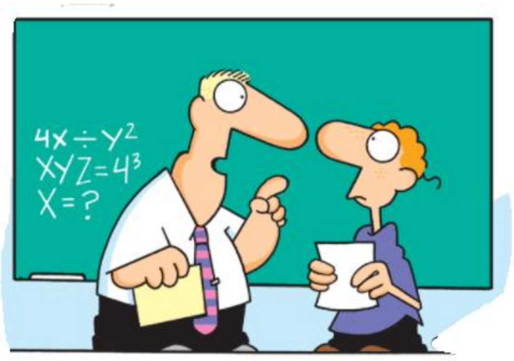
Week 10
Term 1
2022

Learning Brief

MA1
Consumer Arithmetic, Algebra & Matrices, Shape & Measurement

Goals



"Algebra will be important to you later in life because there's going to be a test six weeks from now!"

Unit goals

- Understand the concepts and techniques introduced in consumer arithmetic, algebra and matrices, and shape and measurement.
- Apply reasoning skills and solve practical problems .

This week the work is on

- Simplifying algebraic expressions
- Expanding (removing brackets)
- Substitution

Theoretical Components

Resources:
For this week the theory work is in the *PDF file:* Week 10

Knowledge Checklist

- Terminology associated with algebra; expression, variable, coefficient, terms
- Rules for algebraic expressions
- Collecting like terms
- Common factors
- Distributive Law
- Replacing variables with numbers
- Working with negative numbers
- What is a formula
- Using appropriate units
- Using formula in medicine
- How equations differ from expressions
- Techniques used in solving equations

Practical Components

There are questions to be answered in the booklet *Week 10*

Order

1. Look at the Investigation.
2. Complete the questions in the *Booklet*
3. Complete the Investigation.
4. Show your completed booklet to Toby/Serene and submit the Investigation for marking.

Investigation

See the end of the brief ☺

On-line Quiz

There is a required Mathspace Task.

MATHEMATICAL APPLICATIONS 1

Week 10 Review of Algebra

As in most areas of mathematics, algebra makes use of words that have a specific meaning. Some of these are;

Expression: A mathematical expression is any calculation or formula that involves a combination of numbers and/or variables, as well as operators eg $4x$ and $A = 2\pi r$ are both expressions.

Variable: A symbol for a number we don't know yet. It is usually a letter like x or y. Example: in $x + 2 = 6$, x is the variable.

Coefficient: The numbers in front of the variable eg the 4 in $4x$

Constant terms: Are terms that do not have a variable.

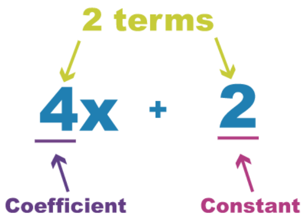
Like terms: Like terms are terms which have exactly the same variable factors eg $2x$ and $4x$ are like terms as they both contain x and b^2 and $\frac{1}{2}b^2$ are like terms as they both contain b^2

Note: Algebraic terms must have the EXACT SAME combination of variables to be like terms. $9ef^2$ and $10ef^2$ are like terms because they both have variables of e and f^2

xy and xy^2 ARE NOT like terms, however, because the powers of y are different between the two terms.

Simplify: To simplify an expression means to collect the like terms and then write as simply as possible.

This picture summarises some of this terminology:



There are a few conventions that we make when writing algebraic expressions

- When we multiply two numbers together, we use a multiplication sign, such as 2×3. When we multiply a number by a variable, or when we multiply variables together, we leave out the multiplication sign. So 2 times $2 \times y$ is written as $2y$, for example, and $a2 \times b$ is written as a^2b.
- When we multiply a number by one or more variables, we write the number first and then the variables. For example, $p\times3\times q$ would be written as $3pq$.
- If we multiply one or more variables by 1, we can leave off the 1. For example, instead of writing $1x$ or $1\times x$, we can just write x.
- We usually avoid using the division symbol, and instead write division using fractions. So instead of writing $12\div t$, we would write $\frac{12}{t}$. This helps to avoid confusion about the order of operations in an expression.
- If we multiply a variable by itself, we usually simplify the expression by using an exponent. So if we have the expression $m\times m\times m$, we would write m^3 instead of mmm.

