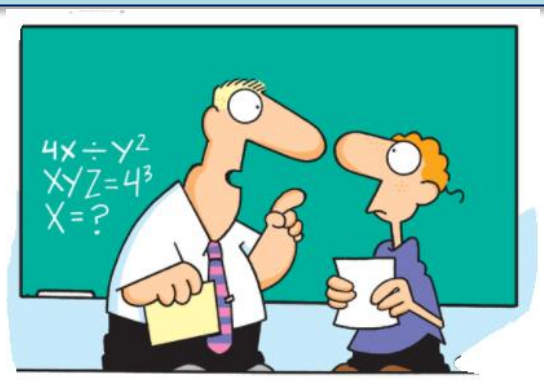
Week 1/2
Term 1
2024

HAWKER COLLEGE
Engage | Inspire | Achieve

MA1
Algebra and matrices

Goals



"Algebra will be important to you later in life because there's going to be a test six weeks from now!"

This fortnight we are going to:

- substitute numerical values into linear algebraic and simple non-linear algebraic expressions, and evaluate
- find the value of the subject of the formula, given the values of the other pronumerals in the formula

Theoretical Components

Resources:
PDF file: Week 1/2 Notes and Exercises

Knowledge Checklist

- Terminology associated with algebra; expression, variable, coefficient, terms
- Rules for algebraic expressions
- Collecting like terms
- Common factors
- Distributive Law
- Replacing variables with numbers
- Working with negative numbers
- What is a formula
- Using appropriate units
- How equations differ from expressions
- Techniques used in solving equations

Order

1. Read through the notes and examples
2. Work through the exercises
3. Complete the investigation at the end of the booklet.
4. Complete the reflection at the end of the booklet
5. Come and see your teacher and make sure you are up to date.

Practical Components

Work through the exercises and show the completed tasks to your teacher.

Be sure to ask for help as you need for the successful completion of all tasks.

Remember to regularly check Google Classroom for messages.

Investigation

Complete the task at the end of the booklet and submit your work for checking. ☺

Other

Remember to check hawkermaths.com **for each week's learning brief.**

Make sure you have joined Google Classroom. If you have not, see your teacher.

MATHEMATICAL APPLICATIONS 1

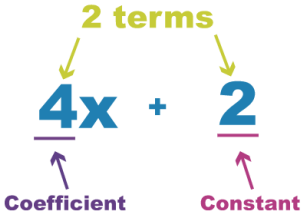
WEEK 1/2 NOTES AND EXERCISES

REVIEW OF ALGEBRA

As in most areas of mathematics, algebra makes use of words that have a specific meaning. Some of these are;

- **Expression:** A mathematical expression is any calculation or formula that involves a combination of numbers and/or variables, as well as operators eg $4x$ is an algebraic expression.
- **Equation:** An equation is a mathematical sentence. Equations always include an equal sign. We can simplify algebraic expressions but we **solve** equations. Eg. $3w - 4$ is an algebraic expression, however, $3w - 4 = 6$ is an algebraic equation.
- **Variable:** A symbol for a number we don't know yet. It is usually a letter like x or y. Example: in $x + 2 = 6$, x is the variable.
- **Coefficient:** The numbers in front of the variable eg the 4 in 4*x*
- **Constant terms:** Are terms that do not have a variable.
- **Like terms:** Like terms are terms which have exactly the same variable factors eg 2*x* and 4*x* are like terms as they both contain *x* and b^2 and $\frac{1}{2}b^2$ are like terms as they both contain b^2
    - Note: Algebraic terms must have the EXACT SAME combination of variables to be like terms. $9ef^2$ and $10ef^2$ are like terms because they both have variables of *e* and f^2
    - xy and xy^2 ARE NOT like terms, however, because the powers of *y* are different between the two terms.
- **Simplify:** To simplify an expression means to collect the like terms and then write as simply as possible.

- This picture summarises some of this terminology:



There are a few conventions that we make when writing algebraic expressions

- When we multiply two numbers together, we use a multiplication sign, such as 2×3. When we multiply a number by a variable, or when we multiply variables together, we leave out the multiplication sign. So $2 \times y$ is written as $2y$, for example, and $a^2 \times b$ is written as a^2b.
- When we multiply a number by one or more variables, we write the number first and then the variables. For example, $p \times 3 \times q$ would be written as $3pq$.
- If we multiply one or more variables by 1, we can leave off the 1. For example, instead of writing $1x$ or $1 \times x$, we can just write x.
- We usually avoid using the division symbol, and instead write division using fractions. So instead of writing $12 \div t$, we would write $\frac{12}{t}$. This helps to avoid confusion about the order of operations in an expression.
- If we multiply a variable by itself, we usually simplify the expression by using an exponent. So if we have the expression $m \times m \times m$, we would write m^3 instead of mmm.

