## Goals Topic 3: Networks and decision mathematics

This brief:

- review networks
- explain the meaning of the terms tree and spanning tree and identify practical examples
- identify a minimum spanning tree in a weighted connected graph either by inspection or by using Prim's algorithm
- use minimal spanning trees to solve minimal connector problems; for example, minimising the length of cable needed to provide power from a single power station to substations in several towns


## Theoretical components

## Content

Content this week is in the slideshow, posted on the Google Classroom.

## Knowledge Checklist

- Review network, matrices, and graphs
- Weight of networks
- Minimum and maximum spanning trees
- Prim's algorithm


## Practical components

There are questions to be answered in the booklet Week 11/12 Notes \& Exercises.

Here is a video explaining Prim's algorithm: https://www.youtube.com/watch?reload=9\&v= cplfcGZmX71


## Investigation

On HawkerMaths and attached to this week's work

WEEK 11 and 12 NOTES \& EXERCISES

## Weights

1. Consider the following network:
a. What is the degree of vertex $A$ ?
b. What is the degree of vertex $C$ ?

2. Consider the following network:
a. What is the degree of vertex $A$ ?
b. What is the degree of vertex C ?

3. Consider the following networks:


A


B


C


D

Complete the table for the graphs

| Graph | Vertices | Faces | Edges | V + F - E | Planar? |
| :---: | :--- | :--- | :--- | :--- | :--- |
| A |  |  |  |  |  |
| B |  |  |  |  |  |
| C |  |  |  |  |  |
| D |  |  |  |  |  |

4. Considering the following networks:

Which network is a subnetwork of the other?


Network 1


Network 2
5. Consider the following two networks:

Is Network 2 a subgraph of Network 1? Justify.


Network 1


Network 2
6. Consider the following network:
a. Is this a connected network? If not, how would you make the network a completed graph?

b. If possible, list a trail from R to U :
7. Find the weights of the following paths in this network:
a. Path Y,E,D:
b. Path $A, B, Z, X$ :

c. Path $X, C, A, B, Z$ :

## Distance optimisation

1. A few friends decided to carpool on the way to school. The following map displays the routes between their homes. Starting from house A, what is the shortest path that visits every house exactly once (Hamiltonian path)?

2. A rock band is planning a tour across several cities. The following table represents the routes and distance between the cities at which they will be performing.

| Starting city | Ending city | Distance |
| :---: | :---: | :---: |
| A | C | 9 |
| A | F | 6 |
| B | E | 11 |
| B | D | 5 |
| C | F | 3 |
| C | D | 7 |
| E | F | 5 |
| E | 8 |  |

a. Construct a network diagram for this information
b. Find the shortest route for the rock band starting at city A and passing by each city only once (Hamiltonian path).
3. The post office needs to make multiple pick-ups from several stores. The following map displays the routes between the stores. Stores A and E need immediate pick up. Starting from store A, then immediately going to Store E, what would be the shortest path to cross if passing over every store only once?

4. The firefighting department received an emergency call about a house fire across town. The following graph shows the local town map. If the firefighting department is located at vertex $L$, and the house is at vertex $A$, what is the shortest route between $L$ and $A$ ?


## Trees and spanning trees

1. In your own words define the difference between a tree and a spanning tree:
2. For each of the following graphs, state whether it is a tree, and if not, why not?
a.

b.

c.

3. Construct a network from the matrix given and explain why it is not a tree.
$A$
$A$
$C$
$D$
$E$$\left[\begin{array}{llllll} & B & C & D & E \\ 0 & 2 & 0 & 1 & 0 \\ 2 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0\end{array}\right]$
4. Rose is writing a report and needs to create a table of contents. Her report consists of four 'chapters', each broken down into sections. The first chapter is her introduction, so it has just one section. Her second and third chapters both have three sections, while the fourth chapter has six sections.
Create a tree that represents the structure of Rose's table of contents.
5. Gabriel wishes to set up a surround sound system in his house. His amplifier is located at vertex C in the graph. Every room must be connected to the amplifier by wiring. Draw a spanning tree showing the edges where the wiring will be needed.