Examples

1.

In the algebraic expression $p - 5qr + 7$ identify the following:

a the number of terms.

b the coefficient of the term containing the variable p.

c the constant term.

Solution

a A term can be a number or a number and one or more pronumerals multiplied together. There are three terms in this expression.

b A coefficient is the number that appears before a pronumeral within the same term. There is no number written in front of the pronumeral p. The coefficient is 1.

c A constant is a term that contains a number only, there is no pronumeral. The constant term is 7.

2. Simplify $8x + 2y - 5x - 3y$

Rearrange the expression to group the like terms; $8x - 5x + 2y - 3y$

Collect the like terms: $3x - y$

3. Simplify $2b \times 6a \times 4c$

Multiply the numbers and pronumerals respectively: $48 \times b \times a \times c$

Write in simplest form; $48abc$

4. Simplify $28xy \div 21yz$

Write as a fraction $\frac{28xy}{21yz}$

Both 28 and 21 are divisible by 7, so $\frac{28}{21}$ becomes $\frac{4}{3}$ and $\frac{y}{y} = 1$

This gives $\frac{28xy}{21yz} = \frac{4x}{3z}$

Note: $\frac{x^2}{x} = x$ and $\frac{c}{c^2} = \frac{1}{c}$

Exercise Set 1

Q1. For the algebraic expression $3m - 4n + 6$ identify;

a) the number of terms

b) the coefficient of the term with variable m

c) the constant term.

Q2. Simplify

a) $3x - 2y + 6x - y$

b) $4ab - 6a - 9ab$

c) $7b - 9a + 3b - 4$

d) $8xy + 3x - 4xy - 6$

e) $15x - 15 + 3x + 5$

f) $3de - 6ed + 2d$

g) $-5 + 7u - 10 - 3u$

h) $3r + 4ar - 2ar - 7r$

Q2. Simplify

a) $3x \times 5y$

b) $-4a \times 2bc$

c) $12mn \times -3n$

d) $-5xy \times 3yz$

e) $4ak \times 4am$

f) $8cd \times -3d$

g) $10ab \times 2ab$

h) $-4bc \times -3cd$

Q3. Simplify

a) $\frac{12xy}{3y}$

b) $\frac{-30mn}{6np}$

c) $\frac{48st}{16rt^2}$

d) $\frac{14yx^2}{2xy}$

e) $\frac{27pr}{-9pq}$

f) $\frac{-4de}{20e}$

g) $\frac{-32abc}{-4bc^2}$

h) $\frac{45yx^2}{9x}$

Q4. If $5x^2 - 10x - 3x^2 - 12x = px^2 + qx$, find the values of p and q

Expanding and Simplifying Algebraic Expressions

Expanding an algebraic expression means that the grouping symbols (brackets) are removed. The distributive law is used which means that every term in the brackets is multiplied by the term outside the brackets.

$$a(b+c) = a \times b + a \times c$$
$$= ab + ac$$

Remember that multiplication signs are omitted in algebra. $a \times b$ means ab

Example

Expand $-5t(3t + 8)$

Using the distributive law gives $-5t \times 3t + -5t \times 8$

which simplifies to $-15t^2 - 40t$

Exercise Set 2

Q1. Expand and simplify

a) $r(r + 5)$

b) $m(1 - m)$

c) $6y(5y - 10)$

d) $2y(7x + 4y)$

e) $y(2y + 3 - y^2)$

f) $3h(h - 7e - 4eh)$

Example

Expand and simplify $3(a - 2b) - 5(2a + b)$

Expand each bracket $3a - 3 \times 2b \;\; -5 \times 2a \; -5 \times b$

Rearrange so like terms are together $3a - 10a - 6b - 5b$

Simplify $-7a - 11b$

Q2. Expand and simplify

a) $5(x + 4) – 2(x + 3)$

b) $3(d – 4) – 2(d + 5)$

c) $(2x – 4) – 5(3x + 5)$

d) $3b(b + 5) – b(b + a)$

e) $4w(w – 7) – w(w + 1)$

f) $2(a – b) + 2(b + a)$

g) $e(3e + 5) – (2e – e^2)$

g) $p(p – q) – q(q – p)$

Substitution

Substitution occurs when we substitute numbers into equations in place of variables to determine a final value. We can substitute in any kinds of numbers, including whole numbers, decimals and fractions.

