Engage | Inspire | Achieve

## EM1

Calculations, percentages and rates

## Goals

Goals for this fortnight:

- identify common usage of rates
- convert units of rates occurring in practical situations to solve problems
- use rates to make comparisons
- use units of energy to describe consumption of electricity, such as kilowatt hours
- use units of energy used for foods, including calories
- use units of energy to describe the amount of energy in activity, such as kilojoules
- convert from one unit of energy to another


## Theoretical Components

## STEP 1

## Resources:

PDF file: Week 7/8 Notes and Exercises
YouTube videos: Linked in the PDF file

## This Fortnight:

We will be looking at:

- Common rates
- The rate at which we use energy kilojoules
- Converting km/hr to mph and vice-versa
- Unit pricing

The following site provides a comprehensive view of the concept of unit pricing:
https://www.accc.gov.au/consumers/pricing/unit -prices-for-groceries

## Practical Components

## STEP 2

Read through Week 7/8 Notes and Exercises for instructions on what to do.

There are 5 Exercises in this booklet. Read any worked examples before you begin.

Remember to regularly check Google Classroom for messages.

## Portfolio Task

## STEP 3

Complete the task at the end of the brief and submit your weekly work. () book" so you may use your folders. Make sure to get your briefs up-to-date and bring a calculator and writing equipment to the test.

## Rate

A relationship between two units


SplashLearn

## ESSENTIAL MATHEMATICS 1

## WEEK $7 / 8$ - RATIO AND RATES

## USING RATIO

Scale models and scale drawings are used to find design errors and improvements for inventions, new designs (e.g. cars, buildings) and engineering projects. A map is a scale copy of the landscape. The scale on a map or a house plan can be used to calculate actual distances.

To convert:


## Exercise 1

1. Use the given scale to calculate the actual distance for each of the scaled distances. Give your answer in appropriate units.
a. Scale $1: 20,000$
i. 5 cm
ii. 150 mm
iii. 22 cm
b. Scale $1: 2,500$
i. 18 cm
ii. 2.75 cm
iii. 22.5 cm
2. Graham and Jean run a sheep farm and, after breeding season, they have calculated that the ewe to lamb ratio is $20: 19$. They currently have a total of 5,265 sheep on their property.
a. Calculate the total number of parts in the ratio.
b. Determine what fraction of the sheep are ewes and what fraction are lambs.
c. Calculate how many ewes and lambs they have in total.

## USING RATES

A rate is a ratio between two quantities that are measured in different units. For example, the rate a tap leaks may be 30 mL every 5 minutes. Rates are often expressed as unitary rates where the second quantity in the rate has a measure of 1. The unitary rate for the leaking tap would be 6 mL for every 1 minute, or $6 \mathrm{~mL} / \mathrm{min}$.

Common examples of rates include:

- Speeds, e.g. $60 \mathrm{~km} / \mathrm{hr}, 20 \mathrm{~m} / \mathrm{s}, 400 \mathrm{~m}$ every 5 minutes
- Growth rates, e.g. $20 \mathrm{~cm} /$ year, $15 \mathrm{~mm} /$ month, 2 kg per week
- Cost of groceries, e.g. $\$ 5$ per kg, $\$ 1.20$ per $100 \mathrm{~g}, \$ 10$ per box


## Exercise 2

1. State the typical units for the following rates.
a. The speed of a 100 m sprinter
b. The cost of buying grapes
c. The cost of petrol
d. A person's typing speed
e. The calorie content in a breakfast cereal
f. The speed of a car
g. The cost of buying land
h. A cricket team's run rate
2. Express the following as rates
a. A pipe costs $\$ 100$ for 20 metres $(\$ / m)$
b. A waiter earns $\$ 90$ for a 6 -hour shift ( $\$ /$ hour)
c. 42 kg of seed is spread over $7 \mathrm{~m}^{2}$ of garden $\left(\mathrm{kg} / \mathrm{m}^{2}\right)$
d. A tanker pumps 1,000 litres of water in 5 minutes (L/min)
e. A vehicle travels 450 km in 6 hours ( $\mathrm{km} / \mathrm{hr}$ )
f. ActewAGL charges 37.4330 cents per kilowatt hour of electricity use (c/kWh)
3. Martin earns $\$ 1,840$ per fortnight
a. How much does Martin earn per week?
b. Calculate the amount that Martin earns annually (i.e. per year).
4. Jarvi was 51 cm when he was born. He grows at a rate of $8 \mathrm{~cm} /$ year until he is 16 years old. Determine Jarvi's height when he turns 16 years old.
5. Brian takes 5 minutes and 48 seconds to complete a three lap event. What is the average time he takes to complete each lap? (answer in seconds)
6. As part of his annual check up, Toby's doctors completed this graph

7. Nectarines are sold for $\$ 3$ per kilogram when they are in season.
a. Determine the cost of 2 kg .
b. Determine the cost of 4.5 kg .
c. Calculate how many kilograms of nectarines can be purchased for $\$ 10.50$.

