Week 5/6
Term 3
2023

HAWKER COLLEGE
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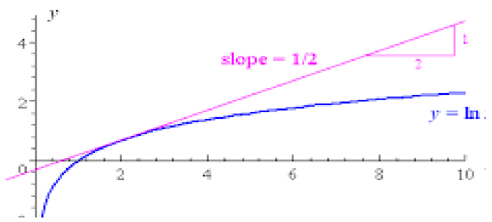
MM4
Logarithmic Functions

Goals



This fortnight we are:

- Investigating Euler's number
- Solving indicial and logarithmic equations using base e
- Further graphing logarithmic functions. Domain and range
- Use natural logarithms to the base e. Learn the notation used
- Using exponential and logarithmic modelling
- Review the chain rule for differentiation of exponential functions of the forms: $y = e^x$, and $y = e^{f(x)}$
- Find the derivatives logarithmic functions of the forms: $y = ln(x)$ and $y = ln[f(x)]$.
- Use derivatives to solve practical problems and further applications of

Theoretical Components

Resources
Maths Quest Year 12 Chapter 3 (pdf – GC)
Chapter 9: Logarithmic functions using calculus (pdf – GC)
What is base e?
Learn more about Euler's number.
Read through to get an insight on the number 'e':
http://www.mathopolis.com/questions/q.php?id=2011&site=1&ref=/numbers/e-eulers-number.html&qs=2011_2012_2013

Watch:
http://bit.ly/w8OiD

You need to remind yourself of exponential functions, their graphs and how to use the product, quotient, and chain rule:

Product rule

(a) If $y = uv$ then $\frac{dy}{dx} = u \times \frac{dv}{dx} + v \times \frac{du}{dx}$.

(b) If $f(x) = u(x) \times v(x)$ then $f'(x) = u(x) \times v'(x) + v(x) \times u'(x)$.

Quotient rule

(a) If $y = \frac{u}{v}$ then $\frac{dy}{dx} = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$.

(b) If $f(x) = \frac{u(x)}{v(x)}$ then $f'(x) = \frac{v(x)u'(x) - u(x)v'(x)}{[v(x)]^2}$.

The chain rule

- The chain rule of differentiation is:

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

- A short way of applying the chain rule is:
If $f(x) = [g(x)]^n$ then $f'(x) = n[g(x)]^{n-1} \times g'(x)$.

Practical Components

Complete the following questions. Organise your solutions neatly in your exercise book.

You will require Chapter 3 of Maths Quest Methods (pdf – Google Classroom).

3E Exponential equations (base *e*)
Qs 1 - 5 (2 or 3 from each), 7, 8

3F Equations with natural (base *e*) logarithms
Qs 1 - 4 (2 or 3 from each), 5 - 10

3G Inverses
Qs 1 - 4 (2 from each), 5 - 7

3I Exponential and logarithmic modelling
As many as you can (at least every 2nd Q)

Do the following questions from **Chapter 9: Logarithmic functions using calculus** (pdf – GC).

Ex 9.2 The derivative of $f(x) = \log_e x$
Qs 1 – 4 (any 2 from each), 6 (all), 7, 13, 14 (a)

Mathspace task - Differentiation of Exponential Functions

Investigation

See the following pages

QFO

Quiz/Forum/Other

Quiz on mathspace: Differentiation of Exponential Functions
https://mathspace.co/student/tasks/SubtopicCustomTask-1437655/

Summary of derivatives:

$f(x)$	$f'(x)$
c	0
ax^n	nax^{n-1}
$[g(x)]^n$	$ng'(x)[g(x)]^{n-1}$
e^x	e^x
e^{kx}	ke^{kx}
$e^{g(x)}$	$g'(x)e^{g(x)}$
$\log_e(x)$	$\frac{1}{x}$
$\log_e(kx)$	$\frac{1}{x}$
$\log_e[g(x)]$	$\frac{g'(x)}{g(x)}$
$\sin(ax)$	$a\cos(ax)$
$\cos(ax)$	$^{-}a\sin(ax)$
$\tan(ax)$	$\frac{a}{\cos^2(ax)}$ $(= a\sec^2(ax))$

- $\log_a(a^x) = x$
- $a^{\log_a(x)} = x$

$$\log_a x = \frac{\ln x}{\ln a}$$

Indicial equations

- To solve indicial equations:
  1. write all terms with the same base, write terms with the smallest possible base or take the logarithm of both sides of the equation
  2. then solve the equation.
- A negative number cannot be expressed in index form.
- If $0 < x < 1$, then $\log_a x < 0$ and if $x > 1$ then $\log_a x > 0$.
- It is not possible to take the logarithm of a negative number.

Exponential equations (base e)

- Euler's number $e = \lim_{h \to \infty}\left(1 + \frac{1}{n}\right)^n = 2.718\,281\,828\,459\ldots$
- The laws of indices and logarithms apply in the same way when using e.

Equations with natural logarithms (base e)

- To solve logarithmic equations use the laws of logarithms and indices.

Continuous growth and decay is modelled by the equation $A = A_0e^{kt}$, A_0 represents the initial value (that is, when $t = 0$) and k represents the rate constant.

Week 5 & 6 Investigation

To verify that two functions are inverses of each other, you need to compose the functions (ie. plug one function into the other function then simplify) and show that the composed function equals x. For the two functions to be inverses of each other, you must show that function composition works both ways. $\boldsymbol{f[g(x)] = x = g[f(x)]}$

Example:

Determine algebraically whether $f(x) = 3x - 2$ and $g(x) = \frac{x+2}{3}$ are inverses of each other.

I will plug the formula for $g(x)$ into every instance of "x" in the formula for $f(x)$:

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= f\left(\frac{x+2}{3}\right) \\ &= 3\left(\frac{x+2}{3}\right) - 2 \\ &= (x+2) - 2 \\ &= x\end{aligned}$$

Now I will plug the formula for $f(x)$ into every instance of "x" in the formula for $g(x)$:

$$\begin{aligned}(g \circ f)(x) &= g(f(x)) \\ &= g(3x - 2) \\ &= \frac{(3x-2)+2}{3} \\ &= \frac{3x}{3} \\ &= x\end{aligned}$$

Since the results both came out with just "x", **$f(x)$ and $g(x)$ are inverses of each other.**

If either or both do not end up with just "x", then the two equations are not inverses of each other.

For you to do:

A: Use the same method to prove $f(x) = e^x$ and $g(x) = log_e(x)$ are the equations of inverse functions.

B: Use graphical methods to show that $y = e^x$ and $y = log_e(x)$ are inverses of each other.