Example

Evaluate $6x – 4$ if $x = 3$. This means that everywhere the letter x has been written, we will replace it with the number 3.

$6x – 4 = 6\times 3 – 4 = 18 – 4 = 14$

The same process applies even if there is more than one unknown value. Evaluate

(i) $2pq - r$ if $p = 2$, $q = -3$ and $r = 8$

$(2pq – r = 2 \times 2 \times -3 – 8 = -12 – 8 = -20$

(ii) $5p(3q – qr)$

$5p(3q – qr) = 15pq – 5pqr = 15 \times 2 \times -3 – (5 \times 2 \times -3 \times 8) = -90 – (-240) = -90 + 240 = 150$

Exercise Set 3

Q1. If m = 6, n = 3 and p = -4 evaluate the algebraic expressions.

a) $4nm$

b) $2np - m$

c) $p^2 + mn$

d) $3mn - 2p$

e) $2n^2 + 3mp$

f) $-5p + 2mn$

g) $24mp - n$

g) $p^2 - n^2 + m^2$

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Substitution in Formula

A **formula** is a rule which describes a relationship between variables. For example: Area = length × width is the formula used to calculate the area of a rectangle. It is usually written as $A = lw$. In this case A is the subject of the formula. If we know l and w, we can find the area by substitution.

Example From a height h above sea level, an observer can see a distance of d km to the horizon, where

$$d = 8\sqrt{\frac{h}{5}}$$

What distance to the nearest km, can be seen from a tower 128 m above sea level.

$d = 8\sqrt{\frac{h}{5}} = 8\sqrt{\frac{128}{5}} = 40.4771....$ = 40 km to the nearest km.

Exercise Set 1

Note: Make sure you use appropriate units as part of your answer.

Q1. Calculate the volume, correct to two decimal places, of a cylinder with a base radius of 4.07 cm and a perpendicular height of 11.58 cm. Use the formula $V = \pi r^2 h$

Q2. The temperature T (in ℃) of the water in a kettle t minutes after it is switched on is given by the formula
$T = 18t + 28$. Find the temperature of the water;

a) 4 minutes after it is turned on

b $1\frac{1}{2}$ minutes after it is turned on

c) When the kettle is first turned on.

Q3. The volume of a sphere with radius r is $V = \frac{4}{3}\pi r^3$. Calculate, correct to one decimal place, the volume of the sphere with radius 14.5 cm.

Q4. The time, T seconds, it takes a swing to go back and forth once is $T = 2\pi\sqrt{\frac{l}{g}}$, where l m is the length of the swing and g is the gravitational acceleration. Find T, correct to two decimal places, if $l = 2.35$ and $g = 10$.

Q5. The speed, V m/s, required for a spacecraft to escape the Earth's gravitational pull during take-off is $V = \sqrt{2gr}$ where g is 9.8 m/s^2 and r is the radius of the Earth (6378000 m). Calculate the escape speed of a spacecraft leaving the Earth's atmosphere.

Using Formulae in Medicine

Nurses regularly use formulas in their work. They calculate such things as:

- Quantities to mix to make a solution for cleaning wounds and infections
- The amount of liquid to put in an injection
- **The number of tablets or capsules to give a patient.**

Example: To solve the following problem, use the formula:

Amount required = $\frac{R}{A} \times v$

Alan is suffering from severe kidney stone pain. His doctor prescribes 75 mg of pethidine. The ampoules of pethidine contain 100 mg (A) per 2 mL (*v*). Calculate the number of millilitres needed for the injection.