6. Consider the following network:
a. What is the weight of the network?
b. Construct three possible trees. Show the weights on the branches and the total weight of each tree.
7. Find the minimum spanning tree for the network below

8. Water pipes need to be laid down in a new area that is currently undergoing construction. The following is a suggested pipe map. To minimise the cost, the council wants to minimise the total length of water pipes used. Find the minimum spanning tree.


## Prim's algorithm

1. Use Prim's algorithm to find the minimum spanning tree for this network.

2. Find the minimum spanning tree for this network. What is the weight?

3. Use Prim's algorithm to find the minimum spanning tree for this network (you have seen this before).

4. The direct flight routes between 5 major cities of the same country are shown in the network below. Each edge is marked with the distance (in km ) travelled on each route. To optimist the investment put into these routes, the operating airline wants to find the minimum spanning tree for the network.

5. The paths between the various cages at the Nolonger Park Zoo are dirt paths and when it rains, they become muddy. The network below shows all the paths with distances in metres. Management has decided to install concrete paths.

a. What is the total length of the path required if each dotted line was to become a concrete path?
b. Using Prim's algorithm, find the minimum length of the concrete path that is required so patrons can see each exhibit and visit the kiosk without walking on a dirt path.
c. Repeat Prim's algorithm using a different starting point and show that it does not matter where you start.

## 2023 MA3 Week 11/12 Investigation

An airline wishes to service six cities. The directors have decided that it is too costly to have direct flights between all the cities. The airline needs to minimise the number of routes which they open and maximise the total number of passengers that they can carry. The table below shows the projected numbers of passengers between these cities.

|  | A | B | C | D | E | F |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A |  | 121 | 317 |  | 456 | 293 |
| B | 121 |  | 356 | 301 |  | 243 |
| C | 317 | 356 |  | 299 | 386 |  |
| D |  | 301 | 299 |  | 267 | 179 |
| E | 456 |  | 386 | 267 |  | 346 |
| F | 293 | 243 |  | 179 | 336 |  |

1. Construct a network diagram from this data.
2. Find the maximum spanning tree that will meet the airlines requirements.
3. Find the total carrying capacity of this tree.

| CRITERIA | EXPECTATIONS | POSS | MULT | GIVEN | TOTAL |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Practical | Student completes practical work, including exercises and any Mathspace and/or other tasks, of the brief to an acceptable standard set by the teacher. | 2 | 3 |  | /6 |
| Investigation Task | Student completes the investigation task of the week to an acceptable standard set by the teacher. | 2 | 2 |  | 14 |
| Communication and Reasoning | Student responses are accurate and appropriate in presentation of mathematical ideas in different contexts, with clear and logical working out shown. | 4 | - |  | 14 |
| Knowledge and Application | Student submitted work selects and applies appropriate mathematical modelling and problem-solving techniques to solve practical problems and demonstrates proficiency in the use of mathematical facts, techniques, and formulae. | 4 | - |  | 14 |
| Submission Guidelines |  |  |  |  |  |
| Timeliness | Student submits the practical work, including exercises and any Mathspace and/or other tasks, and investigation by the set deadline. See scoring guidelines for specific details. | 2 | - |  | 12 |
|  |  |  |  | FINAL | 120 |

Student Reflection: How did you go with this week's work? What was interesting? What did you find easy? What do you need to work on?

## Optional: Not assessed this week.

1. Consider the following network:
a. Is this a connected network? If not, how would you make the network a completed graph?
b. If possible, list a trail from $P$ to $Q$ :

2. Find the weights of the following paths in this network:
a. Path $Y$ to $X$ :
b. Path Z to C:
c. Path $B$ to $X$ :

3. Consider the following network:
a. What is the weight of the subnetwork with vertices $Q, R, S$ and $T$ ?
b. What is the weight of the subnetwork with vertices $P, Q, S, T$ and $U$ ?
4. The city is deciding on a new school bus route for a new area. The following map shows the routes connecting each intersection. The city wants to save as much time as possible on this route. If the route starts at vertex D, what is the shortest path for the bus to travel if the bus is passing over each house once?

5. Find the minimum spanning tree for the network below

6. Find the minimum spanning tree for the network below

7. Find the minimum spanning tree and its weight for this network.

8. A university is planning to connect all the campus buildings using underground fibre optic cables. The network below was suggested by the engineer. The project accountant wanted to minimise the cost of the project. What is the minimum spanning tree?