Example 1

In the algebraic expression $p - 5qr + 7$ identify the following:

a the number of terms.

b the coefficient of the term containing the variable p.

c the constant term.

Solution

a A term can be a number or a number and one or more pronumerals multiplied together.	There are three terms in this expression.
b A coefficient is the number that appears before a pronumeral within the same term.	There is no number written in front of the pronumeral p. The coefficient is 1.
c A constant is a term that contains a number only, there is no pronumeral.	The constant term is 7.

Example 2

Simplify	$8x + 2y - 5x - 3y$
Rearrange the expression to group the like terms	$8x - 5x + 2y - 3y$
Collect the like terms	$3x - y$

Example 3

Simplify	$2b \times 6a \times -4c$
Multiply the numbers and pronumerals respectively	$-48 \times b \times a \times c$
Write in simplest form	$-48abc$

Example 4

Simplify	$28xy^2 \div 21yz$
Write as a fraction	$\frac{28xy^2}{21yz}$

Both 28 and 21 are divisible by 7, so $\frac{28}{21}$ becomes $\frac{4}{3}$ and $\frac{y}{y} = 1$

This gives $\frac{28xy^2}{21yz} = \frac{4xy}{3z}$

Note: $\frac{x^2}{x} = x$ and $\frac{c}{c^2} = \frac{1}{c}$

EXERCISE 1

1. For the algebraic expression $3m - 4n + 6$ identify;

a) the number of terms

b) the coefficient of the term with variable m

c) the constant term.

2. Simplify:

a) $3x - 2y + 6x - y$

b) $4ab - 6a - 9ab$

c) $7b - 9a + 3b - 4$

d) $8xy + 3x - 4xy - 6$

e) $15x - 15 + 3x + 5$

f) $3de - 6ed + 2d$

g) $-5 + 7u - 10 - 3u$

h) $3r + 4ar - 2ar - 7r$

3. Simplify:

a) $3x \times 5y$

b) $-4a \times 2bc$

c) $12mn \times -3n$

d) $-5xy \times 3yz$

e) $4ak \times 4am$

f) $8cd \times -3d$

g) $10ab \times 2ab$

h) $-4bc \times -3cd$

4. Simplify:

a) $\frac{12xy}{3y}$

b) $\frac{-30mn}{6np}$

c) $\frac{48st}{16rt^2}$

d) $\frac{14yx^2}{2xy}$

e) $\frac{27pr}{-9pq}$

f) $\frac{-4de}{20e}$

g) $\frac{-32abc}{-4bc^2}$

h) $\frac{45yx^2}{9x}$

5. If $5x^2 - 10x - 3x^2 - 12x = px^2 + qx$, find the values of p and q

EXPANDING AND SIMPLIFYING ALGEBRAIC EXPRESSIONS

Expanding an algebraic expression means that the grouping symbols (brackets) are removed. The distributive law is used which means that every term in the brackets is multiplied by the term outside the brackets.

$$a(b+c) = a \times b + a \times c$$
$$= ab + ac$$

Remember that multiplication signs are omitted in algebra. $a \times b$ means ab

Example 1

Expand	$-5t(3t + 8)$
Using the distributive law gives	$-5t \times 3t + -5t \times 8$
which simplifies to	$-15t^2 - 40t$

Example 2

Expand and simplify	$3(a - 2b) - 5(2a + b)$
Expand each bracket	$3a - 3 \times 2b \ -5 \times 2a + -5 \times b$
Rearrange so like terms are together	$3a - 10a - 6b - 5b$
Simplify	$-7a - 11b$

EXERCISE 2

1. Expand and simplify

a) $r(r + 5)$

b) $m(1 - m)$

c) $6y(5y - 10)$

d) $2y(7x + 4y)$

e) $y(2y + 3 - y^2)$

f) $3h(h - 7e - 4eh)$

2. Expand and simplify

a) $5(x + 4) - 2(x + 3)$

b) $3(d - 4) - 2(d + 5)$

c) $(2x - 4) - 5(3x + 5)$

d) $3b(b + 5) - b(b + a)$

e) $4w(w - 7) - w(w + 1)$

f) $2(a - b) + 2(b + a)$

g) $e(3e + 5) - (2e - e^2)$

g) $p(p - q) - q(q - p)$

3. State two possible algebraic expressions, with brackets, which expand to:

a) $12a - 6b$

b) $24mn + 36n^2$

SUBSTITUTION

Substitution occurs when we substitute numbers into equations in place of variables to determine a final value. We can substitute in any kinds of numbers, including whole numbers, decimals and fractions.

