the square root.	$c = \sqrt{1000}$ $= 31.62$ m
6 Give a written answer.	The ladder will be long enough to make the rescue, since it is 32 m long.

EXERCISE 4

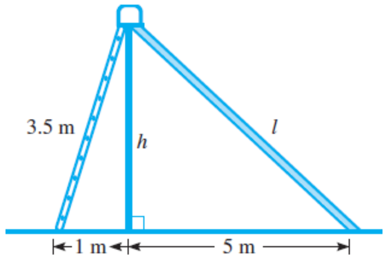
1. A rectangular gate is 3.5 m long and 1.3 m wide. The gate is to be strengthened by a diagonal brace as shown at right. How long should the brace be (correct to 2 decimal places)?



2. A 6 m ladder leans against a house so that its base is 2 m out from the bottom of the house. How far up the house does the ladder reach (to the nearest centimetre)?



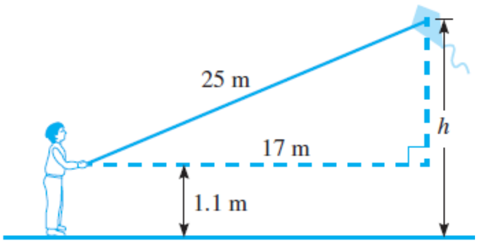
3. A playground slide is made up of two right triangles. Find, correct to the nearest centimetre:



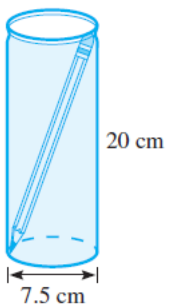
(a) *h*, the height of the slide

(b) *l*, the length of the slide

4. This diagram shows a boy flying a kite. How high is the kite above the ground (correct to 1 decimal place)?



5. Jackie wants to use an old tennis-ball can as a pencil case. If this can has a diameter of 7.5 cm and a height of 20 cm, what is the length of the longest pencil that will fit inside the can (to the nearest millimetre)?

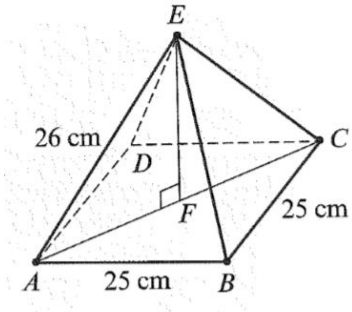


PYTHAGORAS IN THREE DIMENSIONS

We can extend the application of Pythagoras' Theorem into 3 dimensions. These questions always involve at least two steps.

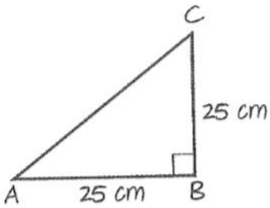
Example 1

Find the height EF of the square pyramid shown.



This needs to be done using two steps.

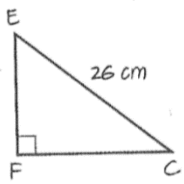
First, find the length of the diagonal AC. To do this use triangle ABC.



$$AC^2 = AB^2 + BC^2$$
$$\therefore AC = \sqrt{25^2 + 25^2}$$
$$= 35.355\ldots$$

Then use triangle EFC to find the height. To find length FC the length AC is halved.

$$FC = \frac{AC}{2}$$
$$= \frac{35.36}{2}$$
$$= 17.68 \text{ cm}$$



Use Pythagoras' theorem

$$EF^2 = EC^2 - FC^2$$
$$\therefore EF = \sqrt{26^2 - 17.68^2}$$
$$= 19.065\ldots$$

The height EF is 19.1m.

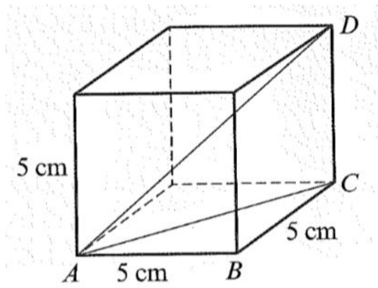
Example 2

The cube on the right has sides of length 5cm.

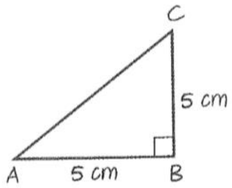
Find the length:

a) AC

b) AD



To find AC use triangle ABC.



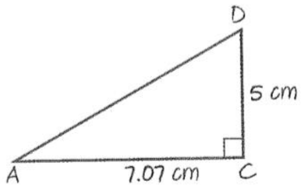
Using Pythagoras' theorem

$\therefore AC = \sqrt{5^2 + 5^2}$

$= 7.071\ldots$

The length AC is 7.1cm

To find AD use triangle ACD.