Amount required = $\frac{R}{A} \times v$

Substitute the values from the question: R = 75, A = 100, *v* = 2

Amount required = $\frac{75}{100} \times 2$

= 1.5 mL

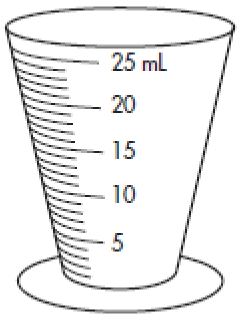
Exercise Set 2

Q1. Ten-year-old Claudia is suffering from nasal and sinus congestion. She is prescribed 12 mg of Demazin syrup three times per day. Demazin syrup contains 3.75 mg (A) in every 5 mL (*v*).

Amount required = $\frac{R}{A} \times v$

a) How many millilitres must Claudia be given in each dose?

b) Mark on this diagram the volume of Demazin Claudia must be given.

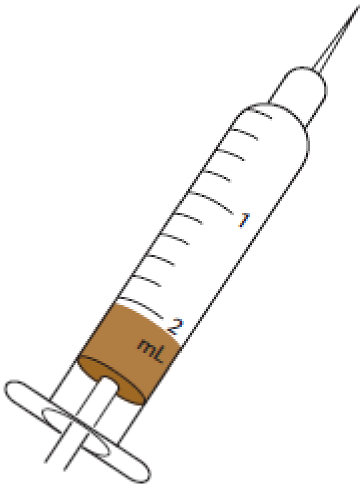


c) Claudia has the first dose at 1015 hours. When can she have the next dose?

Q2. To help thin Kim's blood after a blood clot the doctor prescribes 15000 units ® of Heptin. Each ampoule contains 25000 units (A) in 1 mL (*v*).

a) Using the formula as before calculate the dose Kim requires.

b) Mark on the diagram the level of Heptin Kim requires.



Q3. Fried's rule can be used to calculate an infant's dose of an adult's medicine.

Dosage = $\frac{Age\ of\ infant\ (months) \times adult\ dosage}{150}$

The adult dose of a medicine is 20 mL. Calculate the dosage for these children.

a) Cathy, aged 18 months

b) Todd, aged 2 years

Q4. The medical researchers, Young and Clark, developed rules for approximating the children's dose of adult's medicine.

Young's rule: Dosage = $\frac{Age\ of\ child\ (years) \times adult\ dose}{Age\ of\ child\ (years) + 12}$

Clark's rule: Dosage = $\frac{Weight\ of\ child\ (kg) \times adult\ dose}{70}$

Emma is 8 years old and she weighs 35 kg. She is short and heavy for her age. The adult dose of the medicine her mother wants to give her is 20mL.

a) How much medicine should Emma have according to;

(i) Young's rule

(ii) Clark's rule

b) The rules recommend Emma receive different amounts. Do you think Emma's mother should use the formula based on age or weight? Why?

Q5. A boy's height on his second birthday can be used to predict his adult height. Predicted adult height (cm) = 1.75 × *h* + 20 where *h* = boy's height on 2nd birthday.

On his second birthday Thomas is 88 cm tall. Predict Thomas's adult height.

Solving Equations

An equation is a mathematical sentence. Equations always include an equal sign. In our previous work we simplified algebraic expressions but now we will **solve** equations.

Eg $3w - 4$ is an algebraic expression, however, $3w - 4 = 6$ is an algebraic equation.