Example 1

Evaluate $6x - 4$ if $x = 3$.

This means that everywhere the letter x has been written, we will replace it with the number 3.

$$\begin{aligned} & 6x - 4 \\ = \; & 6 \times 3 - 4 \\ = \; & 18 - 4 \\ = \; & 14 \end{aligned}$$

The same process applies even if there is more than one unknown value.

Example 2

If $p = 2, q = -3 \; and \; r = 8$, evaluate:

(i) $2pq - r$

$$\begin{aligned} & 2pq - r \\ = \; & 2 \times 2 \times -3 - 8 \end{aligned}$$ this can be written as $2(2)(-3) - 8$

$$\begin{aligned} = \; & -12 - 8 \\ = \; & -20 \end{aligned}$$

(ii) $5p(3q - qr)$

$$\begin{aligned} & 5p(3q - qr) \\ = \; & 15pq - 5pqr \\ = \; & 15(2)(-3) - 5\,(2)(-3)(8) \\ = \; & -90 - (-240) \\ = \; & -90 + 240 \\ = \; & 150 \end{aligned}$$

A **formula** is a rule which describes a relationship between variables. For example: $Area = length \times width$ is the formula used to calculate the area of a rectangle. It is usually written as $A = lw$. In this case A is the subject of the formula. If we know l and w, we can find the area by substitution.

Example 3

From a height *h* above sea level, an observer can see a distance of *d* km to the horizon, where $d = 8\sqrt{\frac{h}{5}}$. What distance to the nearest km, can be seen from a tower 128 m above sea level?

$d = 8\sqrt{\frac{h}{5}} = 8\sqrt{\frac{128}{5}} = 40.4771\ldots$ = 40 km to the nearest km.

EXERCISE 3

1. If $m = 6, n = 3$ *and* $p = -4$ evaluate the algebraic expressions.

a) $4nm$

b) $2np - m$

c) $p^2 + mn$

d) $3mn - 2p$

e) $2n^2 + 3mp$

f) $-5p + 2mn$

g) $24mp - n$

g) $p^2 - n^2 + m^2$

2. Calculate the volume, correct to two decimal places, of a cylinder with a base radius of 4.07 cm and a perpendicular height of 11.58 cm.

Use the formula $V = \pi r^2 h$

Note: Make sure you use appropriate units as part of your answer.

3. The temperature *T* (in ℃) of the water in a kettle *t* minutes after it is switched on is given by the formula $T = 18t + 28$. Find the temperature of the water:

a) 4 minutes after it is turned on

b $1\frac{1}{2}$ minutes after it is turned on

c) When the kettle is first turned on.

4. The volume of a sphere with radius *r* is $V = \frac{4}{3}\pi r^3$. Calculate, correct to one decimal place, the volume of the sphere with radius 14.5 cm.

5. The time, *T* seconds, it takes a swing to go back and forth once is $T = 2\pi\sqrt{\frac{l}{g}}$, where *l* m is the length of the swing and *g* is the gravitational acceleration. Find T, correct to two decimal places, if $l = 2.35 \; and \; g = 10$.

6. The speed, V m/s, required for a spacecraft to escape the Earth's gravitational pull during take-off is $V = \sqrt{2gr}$ where $g = 9.8\, m/s^2$ and *r* is the radius of the Earth ($6378000\, m$). Calculate the escape speed of a spacecraft leaving the Earth's atmosphere.

7. Ten-year-old Claudia is suffering from nasal and sinus congestion. She is prescribed 12 mg (R) of Demazin syrup three times per day. Demazin syrup contains 3.75 mg (A) in every 5 mL (v).

Amount required = $\frac{R}{A} \times v$

How many millilitres must Claudia be given for each dose?

8. A boy's height on his second birthday can be used to predict his adult height. Predicted adult height (cm) = $1.75 \times h + 20$ where h = boy's height on 2nd birthday.

On his second birthday Thomas is 88 cm tall. Predict Thomas's adult height.

SOLVING EQUATIONS

An equation is a mathematical sentence. Equations always include an equal sign. In our previous work we simplified algebraic expressions but now we will **solve** equations.

Eg. $3w - 4$ is an algebraic expression, however, $3w - 4 = 6$ is an algebraic equation.