$AD^2 = AC^2 + CD^2$

$\therefore AD = \sqrt{7.07^2 + 5^2}$

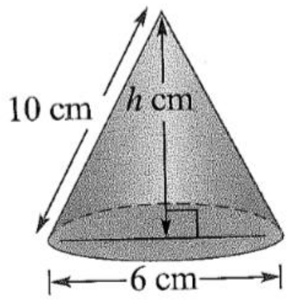
$= 8.659\ldots$

The length of AD is 8.7cm

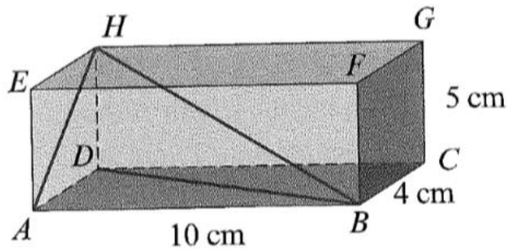
AD is the 'longest' diagonal in the cube. An interpretation could be….the longest rod that would fit in the box
is 8.7cm long.

EXERCISE 5

1. Calculate the height of this cone (correct to 2 decimal places).



2. For this cuboid, calculate the lengths (correct to 2 decimal places):

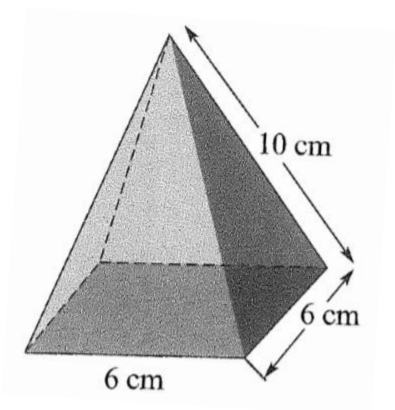


a) DB

b) BH

c) AH

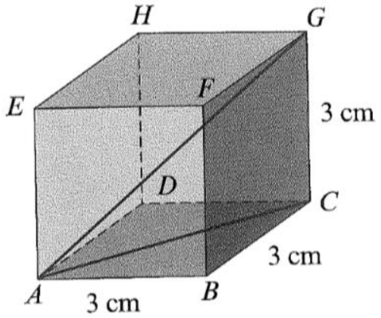
3. For the square-based pyramid shown, find (correct to 2 decimal places):



a) The length of the diagonal of the base.

b) The height of the pyramid.

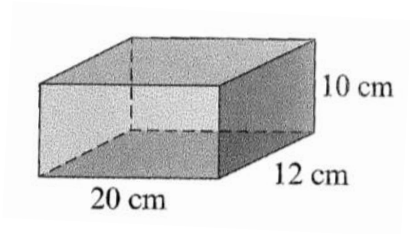
4. For the cube shown, find: (correct to 2 decimal places):



a) AC

b) AG

5. Chris wants to use a rectangular pencil box. What is the length of the longest pencil that would fit inside the box shown (correct to 2 decimal places)?

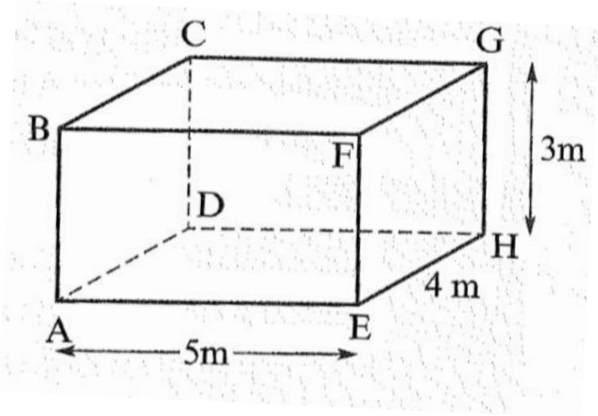


6. In a primate enclosure at the zoo, a rope is to be attached from the bottom corner of the enclosure to the opposite top corner for the monkeys to swing and climb on. If the enclosure measures 8m by 10m by 12m, what is the length of the rope (correct to 2 decimal places). Make sure you draw a diagram.

WEEK 14 INVESTIGATION

A new home entertainment system needs to set in Room with dimensions 4m × 5m × 3m as shown in the diagram. Expensive cabling is used to wire

the room from corner A to corner G.



a) What is the length of cabling required to
Go from A to E to H to G?

b) What length of cabling is required to go from A to F to G?

c) What length of cabling is required to go from A to E to G?

d) If cabling costs $25.50 per metre, which is the cheaper option?

MARKING RUBRIC

CRITERIA	EXPECTATIONS	POSS	MULT	GIVEN	TOTAL
Practical	Student completes practical work of the brief to an acceptable standard set by the teacher.	**2**	**3**		**/6**
Portfolio Task	Student completes the portfolio task of the brief to an acceptable standard set by the teacher.	**2**	**2**		**/4**
Reasoning and communications	Student responses are accurate and appropriate in presentation of mathematical ideas in different contexts, with clear and logical working out shown.	**4**	**-**		**/4**
Concepts and techniques	Student submitted work selects and applies appropriate mathematical modelling and problem solving techniques to solve practical problems, and demonstrates proficiency in the use of mathematical facts, techniques and formulae.	**4**	**-**		**/4**
	Submission Guidelines				
Timeliness	Student submits the exercises and portfolio task by the set deadline. See scoring guidelines for specific details.	**2**	**-**		**/2**
		FINAL			**/20**

Student Reflection:
How did you go with this week's work?

What was interesting?

What did you find easy?

What do you need to work on?