Example Solve the equation $2x - 6 = 14$

Write the equation.	$2x - 6 = 14$
Add 6 to both sides.	$2x - 6 + 6 = 14 + 6$
Simplify both sides.	$2x = 20$
Divide both sides by 2.	$\frac{2x}{2} = \frac{20}{2}$
Simplify to find the value of *x*.	$x = 10$

Solve the equation $\frac{x}{3} + 2 = 7$

Write the equation.	$\frac{x}{3} + 2 = 7$
Subtract two from both sides.	$\frac{x}{3} + 2 - 2 = 7 - 2$
Simplify both sides.	$\frac{x}{3} = 5$
Multiply both sides by 3.	$\frac{x}{3} \times 3 = 5 \times 3$
Simplify to find the value of *x*.	$x = 15$

Solve the equation $\frac{2a\ 6}{5} = 4$

Write the equation.	$\frac{2a-6}{5}=4$
Multiply both sides by 5.	$\frac{2a-6}{5}\times 5=4\times 5$
Simplify both sides.	$2a-6=20$
Add 6 to both sides.	$2a-6+6=20+6$
Simplify both sides.	$2a=26$
Divide both sides by 2.	$\frac{2a}{2}=\frac{26}{2}$
Simplify to find the value of *a*.	$a=13$

Exercise Set 3

Q1. Solve these equations

a) $3d + 2 = 20$

b) $2p – 3 = 2$

c) $4x + 2 = 22$

d) $28 – 5x = 3$

e) $\frac{3h}{9} = 9$

f) $\frac{r-1}{6} = 2$

g) $12 + 2w = -18$

h) $\frac{z}{3} – 11 = 9$

i) $10 - 3a = 16$

j) $\frac{8-2b}{2} = 7$

Solving Equations With Pronumerals on Both Sides

Example Solve $2a - 5 = a + 7$

Write the equation.	$2a - 5 = a + 7$
Subtract a from both sides.	$2a - 5 - a = a + 7 - a$
Simplify both sides.	$a - 5 = 7$
Add 5 to both sides.	$a - 5 + 5 = 7 + 5$
Simplify to find the value of a.	$a = 12$

Exercise Set 4

Q1. Solve these equations.

a) $2x - 5 = x + 4$

b) $5k - 13 = 3k + 9$

c) $2(m - 4) = m + 12$

d) $\frac{2a-8}{5} = 2a$

Q2. Change these sentences into equations and solve.

a) Twice the number less 6 equals the sum of the number and 4.

b) Eight times the number plus 7 equals 14 less than 5 times the number.

c) Three times the difference between the number and 1 equals twice the number plus 8.

2022 MA1 Week 10 Investigation

Q1. Subtract the sum of $2x^2 - 3(x-1)$ and $2x + 3(x^2 - 2)$ from the sum of $5x^2 - (x-2)$ and $x^2 - 2(x+1)$.

Q2. The volume of a cone with base radius *r* and perpendicular height *h* is

$$V = \frac{4}{3}\pi r^2 h$$

If a cylinder has a volume of 256 cm^3 and a radius of 7 cm, find its height correct to two decimal places.

Q3. The mean, *M*, of three numbers *x, y* and *z* is calculated using the formula

$$M = \frac{x+y+z}{3}$$

If three numbers have a mean of 17 and two of the numbers are 10 and 20, find the third number.

MARKING RUBRIC **Week 10**

CRITERIA	EXPECTATIONS	POSS	MULT	GIVEN	TOTAL
Practical	Student completes practical work, including exercises and Mathspace task, of the brief to an acceptable standard set by the teacher.	**2**	**3**		**/6**
Investigation Task	Student completes the investigation task of the week to an acceptable standard set by the teacher.	**2**	**2**		**/4**
Communication and Reasoning	Student responses are accurate and appropriate in presentation of mathematical ideas in different contexts, with clear and logical working out shown.	**4**	**-**		**/4**
Knowledge and Application	Student submitted work selects and applies appropriate mathematical modelling and problem solving techniques to solve practical problems and demonstrates proficiency in the use of mathematical facts, techniques and formulae.	**4**	**-**		**/4**
	Submission Guidelines				
Timeliness	Student submits the exercises, Mathspace task and investigation by the set deadline. See scoring guidelines for specific details.	**2**	**-**		**/2**
				FINAL	**/20**

udent Reflection: How did you go with this week's work? What was interesting? What did you
nd easy? What do you need to work on?