

Goals



This week:

- calculate the effective annual rate of interest and use the results to compare investment returns and cost of loans when interest is paid or charged daily, monthly, quarterly or six-monthly
- with the aid of a calculator or computer-based financial software, solve problems involving compound interest loans or investments; for example, determining the future value of a loan, the number of compounding periods for an investment to exceed a given value, the interest rate needed for an investment to exceed a given value

Theoretical Components

Resources:

Week 6: Loan Repayment Spreadsheet

Knowledge Checklist

- Reducing balance loans
- Annuities formula
- Interest charged per period
- Calculating r and R
- Calculating n
- Meaning of the variables A , P and Q
- Number of weeks, fortnights, months and quarters in a year

Practical Components

There are questions to be answered in the booklet *Week 6/7 Notes & Exercises*.

Investigation

End of Brief

Number of repayments

The situation often arises in reducing balance loans where a potential borrower knows how much money that needs to be borrowed as well as the amount that can be repaid each month. The person also wants to know how long the loan needs to be to accommodate for these conditions, that is, to determine the number of repayments, n , required.

n can be calculated by trial and error. The annuities formula is used to calculate the amount owing, A , after various times until a value of n gives a value for A close to zero (loan repaid). The values of n chosen will be high if the repayment value, Q , is low, the principal is high, the interest rate is high or if the repayments are made often.

As a guide, remember that the principal is repaid more quickly towards the latter stages of the loan. This will also influence what n values are chosen.

Example 1

A reducing balance loan of \$60,000 is to be repaid with monthly instalments of \$483.36 at an interest rate of 7.5% p.a. (debited monthly). Calculate:

- The number of monthly repayments (and, hence, the term of the loan in more meaningful units) needed to repay the loan in full.
- The total interest charged.

Solution

The number of monthly repayments (and, hence, the term of the loan in more meaningful units) needed to repay the loan in full.

Step 1: State the loan amount, P , regular repayments, Q , and growth factor, R .

$$\begin{aligned} P &= 60000 \\ Q &= 483.36 \\ r &= \frac{7.5}{12} = 0.625 \\ R &= 1 + \frac{0.625}{100} = 1.00625 \end{aligned}$$

Step 2: Since the loan amount, P , is high. It is expected the frequency of repayments, n , is also high. Use trial and error and the spreadsheet provided in *Week 5*.

Try $n = 200$, $A = \$17,058.38$. n needs to be larger.
Try $n = 220$, $A = \$9,058.86$. n needs to be larger.
Try $n = 240$, $A = -\$2.26$. since A is negative, this means that the loan is paid off.

$$\begin{aligned} n &= 240 \\ \text{Time} &= \frac{240}{12} = 20 \end{aligned}$$

Step 3: Write your answer.

It will take 20 years for the loan to be paid off

The total interest charged.

Step 1: Total interest = Total repayments – Amount borrowed

$$\begin{aligned} \text{Total Interest} &= 483.36 \times 240 - 60000 \\ \text{Total Interest} &= 116006.4 - 60000 \\ \text{Total Interest} &= 56006.4 \end{aligned}$$

Step 2: Write your answer.

The total interest on a \$60,000 loan is \$56,006.40

Example 2

Laura borrowed \$14,000 to buy a car some time ago. The interest on this reducing balance loan has been charged at 9.2% p.a. (debited monthly) and she has been paying \$446.50 each month to service the loan. She currently still owes \$9,753.92. How long ago did Laura borrow the money?

Solution

Step 1: Find P , A , Q and R .

$$P = 14000, A = 9753.92, Q = 446.50$$

$$r = \frac{9.2}{12} = 0.76667$$

$$R = 1 + \frac{0.76667}{100} = 1.0076667$$

Step 2: Use trial and error and the spreadsheet.

$$A_{20} = \$6699.13$$

$$A_{10} = \$10488.90$$

$$A_{11} = \$10122.81$$

$$A_{12} = \$9753.92$$

Step 3: Write your answer.

$n = 12$ months, Laura borrowed the money a year ago.

Example 3

A loan of \$11,000 is being repaid by monthly instalments of \$362.74 with interest being charged at 11.5% p.a. (debited monthly). Currently, the amount owing is \$7,744.05. How much longer will it take to:

- Reduce the amount outstanding to \$2,105.11?
- Repay the loan in full?

Solution

Reduce the amount outstanding to \$2,105.11?

Step 1: Find P , A , Q and R .

$$P = 7744.05, A = 2105.11, Q = 362.74$$

$$r = \frac{11.5}{12} = 0.958333$$

$$R = 1 + \frac{0.958333}{100} = 1.00958333$$

Step 2: Using the spreadsheet in Week 5.

$$A_{20} = \$1416.69$$

$$A_{15} = \$3113.42$$

$$A_{18} = \$2105.11$$

Step 3: Write your answer.

It will take a year and a half to reduce the loan to \$2,105.11

Repay the loan in full?

Step 1: Find P , A , Q and R .

$$P = 7744.05, Q = 362.74$$

$$r = \frac{11.5}{12} = 0.958333$$

$$R = 1 + \frac{0.958333}{100} = 1.00958333$$

A needs to be as close to zero.

