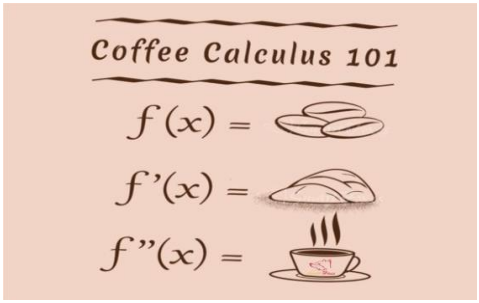
Week 5 & 6
Term 1
2024

HAWKER COLLEGE
Engage | Inspire | Achieve

MM3
Further differentiation and applications

Goals



By the end of this fortnight, you should be able to:

- establish the formulas $\frac{d}{dx}(\sin x) = \cos x$, and $\frac{d}{dx}(\cos x) = -\sin x$ by numerical estimations of the limits and informal proofs based on geometric constructions
- use trigonometric functions and their derivatives to solve practical problems.
- understand and use the product and quotient rules
- understand the notion of composition of functions and use the chain rule for determining the derivatives of composite functions
- apply the product, quotient and chain rule to differentiate functions such as xe^x, $\tan x$, $\frac{1}{x^n}$, $x \sin x$, $e^{-x} \sin x$ and $f(ax + b)$.

Theoretical Components

Common Functions	Function	Derivative
Constant	c	0
Line	x	1
	ax	a
Square	x^2	2x
Square Root	√x	$(½)x^{-½}$
Exponential	e^x	e^x
	a^x	$\ln(a)\, a^x$
Logarithms	ln(x)	1/x
	$\log_a(x)$	1 / (x ln(a))
Trigonometry (x is in radians)	sin(x)	cos(x)
	cos(x)	−sin(x)
	tan(x)	$\sec^2(x)$
Inverse Trigonometry	$\sin^{-1}(x)$	$1/\sqrt{(1-x^2)}$
	$\cos^{-1}(x)$	$-1/\sqrt{(1-x^2)}$
	$\tan^{-1}(x)$	$1/(1+x^2)$

Rules	Function	Derivative
Multiplication by constant	cf	cf′
Power Rule	x^n	nx^{n-1}
Sum Rule	f + g	f′ + g′
Difference Rule	f - g	f′ − g′
Product Rule	fg	f g′ + f′ g
Quotient Rule	f/g	$\frac{f'g - g'f}{g^2}$
Reciprocal Rule	1/f	$-f'/f^2$
Chain Rule (using $\frac{d}{dx}$)		$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

Practical Components

Complete the following questions. Organise your solutions neatly in your exercise book.

You will require Chapter 7 of Maths Quest Methods (pdf – Google Classroom).

Ex 7J Mixed problems on differentiation
Qs 1, 2 and 3 (all non-log problems)

Resources:

- *Year 12 Maths Quest Methods* Chapter 7

Investigation

See next page.

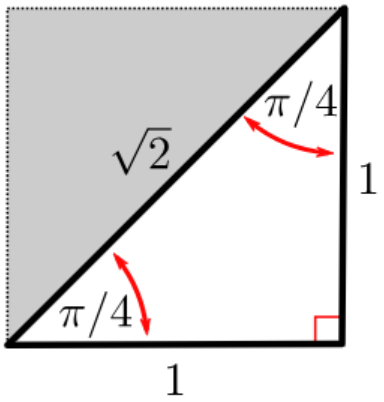
QFO
Quiz/Forum/Other

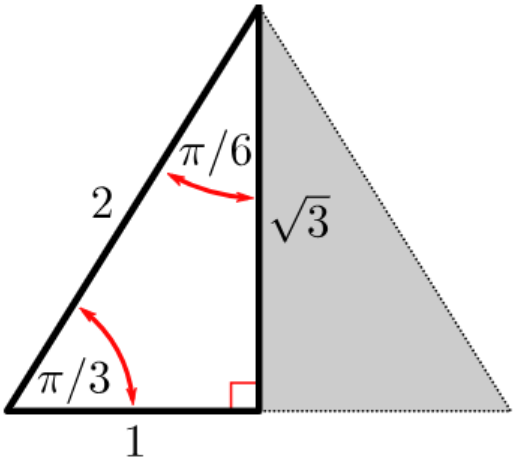
Remember to check-in with Serene each lesson and get your name marked off.
Our in-class problem solving task will be during your double in Week 6.

MM3 Week 5/6 Investigation

Review and complete the following rules to do with the exact values of trigonometric functions at values of θ. Some are already filled in for you. Assume all values of θ are in radians.

Remember: $\tan(\theta) = \sin(\theta)/\cos(\theta)$





θ value–> Trig Function ↓	0/2π	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$
sin(θ)	0	$\frac{1}{2}$			1	$\frac{\sqrt{3}}{2}$		
cos(θ)	1		$\frac{1}{\sqrt{2}}$		0		$\frac{-1}{\sqrt{2}}$	
tan(θ)	0			$\frac{\sqrt{3}}{1}$	UNDEFINED			$\frac{-1}{\sqrt{3}}$
θ value–> Trig Function ↓	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$
sin(θ)	0	$\frac{-1}{2}$			-1	$\frac{-\sqrt{3}}{2}$		
cos(θ)	-1		$\frac{-1}{\sqrt{2}}$		0		$\frac{1}{\sqrt{2}}$	
tan(θ)	0			$\frac{\sqrt{3}}{1}$	UNDEFINED			$\frac{-1}{\sqrt{3}}$

Now, answer the following question algebraically.

f(x) = $\sin(x) + \cos(x)$. Find a value of x where f(x) and f'(x) = 1.