Week 13-14
Term 2
2024

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MM3
Integrals

x

Goals

By the end of this fortnight, you should be able to:

- interpret the definite integral $\int_a^b f(x)dx$ as area under the curve $y = f(x)$ if $f(x) > 0$
- recognise the definite integral $\int_a^b f(x)dx$ as a limit of sums of the form $\sum_i f(x_i)\,\delta x_i$
- interpret $\int_a^b f(x)dx$ as a sum of signed areas
- recognise and use the additivity and linearity of definite integrals. (MMT31)
- understand the concept of the signed area function $F(x) = \int_a^x f(t)dt$
- understand and use the theorem: $F'(x) = \frac{d}{dx}\left(\int_a^x f(t)dt\right) = f(x)$, and illustrate its proof geometrically
- understand the formula $\int_a^b f(x)dx = F(b) - F(a)$ and use it to calculate definite integrals

Theoretical Components

Exact area under the curve using definite integral:

The fundamental theorem of calculus is $\int_a^b f(x)\,dx = [F(x)]_a^b = F(b) - F(a)$,
where $F(x)$ is an antiderivative of $f(x)$.

https://www.3blue1brown.com/lessons/essence-of-calculus

http://www.youtube.com/watch?v=ODwkTt0RMDg&feature=relmfu

https://www.youtube.com/watch?v=GtCYrxxTjH4

https://www.khanacademy.org/math/ap-calculus-ab/fundamental-theorem-of-calculus-ab/fundamental-theorem-of-calculus-tut-ab/v/fundamental-theorem-of-calculus

Practical Components

Complete the following questions. Organise your solutions neatly in your exercise book.

You will require Chapter 9 of Maths Quest Methods (pdf – Google Classroom).

Ex 9E The fundamental theorem of integral calculus
- Qs 1 (1st col), 2 (b,h,k,q), 5 (a,c,d), 6, 10

Ex 9F Signed areas
- Qs 3(a, b, c) 4(a, b , c) , 5(e), 6(e,i), 10

Ex 9G Further areas
- Qs 2 (a,e,f), 4, 11, 14, 17
-

Ex 9H Areas between two curves
- Qs 1 (a,e,f,g), 5, 11, 14

Ex 9J Further applications of integration
- Qs 1, 5, 7, 10, 11

Investigation

See separate page.

QFO
Quiz/Forum/Other

Complete the Cambridge Task.

Don't forget to check in with Serene at the start and end of every lesson.

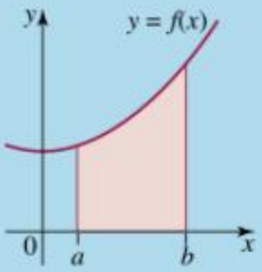
Properties of definite integrals

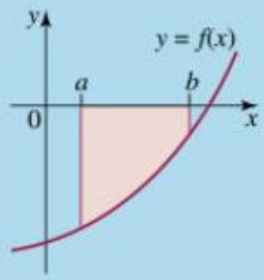
Definite integrals have the following five properties.

1. $\int_a^a f(x)\,dx = 0$
2. $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx,\ a < c < b$
3. $\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$
4. $\int_a^b [f(x) + g(x)]\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$
5. $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

Area under curves

- Area $= \int_a^b f(x)\,dx$, if $f(x) > 0$ for $x \in [a, b]$



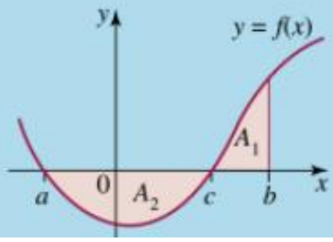


- Area $= -\int_a^b f(x)\,dx$, if $f(x) < 0$ for $x \in [a, b]$

- Area $= \int_c^b f(x)\,dx - \int_a^c f(x)\,dx$

  $= \int_c^b f(x)\,dx + \left|\int_a^c f(x)\,dx\right|$, if $f(x) > 0$ for $x \in [c, b]$

  and $f(x) < 0$ for $x \in [a, c]$



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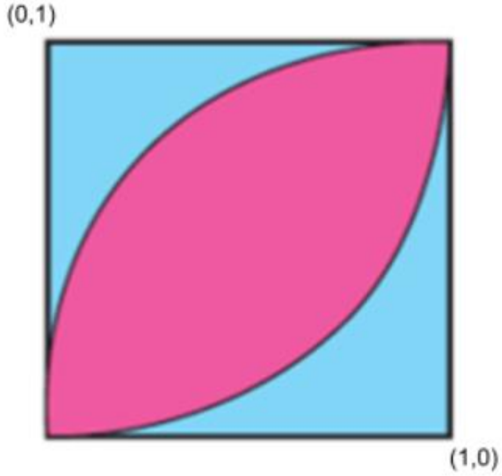
MM3 Week 13-14 Investigation

The square below has an area of 1 square unit.



Assuming that the curves are arcs of circles where the centres are at the opposite corners of the square:

a) Find the equations for the two arcs in general form ($y = \cdots$)

b) Determine the area of the enclosed darker shaded shape in the square **in terms of integrals**. *You do not need to calculate the numerical value of the area.*

> *Hint:* The centre-radius form of the circle equation is in the format $(x - h)^2 + (y - k)^2 = r^2$, with the centre being at the point (h, k) and the radius being "r". *The (h, k) values are given, as if placed on the cartesian plane.*