## UNITS OF ENERGY - KILOJOULES

If you use more energy than you eat, then you will lose body weight. Similarly. If you eat more energy than your body uses, this energy will be stored as fat.

The number of kilojoules (a measurement of energy) your body requires each day depends on your age, gender and lifestyle.

| Age (years) | Lifestyle | Men (kJ/day) | Women (kJ/day) |
| :---: | :--- | :---: | :---: |
| $18-35$ | Inactive | 10,500 | 8,000 |
|  | Active | 12,500 | 9,000 |
|  | Very active | 14,800 | 10,500 |
|  | Inactive | 10,000 | 8,000 |
|  | Active | 11,800 | 8,800 |
|  | Very active | 14,300 | 10,400 |
| Pregnant women |  |  | 10,100 |
|  | Breast-feeding |  | 11,800 |

## Exercise 3

1. Estelle is 18 years old. During the week she works in an office and is inactive, but on the weekends, she is very active: jogging, swimming, and cycling.
a. How many kilojoules does Estelle use per day on weekdays?
b. How many more kilojoules does Estelle use per day on the weekend than on weekdays? (Show working)
c. How many kilojoules does Estelle use per week? (Show working)
2. How many kilojoules does a very active 40 year old male require per week?
3. According to the above table, how many kilojoules per day does Joanne need, now that she is pregnant, then her twin sister Alice, who is an active 20 year old woman? (Show working)
4. This table shows the average length of time it takes a typical 18 year old to burn $1,000 \mathrm{~kJ}$.

| Activity | Time required to use $\mathbf{1 0 0 0}$ kJ |
| :--- | :---: |
| Sleeping | 4 hours |
| Eating | 3 hours |
| Working in class, <br> studying, watching TV | $2 \frac{1}{2}$ hours |
| Walking | 1 hour |
| Bike riding | 50 minutes |
| Swimming | 30 minutes |

a. How many kilojoules are used in swimming for 30 minutes?
b. How long does it take to burn 500 kJ while sleeping? (Show working)
c. Suzie, aged 18 years, leads a very active life. This is how she usually spends her day:

8 hours Sleeping
5 hours Working in class
3 hours Swimming
$2 \frac{1}{2}$ hours Studying
$2 \frac{1}{2}$ hours Watching TV
$1 \frac{1}{2}$ hours Walking
$1 \frac{1}{2}$ hours Eating
i. According to the above table, how many kilojoules does Suzie use each day? (Show working)
ii. If Suzie restricts her daily kilojoule intake to the amount shown in the table on Page 8, would she have a sufficient amount of kilojoules to meet her energy requirements? Explain.
5. This table shows another way of calculating kilojoule requirements.

| Activity | Energy used <br> $(\mathbf{k J} / \mathbf{m i n})$ |
| :--- | :---: |
| Sleeping | 4 |
| Cleaning | 15 |
| Ironing | 17 |
| Bricklaying | 17 |
| Playing tennis | 31 |
| Gardening | 23 |
| Circuit training | 53 |
| Walking | 23 |

## Calories and kilojoules

A calorie is another unit that measures energy. It is usually used to measure the energy content of foods and beverages.

Calories are bigger than kilojoules.
1 calorie $=4.18$ kilojoules
*To convert kilojoules to calories, divide by 4.18
*To convert calories to kilojoules, multiply by 4.18
a. How long would it take you to use 230 kJ walking?
b. Jodie uses $1,240 \mathrm{~kJ}$ playing tennis. For how long does she play tennis?
c. How many kilojoules would you use doing these activities? How many calories would need to be consumed?

|  |  | Kilojoules |
| :--- | :--- | :--- |
| i. | Cleaning for one hour |  |

## CONVERSION TABLES

Converting quantities such as metres per second ( $\mathrm{m} / \mathrm{s}$ ) to kilometres per hour ( $\mathrm{km} / \mathrm{hr}$ or $\mathrm{km} / \mathrm{h}$ ), temperatures from degrees Fahrenheit to degrees Celsius, are easy with a conversion table.

