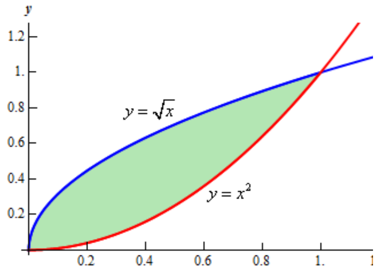
Week 13/14
Term 2
2020

HAWKER COLLEGE
Engage | Inspire | Achieve

Learning Brief:
SMM3
Integrals

Goals



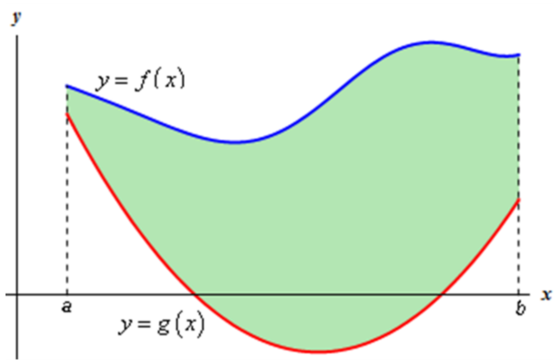
By the end of this brief, you should be able to:

- Understand and use definite integrals to find the further areas area between curves and the x axis and the y axis
- Understand and use definite integrals to find the area between two curves
- Understand and use integrals to find the volumes of the solids of revolution

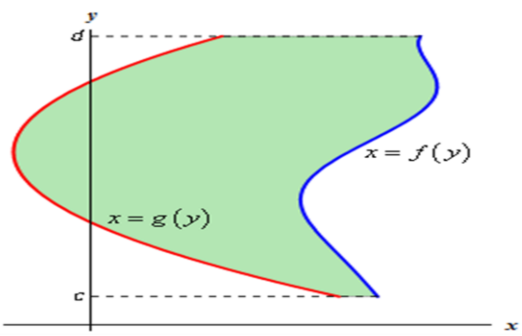
Begin Assignment.

Theoretical Components

JacPlus eBook *Year 12 Maths Quest Methods*
Chapter 9
View the following video
Areas about the x axis:
http://www.youtube.com/watch?v=DRFyNHdVgUA



Read worked examples 32 to 34
View the following video
Areas about the y axis:
http://www.youtube.com/watch?v=70NQ3ISYihw



Read the notes on the **Volumes of solids of revolution.** See the next page.

See the **PowerPoint** posted on GC**.**

Practical Components

JacPlus eBook *Year 12 Maths Quest Methods*
Exercises 9F, 9G, 9H and 9J
(Do every 2^{nd} or 3^{rd} question i.e. 1a, c, e; 2a, c, f; 3 etc)

Try the questions (there are solutions provided) on this website:
http://tutorial.math.lamar.edu/Problems/CalcI/AreaBetweenCurves.aspx

Try the Exercises on the **Volumes of solids of revolution.**
See the next page.
Solutions will be published later.

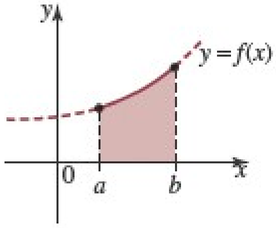
Investigation

Work on your assignment

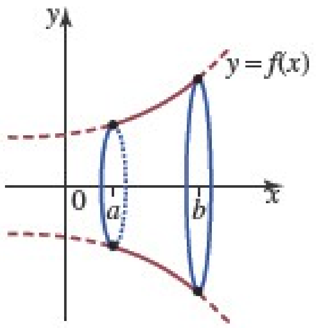
2020 SMM3 Notes

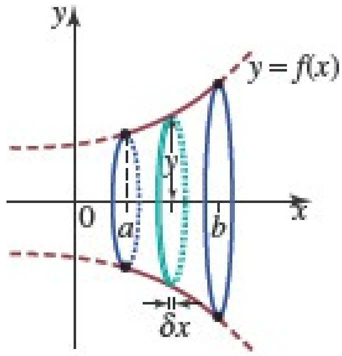
Volumes of solids of revolution

If part of a curve is rotated about the x-axis, or y-axis, a figure called a *solid of revolution* is formed. For example, a solid of revolution is obtained if the shaded region in the figure below is rotated about the x-axis.