Example 1

Solve the equation $2x - 6 = 14$

Write the equation.	$2x-6=14$
Add 6 to both sides.	$2x-6+6=14+6$
Simplify both sides.	$2x=20$
Divide both sides by 2.	$\frac{2x}{2}=\frac{20}{2}$
Simplify to find the value of x.	$x=10$

Example 2

Solve the equation $\frac{x}{3} + 2 = 7$

Write the equation.	$\frac{x}{3}+2=7$
Subtract two from both sides.	$\frac{x}{3}+2-2=7-2$
Simplify both sides.	$\frac{x}{3}=5$
Multiply both sides by 3.	$\frac{x}{3}\times 3=5\times 3$
Simplify to find the value of x.	$x=15$

Example 3

Solve the equation $\frac{2a\ 6}{5} = 4$

Write the equation.	$\frac{2a-6}{5}=4$
Multiply both sides by 5.	$\frac{2a-6}{5}\times 5=4\times 5$
Simplify both sides.	$2a-6=20$
Add 6 to both sides.	$2a-6+6=20+6$
Simplify both sides.	$2a=26$
Divide both sides by 2.	$\frac{2a}{2}=\frac{26}{2}$
Simplify to find the value of a.	$a=13$

EXERCISE 4

1. Solve these equations

a) $3d + 2 = 20$

b) $2p - 3 = 2$

c) $4x + 2 = 22$

d) $28 - 5x = 3$

e) $\frac{3h}{9} = 9$

f) $\frac{r-1}{6} = 2$

g) $12 + 2w = -18$

h) $\frac{z}{3} - 11 = 9$

i) $10 - 3a = 16$

j) $\frac{8-2b}{2} = 7$

2. In which line was an error made in solving the following equation?

$$\frac{c-4}{8} + 2 = 6 \qquad \text{Line 1}$$

$$\frac{c-4}{8} = 8 \qquad \text{Line 2}$$

$$c - 4 = 64 \qquad \text{Line 3}$$

$$c = 68 \qquad \text{Line 4}$$

3. Let the number represented in the following be n. For each statement below, write an equation to match the words and solve this equation:

a) twice a number less 8 is equal to 18

b) twelve less three times a number equals 39

c) one third of a number less 4 is equal to 1

d) twice a number less 4 is divided by 5 and equals 4

4. A paddock is 40 metres long and x metres wide. The perimeter of the paddock is 300 metres. Write an equation for the perimeter of the paddock and calculate the width of the paddock.

5. The volume of a cone with base radius *r* and perpendicular height *h* is

$$V = \frac{1}{3}\pi r^2 h$$

If a cone has a volume of 256 cm^3 and a radius of 7 cm, find its height correct to two decimal places.

SOLVING EQUATIONS WITH PRONUMERALS ON BOTH SIDES

Example

Solve $2a - 5 = a + 7$

Write the equation.	$2a-5=a+7$
Subtract a from both sides.	$2a-5-a=a+7-a$
Simplify both sides.	$a-5=7$
Add 5 to both sides.	$a-5+5=7+5$
Simplify to find the value of a.	$a=12$

EXERCISE 5

1. Solve these equations.

a) $2x - 5 = x + 4$

b) $5k - 13 = 3k + 9$

c) $2(m - 4) = m + 12$

d) $\frac{2a-8}{5} = 2a$

2. Change these sentences into equations and solve.

a) Twice the number less 6 equals the sum of the number and 4.

b) Eight times the number plus 7 equals 14 less than 5 times the number.

c) Three times the difference between the number and 1 equals twice the number plus 8.

WEEK 1/2 INVESTIGATION

1. Subtract the sum of $2x^2 - 3(x - 1)$ and $2x + 3(x^2 - 2)$

from the sum of $5x^2 - (x - 2)$ and $x^2 - 2(x + 1)$.

2. The mean, M, of three numbers $x, y \; and \; z$ is calculated using the formula

$$M = \frac{x+y+z}{3}$$

If three numbers have a mean of 17 and two of the numbers are 10 and 20, find the third number.

MARKING RUBRIC

CRITERIA	EXPECTATIONS	POSS	MULT	GIVEN	TOTAL
Practical	Student completes practical work (including exercises) of the brief to an acceptable standard set by the teacher.	**2**	**3**		**/6**
Investigation	Student completes the investigation task of the brief to an acceptable standard set by the teacher.	**2**	**2**		**/4**
Reasoning and Communications	Student responses are accurate and appropriate in presentation of mathematical ideas in different contexts, with clear and logical working out shown.	**4**	**-**		**/4**
Concepts and techniques	Student submitted work selects and applies appropriate mathematical modelling and problem solving techniques to solve practical problems, and demonstrates proficiency in the use of mathematical facts, techniques and formulae.	**4**	**-**		**/4**
	Submission Guidelines				
Timeliness	Student submits the practical work and investigation by the set deadline. See scoring guidelines for specific details.	**2**	**-**		**/2**
		FINAL			**/20**

Student Reflection:

How did you go with this week's work?

What was interesting?

What did you find easy?

What do you need to work on?