Conversion from $\mathrm{km} / \mathrm{h}$ to $\mathrm{m} / \mathrm{s}$

| Ikm/h | $\mathrm{m} / \mathrm{s}$ |
| :---: | :---: |
| $\\|$ | 0.28 |
| 2 | 0.56 |
| 3 | 0.83 |
| 4 | 1.11 |
| 5 | 1.39 |
| 6 | 1.67 |
| 7 | 1.94 |
| 8 | 2.22 |
| 9 | 2.50 |
| 10 | 2.78 |


| $\mathrm{km} / \mathrm{h}$ | $\mathrm{m} / \mathrm{s}$ |
| :---: | :---: |
| 12 | 3.3 |
| 14 | 3.9 |
| 15 | 4.2 |
| 20 | 5.6 |
| 25 | 6.9 |
| 30 | 8.3 |
| 35 | 9.7 |
| 40 | 11.1 |
| 45 | 12.5 |
| 50 | 13.9 |


| $\mathrm{km} / \mathrm{h}$ | $\mathrm{m} / \mathrm{s}$ |
| :---: | :---: |
| 55 | 15.3 |
| 60 | 16.7 |
| 65 | 18.1 |
| 70 | 19.4 |
| 75 | 20.8 |
| 80 | 22.2 |
| 85 | 23.6 |
| 90 | 25.0 |
| 95 | 26.4 |
| 100 | 27.8 |


| $\mathrm{km} / \mathrm{h}$ | $\mathrm{m} / \mathrm{s}$ |
| :---: | :---: |
| 105 | 29.2 |
| 110 | 30.6 |
| 115 | 31.9 |
| 120 | 33.3 |
| 125 | 34.7 |
| 130 | 36.1 |
| 135 | 37.5 |
| 140 | 38.9 |
| 145 | 40.3 |
| 150 | 41.6 |

Example: Use the conversion table above to change $45 \mathrm{~km} / \mathrm{hr}$ to $\mathrm{m} / \mathrm{s}$.
Solution: The third column from the left lists speeds from $12 \mathrm{~km} / \mathrm{hr}$ to $50 \mathrm{~km} / \mathrm{hr}$. the fourth column, next to 45 in the third column, is 12.5. A speed of $45 \mathrm{~km} / \mathrm{hr}$ is the same as $12.5 \mathrm{~m} / \mathrm{s}$

Example: Use the conversion formula ( $\mathbf{m} / \mathbf{s}=\mathbf{k m} / \mathbf{h r} \times \mathbf{0 . 2 7 8}$ ) to convert a speed of $32 \mathrm{~km} / \mathrm{hr}$ to $\mathrm{m} / \mathrm{s}$.

Solution: $32 \mathrm{~km} / \mathrm{hr}=32 \times 0.278 \mathrm{~m} / \mathrm{s}$

$$
\text { = } 8.896 \mathrm{~m} / \mathrm{s} \text { or } 8.9 \mathrm{~m} / \mathrm{s} \text {, correct to } 1 \text { decimal place. }
$$

$32 \mathrm{~km} / \mathrm{hr}$ is the same as $8.9 \mathrm{~m} / \mathrm{s}$

## Exercise 4

1. Use the conversion table above to express the following speeds in metres per second.
a. $20 \mathrm{~km} / \mathrm{hr}$
b. $9 \mathrm{~km} / \mathrm{hr}$
c. $65 \mathrm{~km} / \mathrm{hr}$
2. Use the conversion table above to approximate the following speeds in kilometres per hour.
a. $8 \mathrm{~m} / \mathrm{s}$
b. $32 \mathrm{~m} / \mathrm{s}$
c. $35 \mathrm{~m} / \mathrm{s}$
3. Which is faster?
a. $40 \mathrm{~km} / \mathrm{hr}$ or $12 \mathrm{~m} / \mathrm{s}$
b. $75 \mathrm{~km} / \mathrm{hr}$ or $20 \mathrm{~m} / \mathrm{s}$
4. Julia rides 8 km on her bike in 45 minutes. Calculate Julia's average speed in $\mathrm{km} / \mathrm{hr}$.
5. The average sloth moves at a speed of $240 \mathrm{~m} / \mathrm{hr}$. Convert this rate to $\mathrm{km} / \mathrm{hr}$ and then determine after how many hours the sloth travels 0.6 km .
6. Kangaroos can bound at a top speed of $48 \mathrm{~km} / \mathrm{hr}$. Approximately how many metres can they bound in a second?
7. A Peregrine falcon's top flying speed is 200 miles per hour. Use the formula:

$$
\mathrm{m} / \mathrm{s}=\mathrm{miles} / \mathrm{hour} \times 1.609 \times 0.278
$$

to convert this speed into m/s. (Show working)
8. Some of the slowest-moving creatures on Earth are the sloth, snail, tortoise and worm. Use suitable calculations to determine which of the creatures is the slowest.
The average speeds are:
$>$ Snail: $0.013 \mathrm{~m} / \mathrm{s}$
$>$ Worm: $0.2 \mathrm{~cm} / \mathrm{s}$
$>$ Tortoise: $0.48 \mathrm{~km} / \mathrm{hr}$
$>$ Sloth: $240 \mathrm{~m} / \mathrm{hr}$

## UNIT PRICES

It is a good idea to compare prices when shopping if you want to get as much value for your money as possible. It is easier to compare prices for items that are identical in quantity than those that come in varying quantities. For example, how would you know which was better value for money - a 150 g chocolate bar for $\$ 2.50$ or a 375 g block for $\$ 6.20$ ? One way to determine which item is better value for money is to calculate unit prices for the item. We will work in cost per 100 g or cost per 100 mL for this section.

