**Week 8 & 9**
**Term 1**
**2024**

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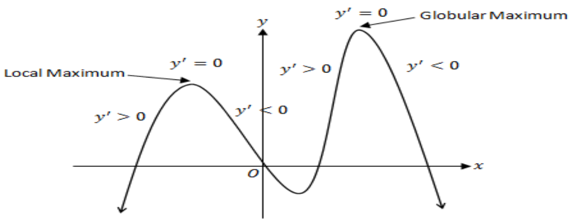
**MM3**
Further differentiation and applications

# Goals




By the end of this brief, you should be able to:

- sketch the graph of a function using first and second derivatives to locate stationary points and points of inflection
- solve optimisation problems from a wide variety of fields using first and second derivatives

# Theoretical Components

**Resources:**

- *Year 12 Maths Quest Methods* Chapter 8

## Second – Derivative Test

Let f ´(c) = 0 and let f ´´exist on an open interval containing c.

1. If f ´´(c) > 0, then f(c) is a relative minimum.
2. If f ´´(c) < 0, then f(c) is a relative maximum.
3. If f ´´(c) = 0 then the test fails. Use the First Derivative Test.

# Practical Components

**Complete the following questions.** Organise your solutions neatly in your exercise book.

You will require Chapter 8 of Maths Quest Methods (pdf – Google Classroom).

**Ex 8C: Maximum and minimum problems when the function is known**

- Qs 4, 6, 7, 9 – 11

**Ex 8D: Maximum and minimum problems when the function is unknown**

- Qs 1, 3, 6, 8, 10

**Ex 8E: Rates of change**

- Qs 3, 4, 6 – 8, 12 – 14

# Investigation

See next two pages.

Don't forget to create a double-sided A4 summary sheet for the exam!

# QFO
Quiz/Forum/Other

Remember to check-in with Serene every lesson, and get your name marked off.

Do the Cambridge task!

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# MM3 Week 8/9 Investigation (notes and example)

An application of differentiation in the world of economics is that which involves marginal cost, marginal revenue and marginal profit.

All of these relate to an instantaneous rate of change (which we now know to be called the derivative) of some cost, revenue or profit functions. That is, the cost, revenue or profit of producing one extra item. (Note: Profit = Revenue – Cost)

**Definitions**

- Marginal Cost: the derivative of the cost function with respect to the production level
- Marginal Revenue: the derivative of the revenue function with respect to the production level
- Marginal Profit: the derivative of the profit function with respect to the production level

Example:

The revenue function for the production of watches is given by $R(n) = n[14 - (\frac{n}{1\,000}]$, and the cost function for the watches is given by $C(n) = 4n + 7\,000$

a) What is the profit function?

The profit is the total revenue (in) less the total costs (out). So $P(n) = R(n) = C(n)$

$$P(n) = n\left(14 - \frac{n}{1\,000}\right) - (4n + 7\,000)$$

$$P(n) = \left(14n - \frac{n^2}{1\,000}\right) - (4n + 7\,000)$$

$$P(n) = 14n - \frac{n^2}{1\,000} - 4n - 7\,000$$

$$P(n) = 10n - \frac{n^2}{1\,000} - 7\,000$$

b) What is the marginal profit function?

Marginal profit means to find $P'(n)$

$$P'(n) = 10 - \frac{n}{500}$$

c) What is the marginal profit for $2\,000$ watches? Interpret this result.

This means we need to find $P'(2000)$

$$P'(2000) = 10 - \frac{2\,000}{500} = 10 - 4 = 6$$

This means that the change in profit from making 2000 watches to 2001 watches is $6.

d) What is the optimum number of watches needed to maximise the profit?

So we need to maximise the profit function. The maximum profit happens when the marginal profit function is $0$. This means that at that point, there is no extra profit to be made to make another watch.

Set $P'(n) = 0$ and solve.

$$P'(n) = 10 - \frac{n}{500}$$

$$10 - \frac{n}{500} = 0$$

$$10 = \frac{n}{500}$$

$$5\,000 = n$$

So to maximise profit we should aim to manufacture $5\,000$ watches.

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**MM3 Week 8/9 Investigation**

The cost of producing $x$ thousand pairs of shoes is given by $x^3 - 6x^2 + 15x$, measured in dollars. Each pair is sold for $10.00 to a wholesaler.

1) Let $P(x)$ be the profit from selling $x$ thousand pairs of shoes.
   Determine the expression for marginal profit.

2) What is the optimum number of pairs of shoes needed to maximise the profit?