Week 2
Term 1
2024

HAWKER COLLEGE
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Learning Brief MM3:
Further differentiation and applications

Goals

By the end of this week, you should be able to:

- learn and use the chain rule for differentiation
- find the derivatives of exponential functions of the forms: $y = e^x$, and $y = e^{f(x)}$
- estimate the limit of $\frac{a^h - 1}{h}$ as $h \to 0$ using technology, for various values of $a > 0$
- recognise that e is the unique number a for which the above limit is 1
- establish and use the formula $\frac{d}{dx}(e^x) = e^x$
- use exponential functions and their derivatives to solve practical problems.

Theoretical Components

STEP 1
Resources:
Maths Quest Year 12 Chapter 7
Chain Rule Proof

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

https://www.khanacademy.org/math/ap-calculus-ab/product-quotient-chain-rules-ab/chain-rule-proof-ab/v/chain-rule-proof

What is base e?
Read through to get an insight on the number 'e':
https://www.investopedia.com/terms/e/eulers-constant.asp

https://www.youtube.com/watch?v=m2MIpDrF7Es

https://www.mathsisfun.com/numbers/e-eulers-number.html

http://www.mathopolis.com/questions/q.php?id=2011&site=1&ref=/numbers/e-eulers-number.html&qs=2011_2012_2013

Derivative of $y = e^x$, and $y = e^{f(x)}$ from first principles
See the following page

Practical Components

STEP 2
Read and make notes examples 13, 14 and 15 from Chapter 7

Read and make notes for examples 16 - 19 from Chapter 7

Ex 7D The Chain Rule
Q's 1, 2, 4, 5, 6, 7 (b,d,f,h), 8, 10 (c,d), 13

Ex 7E The Derivative of e^x and $e^{f(x)}$
Q's 1 (a,d,g), 2 (b,e,h,k), 3, 4 (a,d,g), 5 (c,f,i,l), 6, 7, 8, 9

Investigation

STEP 3
See below. Be sure to show your results to Serene.

QFO
Quiz/Forum/Other

Complete the Cambridge Task!

Further Notes:

Chain rule

1. A composite function is a function composed of two (or more) functions.
2. Composite functions can be differentiated using the chain rule, $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$.
3. A short way of applying the chain rule is:
   If $f(x) = [g(x)]^n$ then $f'(x) = n[g(x)]^{n-1} \times g'(x)$.

The derivative of e^x

1. If $f(x) = e^x$, $f'(x) = e^x$.
2. If $f(x) = e^{kx}$, $f'(x) = ke^{kx}$.
3. If $f(x) = ae^{kx+c}$, $f'(x) = ake^{kx+c}$.
4. If $f(x) = ae^{g(x)}$, $f'(x) = g'(x) \times ae^{g(x)}$.

Derivative of $f(x) = e^x$ from first principles:

$$\begin{aligned} f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}, h \neq 0 \\ &= \lim_{h \to 0} \frac{e^{x+h} - e^x}{h} \\ &= \lim_{h \to 0} \frac{e^x e^h - e^x}{h} \\ &= \lim_{h \to 0} \frac{e^x(e^h - 1)}{h} \\ &= e^x \lim_{h \to 0} \frac{e^h - 1}{h} \end{aligned}$$

Note that $\lim_{h \to 0} \frac{e^h - 1}{h}$ can be deduced by using a calculator and substituting values of h close to zero.

h	$\frac{e^h - 1}{h}$
0.01	1.0050
0.0001	1.00005
0.000001	1.000000

That is, $\lim_{h \to 0} \frac{e^h - 1}{h} = 1$.

Therefore, $f'(x) = e^x \times 1$
$= e^x$

If $f(x) = e^x$ then $f'(x) = e^x$.

Note: e^x is the only function which has itself as a derivative.

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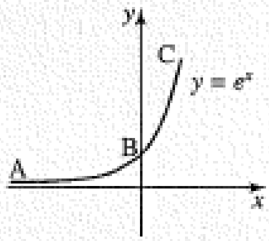
MM3 Week 2 Investigation **Name:**

Please submit your response for checking 😊
You can use https://www.desmos.com/calculator or CAS calculator for your graphs.

The graph of $y = Ne^{kx}$

The diagram at right shows the graph of $y = e^x$ and uses the letters A, B and C to indicate key parts of the graph.

In this investigation you will use your graphics calculator to observe and report on the effect of changing N and k in the equation $y = Ne^{kx}$.



1 $N = 2$. On the same axes, graph the equations $y = 2e^x$ and $y = e^x$.
In your book, sketch the view window.
Write a sentence summarising the effect of changing N from 1 to 2.

2 $N = -1$. On the same axes, graph the equations $y = -1 \times e^x$ and $y = e^x$.
In your book, sketch the view window.
Write a sentence summarising the effect of changing N from 1 to -1.

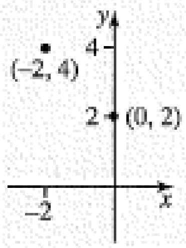
3 $k < 0$. On the same axes, graph the equations $y = e^x$ and $y = e^{-x}$.
In your book, sketch the view window.
Write a sentence summarising the effect of changing k from 1 to -1.

4 $0 < k < 1$. On the same axes, graph the equations $y = e^x$ and $y = e^{0.5x}$.
In your book, sketch the view window.
Write a sentence summarising the effect of changing k from 1 to 0.5.

5 $k > 1$. On the same axes, graph the equations $y = e^x$ and $y = e^{2x}$.
In your book, sketch the view window.
Write a sentence summarising the effect of changing k from 1 to 2.

Challenge

Use your calculator to obtain a guess-and-check solution to the following problem. Find the values of k and N such that the graph of $y = Ne^{kx}$ passes through $(-2, 4)$ and $(0, 2)$, the points shown.



Write down your guesses and the adjustments you made to find the equation of the challenge question.

Complete the challenge algebraically as well.