## Goals



By the end of this week, you will:

- Understand the notion of a combination as an unordered set of $r$ objects taken from a set of $n$ distinct objects
- Use the notation $\binom{n}{r}$ and the formula $\binom{n}{r}=\frac{n!}{r!(n-r)!}$ for the number of combinations of $r$ objects taken from a set of $n$ distinct objects
- Expand $(x+y)^{n}$ for small positive integers $n$
- Recognise the numbers $\binom{n}{r}$ as binomial coefficients, (as coefficients in the expansion of $\left.(x+y)^{n}\right)$
- Use Pascal's triangle and its properties


## Theoretical components

## Practical components

## Knowledge Checklist:

- Use ${ }^{n} P_{r}$ to count number of possible arrangements where order is important (permutations)
- Use ${ }^{n} \mathrm{C}_{r}$ to represent selections where order is not important (combinations)
- Make connections between the number of combinations and the number of permutations.
- Investigate patterns in Pascal's triangle and the relationship to combinations, establish counting principles and use them to solve simple problems involving numerical values for $n$ and $r$.
- Use CAS to compute ${ }^{n} \mathrm{C}_{\mathrm{r}}$ for a given n and a given $r$


## Online Links

- Combinations -
https://www.youtube.com/watch?v=Ej 40Sipges \&feature=youtu.be
- Binomial Expansion theorem https://www.youtube.com/watch?v=ajaAk1CP5p w
- http://www.mathsisfun.com/combinatorics/combi nations-permutations.html
- http://www.mathsisfun.com/pascalstriangle.html


## Investigation

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Make sure you have joined the Google Classroom. If you have not, see your teacher.
Fun fact: How many randomly chosen people do you need before there is a good chance at least two share a birthday? Counter-intuitively, the probability of a shared birthday exceeds $50 \%$ with only 23 people. This might seem like just a bit of fun mathematical trivia, but the ideas behind this paradox have serious applications in cryptography and information security.

## Week 3 Investigation

Pascal's triangle is a triangular array where each number is the sum of the two numbers above it (except for the edges, which are all " 1 "). It is named after the $17^{\text {th }}$ century French mathematician, Blaise Pascal (1632-1662).


The triangle is constructed in the following manner: in row 0 (the topmost row), there is a unique nonzero entry 1 . Each entry of each subsequent row is constructed by adding the two numbers above it.

For example, numbers 1 and 3 in the third row are added to produce the number 4 in the fourth row.

The numbers in Pascal's Triangle have a special relationship with the coefficients of binomial expansions (binomial coefficients) and combinations.

| $(a+b)^{n}$ | Binomial expansion | Pascals Triangle as combinations ${ }^{n} C_{r}=\binom{n}{r}$ |
| :---: | :---: | :---: |
| $(a+b)^{0}=$ | 1 | $\binom{0}{0}$ |
| $(a+b)^{1}=$ | $a+b$ | $\binom{1}{0} \quad\binom{1}{1}$ |
| $(a+b)^{2}=$ | $a^{2}+2 a b+b^{2}$ | $\binom{2}{0} \quad\binom{2}{1} \quad\binom{2}{2}$ |
| $(a+b)^{3}=$ | $a^{3}+3 a^{2} b+3 a b^{2}+b^{3}$ | $\binom{3}{0} \quad\binom{3}{1} \quad\binom{3}{2} \quad\binom{3}{3}$ |
| $(a+b)^{4}=$ | $a^{4}+4 a^{3} b+6 a^{2} b^{2}+4 a b^{3}+b^{4}$ | $\binom{4}{0} \quad\binom{4}{1} \quad\binom{4}{2} \quad\binom{4}{3} \quad\binom{4}{4}$ |
| $(a+b)^{5}=$ | $a^{5}+5 a^{4} b+10 a^{3} b^{2}+10 a^{2} b^{3}+5 a b^{4}+b^{5}$ | $\binom{5}{0} \quad\binom{5}{1} \quad\binom{5}{2} \quad\binom{5}{3} \quad\binom{5}{4} \quad\binom{5}{5}$ <br> $n$ is the row number (starting at row 0 ) and $r$ is the element in the row (also starting at 0 ) |

Consider the expansion above of $(a+b)^{n}$. Particularly note the following patterns.

- For each expansion to the power of $n$, there is $n+1$ elements
- For each term, the sum of the exponents is $n$
- Powers of $a$ decrease from left to right, from $n$ down to 0
- Powers of $b$ increase from left to right, from 0 up to $n$
- The coefficients start at 1, end at 1, and are the terms of the relevant row from Pascal's triangle

The pattern in the expansions observed is summaries in the formula called the binomial theorem.

Any particular term in the expansion of $(a+b)^{n}$ can be found using $\binom{n}{r} a^{n-r} b^{r}$
Binomial Theorem

$$
\begin{gathered}
(a+b)^{n}=\binom{n}{0} a^{n} b^{0}+\binom{n}{1} a^{n-1} b^{1}+\binom{n}{2} a^{n-2} b^{2}+\ldots+\binom{n}{n} a^{0} b^{n} \\
\text { where }\binom{n}{r}={ }^{n} C_{r}=\frac{n!}{r!(n-r)!}
\end{gathered}
$$

Example: What is the seventh term in the expansion of $(m-2 n)^{12}$ ?
We need to construct the seventh term from $\binom{n}{r} a^{n-r} b^{r}$ where $n=12$ and $r=6$.
The coefficient $\binom{n}{r}$ where $n=12$ and $r=6$ is $\binom{12}{6}=924$.
The term will have both $m$ and $2 n$ components. The $m$ component would be $m^{12-6}=$ $m^{6}$. The $2 n$ component would be $(2 n)^{6}=64 n^{6}$.

Therefore, the seventh term is $924 m^{6} \times 64 n^{6}=59136 m^{6} n^{6}$.

## Questions

1. Research and write down at least 3 interesting things about the number patterns in Pascals Triangle.
2. The $4^{\text {th }}$ term in the expansion of $\left(3 a^{2}+\frac{p}{b}\right)^{4}$ is $\frac{96 a^{2}}{b^{3}}$, for some constant $p$. Find the value of $p$.