The solid generated is shown below and is symmetrical about the x-axis and any vertical cross-section is circular, with a radius equal to the value of y at that point. For example, the radius at $x = a$ is $f(a)$.





Any thin vertical slice may be considered to be cylindrical, with radius y and height (thickness) δx. The volume of the solid of revolution generated between $x = a \text{ to } x = b$ is found by allowing the height of cylinder, δx, to be as small as possible and adding the volumes of all the cylinders formed between $x = a \text{ to } x = b$. That is, the volume of a typical strip is equal to $\pi y^2 \delta x$. This is equivalent to $\pi r^2 h$ as the volume of a cylinder.
Therefore the volume of the solid contained from $x = a \text{ to } x = b$ is the sum of all the infinitesimal volumes:

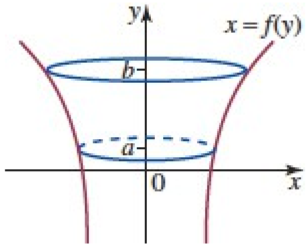
$$V = \lim_{\delta x \to 0} \sum_{x=a}^{x=b} \pi y^2 \, \delta x$$

$$= \int_a^b \pi y^2 \, dx$$

The value of y must be expressed in terms of x so that the integral can be evaluated. As $y = f(x)$ the volume of revolution of a curve $y = f(x)$ from $x = a \text{ to } x = b$ is

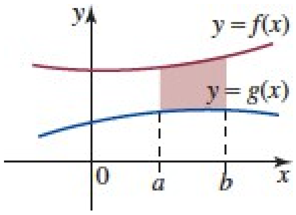
$$V = \pi \int_a^b [f(x)]^2 \, dx$$

Similarly, if a curve is rotated about the y-axis, the solid of revolution shown below is produced.



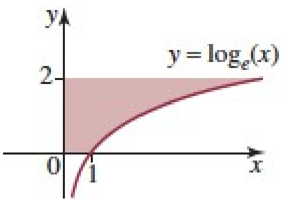
The volume of the solid of revolution is $V = \pi \int_a^b [f(y)]^2 \, dy$.

For regions between two curves that are rotated about the x-axis.



$$V = \pi \int_a^b ([f(x)]^2 - [g(x)]^2) \, dx$$

Example



This diagram represents the region bounded by $y = \ln x$, the x-axis, the y-axis and the line $y = 2$. Find the volume when this region is rotated about the y-axis.

$y = \ln x$ which gives $x = e^y$

So $V = \pi \int_0^2 (e^x)^2 dy$

$= \pi [\frac{e^{2y}}{2}]_0^2$

$= \frac{\pi}{2}(e^4 - 1)$ cubic units.

2020 SMM3 Exercises

Volumes of solids of revolution

Q1. Find the volume generated by rotating the area bounded by $y = e^{2x}$, the y-axis and the line y = 2 about the x-axis.

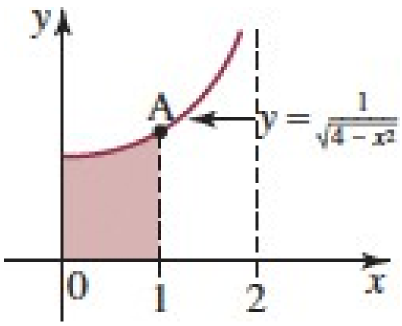
Q2. Find the volume generated by rotating the ellipse with equation

$\frac{x^2}{4} + \frac{y^2}{9} = 1$ about

(i) the x-axis and
(ii) the y-axis.

Q3. Find the volume generated by the rotation of the area bounded by the curves $y = x^3$ and $y = x^2$ around (i) the x-axis and (ii) the y-axis.

Q4.



For the graph above:

a) Find the co-ordinates of A
b) Find the volume generated when the shaded region is rotated about the x-axis.
c) Find the volume generated when the shaded region is rotated about the y-axis.

Q5. A hemispherical bowl of radius 10 cm contains water to a depth of 5 cm. What is the volume of water in the bowl?

Q6. A solid sphere of radius 6 cm has a cylindrical hole of radius 1 cm bored through its centre. What is the volume of the remainder of the sphere?