Week 9/10
Term 3 •
2023

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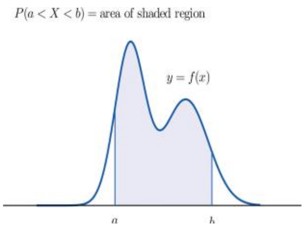
MM4
Continuous Random Variables

Goals



By the end of this fortnight, you should be able to:

• distinguish between discrete and continuous random variables (CRV)

• understand probability density functions and cumulative distributions for continuous random variables

• compute the central tendency and variability (spread) of continuous distributions

Theoretical Components

Resources:

- Quest Mathematical Methods 12, Chapter 12 (see pdf on Google Drive)

1. $f(x) \geq 0, \quad for \ all \ x$
2. $\int_{-\infty}^{\infty} f(x)dx = 1$
3. $P(a \leq X \leq b) = \int_{a}^{b} f(x)dx$

For a continuous random variable X with probability density function f:

- the **mean** or **expected value** of X is given by $\mu = E(X) = \int_{-\infty}^{\infty} xf(x)\,dx$
- the expected value of $g(X)$ is given by $E[g(X)] = \int_{-\infty}^{\infty} g(x)f(x)\,dx$
- the **median** of X is the value m such that $\int_{-\infty}^{m} f(x)\,dx = 0.5$

- The **variance** of a continuous random variable X with probability density function f is defined by

$$\sigma^2 = \text{Var}(X) = E[(X-\mu)^2]$$
$$= \int_{-\infty}^{\infty}(x-\mu)^2 f(x)\,dx$$

provided the integral exists. To calculate the variance, use

$$\text{Var}(X) = E(X^2) - \mu^2$$

- The **standard deviation** of X is defined by

$$\sigma = \text{sd}(X) = \sqrt{\text{Var}(X)}$$

- The **interquartile range** of X is

$$\text{IQR} = b - a$$

where a and b are such that

$$\int_{-\infty}^{a} f(x)\,dx = 0.25 \quad \text{and} \quad \int_{-\infty}^{b} f(x)\,dx = 0.75$$

and where f is the probability density function of X.

Khan Academy quiz:
https://www.khanacademy.org/math/statistics-probability/random-variables-stats-library/random-variables-continuous/e/probability-density-curves

Practical Components

Do the following questions from **Chapter 12: Continuous random variables and their probability distributions** (pdf – GC). Organise your solutions neatly in your exercise book.

Ex 12A Continuous random variables
- Qs 1 a, d, g; 2 a, g; 4; 9

Ex 12B Using a probability density function to find probabilities of continuous random variables
- Qs 1; 3; 5; 6; 9; 12; 17 (Use technology to find the integral for this question)

Ex 12C Measures of central tendency and spread
- Qs 1; 3; 5; 9

Investigation

X is a random variable denoting the number of minutes in excess of two hours which a person takes to travel from one town to another. The probability density function is defined by:

$$f(x) = \begin{cases} k(10+x), & -10 \leq x \leq 0 \\ k(10-x), & 0 \leq x \leq 10 \\ 0, & elsewhere \end{cases}$$

(a) Find the value of k
(b) Sketch the graph of f
(c) Find the probability that:
(i) X is less than 5
(ii) X is less than 0, given that X is less than 5
(iii) -2 ≤ X < 3

QFO

Quiz/Forum/Other

No Mathspace this brief 😉

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Probability revision

- *Outcomes* are results of experiments.
- The set of all possible outcomes of an experiment is called the *sample space* and is denoted by ε, and each possible outcome is called a *sample point*.
- A subset of the sample space is known as an *event*.
- The *union* (symbol ∪) of two events *A* and *B* implies a combined event, that is, either event *A* or event *B* or both occurring. Common elements are written only once.
- The *intersection* (symbol ∩) of two events *A* and *B* is represented by the common sample points of the two events.
- Venn diagrams involve drawing a rectangle that represents the sample space and a series of circles that represent subsets of the sample space. They provide a visual representation of the information at hand and clearly display the relationships between sets.
- The probability of an event occurring is defined by the rule:

$$\Pr(A) = \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}.$$

- The probability of an event occurring lies within the restricted interval $0 \leq \Pr(A) \leq 1$.
- The individual probabilities of a particular experiment will sum to 1; that is, $\Sigma\, p(x) = 1$.
- Conditional probability is defined by the rule $\Pr(A \mid B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$ where $\Pr(B) \neq 0$.
- This can be transposed to $\Pr(A \cap B) = \Pr(A \mid B) \times \Pr(B)$.