## Use these unit pricing tips to help get better value for money:

1. Compare the unit price of different sizes of the same brand's product, as well as products from different brands of the same product. The labels on the shelf that show the price of an item also show the unit price of that item.
2. Look out for special offers which might temporarily have the lowest unit price - but not always.
3. The unit price of large packs is often lower than small or medium size packs. But avoid buying a bigger pack if it's likely to go to waste.
4. If a product is available loose or pre-packaged, check the unit price of both.
5. Compare unit prices in different parts of the supermarket. The same product may be sold in different sections, for example, cheese, meats, seafood, nuts, fruit and vegetables.

Example: A 200 g item sells for $\$ 5.95$. Calculate the unit price for a 100 g quantity.

## Solution:

Divide the cost of the item by the quantity. $\quad \frac{\$ 5.95}{220}=\$ 0.027$ per g
Multiply by 100 to get the cost per 100 g . $0.027 \times 100$
Write the answer as $\$$ per 100 g . $\quad \$ 2.70$ per 100 g
Example: Which is better value for money - a 350 mL carton of milk for $\$ 1.75$ or a 1.5 L bottle of milk for $\$ 4.50$ ?

## Solution:

Calculate the unit price for 100 mL for each item. Remember to change litres into millilitres first. $1.5 \mathrm{~L} \times 1000=1500 \mathrm{~mL}$

$$
\frac{\$ 1.75}{350 \mathrm{~mL}}=0.005 \quad \frac{\$ 4.50}{1500 \mathrm{~mL}}=0.003
$$

Compare the unit price per 100 mL . $0.005 \times 100=\$ 0.50$ per 100 mL

$$
0.03 \times 100=\$ 0.30 \text { per } 100 \mathrm{~mL}
$$

State which is better value.
$\$ 0.30$ per 100 mL is cheaper than $\$ 0.50$ per 100 mL , so the 1.5 L bottle of milk is better value for money.

## Exercise 5

For this exercise, remember to change kilograms into grams: $1 \mathrm{~kg}=1000 \mathrm{~g}$

1. Calculate the unit price per 100 g for the following items. Explain which is better value.

|  |  | Which is better value? |
| :--- | :--- | :--- |
| 200 g tin of Milo for $\$ 4.50$ | 460 g tin of Milo for $\$ 8.80$ |  |
|  |  |  |
| 115 g of toothpaste for | 200 g of toothpaste for <br> $\$ 5.50$ <br> $\$ 8.00$ |  |
|  |  |  |
| 500 g packet of spaghetti | 1 kg packet of spaghetti for |  |
| for $\$ 2.95$ |  |  |
|  |  |  |
|  |  |  |


| 2 kg bag of potatoes for <br> $\$ 7.50$ | 1.5 kg bag of potatoes for <br> $\$ 6.00$ |  |
| :--- | :--- | :--- |
|  |  |  |
| Box of 18 dishwasher <br> tablets for $\$ 15$ | Box of 72 dishwasher <br> tablets for $\$ 42$ |  |

## Week 7/8 Portfolio Task

1. At the 2020 Olympic games, the qualifying standards for the women's 100 metres race was 11.15 s . How does this compare with the speed of a bus travelling at 40 $\mathrm{km} / \mathrm{hr}$ through a school zone?
2. Imagine that you raced in the 200 m with Usain Bolt. By what length would he beat you? (Note: In the 2009 IAAF World championship, Usain Bolt ran the 100 m in 9.58 s .)

MARKING RUBRIC

| CRITERIA | EXPECTATIONS | POSS | MULT | GIVEN | TOTAL |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Practical | Student completes practical work, including exercises of the brief to an acceptable standard set by the teacher. | 2 | 3 |  | /6 |
| Portfolio Task | Student completes the portfolio task of the week to an acceptable standard set by the teacher. | 2 | 2 |  | 14 |
| Reasoning and Communications | Student responses are accurate and appropriate in presentation of mathematical ideas, with clear and logical working out shown. | 4 | - |  | 14 |
| Concepts and Techniques | Student submitted work selects and applies appropriate mathematical modelling and problem solving techniques to solve practical problems, and demonstrates proficiency in the use of mathematical facts, techniques and formulae | 4 | - |  | 14 |
|  | Submission Guidelines |  |  |  |  |
| Timeliness | Student submits the exercises and portfolio tasks by the set deadline. See scoring guidelines for specific details. | 2 | - |  | /2 |
|  |  | FINAL |  |  | /20 |

## Student Reflection:

How did you go with this week's work?

What was interesting?

What did you find easy?

What do you need to work on?

