Week 5/6
Term 1
2020

HAWKER COLLEGE
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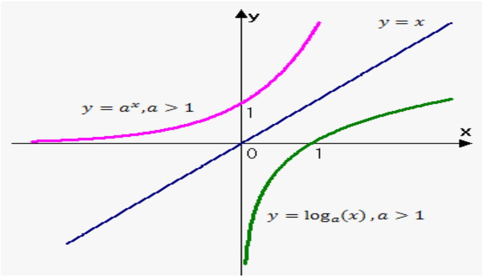
SMM3
Logarithmic Functions and Further differentiation and applications

Goals



By the end of this fortnight, you should be able to:

- Learn and use the chain rule, product and quotient rule for differentiation
- Find the derivatives of exponential functions of the forms: $y = e^x$, and $y = e^{f(x)}$
- Find the derivatives logarithmic functions of the forms; $y = ln(x)$ and $y = ln[f(x)]$.
- Use derivatives to solve practical problems

Theoretical Components

STEP 1
Text book Maths Quest Maths B Year 12 Chapter 4

If you need to remind yourself of exponential functions and their graphs review:
https://mathspace.co/learn/ac-methods-12/calculus-of-exponential-functions-3232/

What is base e?
Read through to get an insight on the number 'e':
http://bit.ly/w8OiD

http://www.mathopolis.com/questions/q.php?id=2011&site=1&ref=/numbers/e-eulers-number.html&qs=2011_2012_2013

Chain Rule Proof

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

https://www.khanacademy.org/math/ap-calculus-ab/product-quotient-chain-rules-ab/chain-rule-proof-ab/v/chain-rule-proof

Derivative of $y = e^x$, and $y = e^{f(x)}$ from first principles
See the following Chapter 4 page 123-127
Read and make notes examples 4 - 7 from

Derivative of $\log_e x$
Read and make notes examples 8 - 11from Chapter 4

Practical Components

STEP 2
Differentiating exponential and logarithmic functions

- Exercise 4A – 4E

Know:

The chain rule: $$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

The product rule: If $y = u(x) \times v(x)$

then $$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

The quotient rule: If $y = \frac{u(x)}{v(x)}$

then $$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Practice sketching Log Functions

- Collect worksheet from Jacqueline.

Investigation

STEP 3
Work on your assignment

What do we know about base e? (Remember this from Week 3/4)

http://www.mathsisfun.com/numbers/e-eulers-number.html

Also, the derivative of $f(x) = e^x$ from first principles is as follows:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}, h \neq 0$$

$$= \lim_{h \to 0} \frac{e^{x+h} - e^x}{h}$$

$$= \lim_{h \to 0} \frac{e^x e^h - e^x}{h}$$

$$= \lim_{h \to 0} \frac{e^x (e^h - 1)}{h}$$

$$= e^x \lim_{h \to 0} \frac{e^h - 1}{h}$$

Note that $\lim_{h \to 0} \frac{e^h - 1}{h}$ can be deduced by using a calculator and substituting values of h close to zero.

h	$\frac{e^h - 1}{h}$
0.01	1.0050
0.0001	1.00005
0.000001	1.000000

That is, $\lim_{h \to 0} \frac{e^h - 1}{h} = 1$.

Therefore, $f'(x) = e^x \times 1$
$= e^x$

If $f(x) = e^x$ then $f'(x) = e^x$.

Note: e^x is the only function which has itself as a derivative.