Step 2: Using the spreadsheet in Week 5.

$$A_{20} = \$1416.69$$

$$A_{24} = -\$0.17$$

Step 3: Write your answer.

It will take another 2 years to repay the loan in full.

Exercise 1

1. Using the annuities formula, find the number of monthly repayments, n , and hence the term of the loan in more meaningful units, needed to repay a loan in full, with interest (debited monthly) charged at the rate of:
 - a. 9% p.a., if $P = \$70,000$ and $Q = \$629.81$
 - b. 12% p.a., if $P = \$45,000$ and $Q = \$540.08$
2. Jim has a reducing balance loan of \$3,500 that he is using for a holiday and has agreed to repay it by monthly instalments of \$206.35 at a rate of 7.6% p.a. (interest debited monthly). Calculate:
 - a. The number of repayments needed to repay in full and determine this in years
 - b. The total interest charged

3. Given $A = \$4,297.63$, $P = \$8,000$, $Q = \$387.89$ per month, $r = 15\%$ p.a. (debited monthly). Use the annuities formula to calculate:
- n and the time in years
 - how long it would take to fully pay off the loan
4. Grace has borrowed \$48,000 to set up a florist and is repaying the reducing balance loan by monthly instalments of \$401.17 at 5.85% p.a. (debited monthly). At present, she still owes \$36,381.40.
- How much longer will it take to pay off this loan?
 - What was the term of the loan?

Effects of changing the repayment

Since most loans are taken over a long period of time, it is probable that a borrower's financial situation will change during this time. For instance, a borrower may receive a pay rise and so the take home pay is greater per week or fortnight. The person may choose to increase the value of the repayments made to service the loan. It may also be that a borrower's financial situation deteriorates, in which case they may request from their institution for the repayment value to be decreased.

Example 1

A reducing balance loan of \$16,000 has a term of 5 years and it is to be repaid by monthly instalments at a rate of 8.4% p.a. (debited monthly).

- Find the repayment value.
- What will be the term of the loan if the repayment is increased to \$393.62?
- Calculate the total interest paid for repayments of \$393.62.
- By how much does the interest figure in part c differ from the interest paid for the original offer?

Solution

Find the repayment value.

Step 1: Find P , n , r and R .

$$\begin{aligned}P &= 16000, n = 5 \times 12 = 60 \\r &= \frac{8.4}{12} = 0.7 \\R &= 1 + \frac{0.7}{100} = 1.007\end{aligned}$$

Step 2: Using the formula for Q in the spreadsheet. $Q = 327.49$

Step 3: Write your answer.

A \$16,000 loan will be paid off in 5 years at 8.4% p.a. with a monthly repayment of \$327.49

What will be the term of the loan if the repayment is increased to \$393.62?

Step 1: P , n , r and R are the same as part a but $Q = A_{48} = -\$0.12$ 327.49. Using the annuities formula in the spreadsheet (or trial and error).

Step 2: Write your answer.

When the monthly instalments increase to \$393.62, the term of the loan will be 4 years.

Calculate the total interest paid for repayments of \$393.62.

Step 1: Total interest = Total repayments – Amount borrowed

$$\begin{aligned}\text{Total Interest} &= 393.62 \times 48 - 16000 \\ \text{Total Interest} &= 2893.76\end{aligned}$$

Step 2: Write your answer.

The total interest over 4 years will be \$2,893.76

By how much does the interest figure in part c differ from the interest paid for the original offer?

Step 1: Total interest = Total repayments – Amount borrowed, using $Q = 327.49$ and a term of 5 years

$$\begin{aligned}\text{Total Interest} &= 327.49 \times 60 - 16000 \\ \text{Total Interest} &= 3649.40\end{aligned}$$

Step 2: Total interest – Part c

$$3649.40 - 2893.76 = 755.64$$

Step 3: Write your answer.

If the repayment is increased from \$327.49 to \$393.62 per month, then \$755.64 is saved in interest payments.

If a borrower does increase the value of each repayment and if all other variables remain the same, then the term of the loan is reduced. Conversely, a decrease in the repayment value increases the term of the loan.

Example 2

Brad borrowed \$22,000 for a gift shop and agreed to repay the loan over 10 years with quarterly instalments of \$783.22 and interest debited at 7.4% p.a. However, after 6 years of the loan Brad decided to increase the repayment value to \$879.59. Calculate:

- The actual term of the loan
- The total interest paid for both repayments
- The interest savings achieved by increasing the repayment value

Solution

The actual term of the loan

Step 1: Find P , Q , n , r and R .

$$P = 22000, n = 6 \times 4 = 24, Q = 783.22$$

$$r = \frac{7.4}{4} = 1.85$$

$$R = 1 + \frac{1.85}{100} = 1.0185$$

Step 2: Using the formula for A in the spreadsheet to find the amount still owing after 6 years.

$$A_{24} = 10761.83$$

Step 3: After 6 years, the amount still owing will be \$10,761.83. This will be our new P , $Q = 879.59$. R remains the same and A is close to zero. Use the same formula in Step 2 to find n .

$$A_{14} = 0.02$$

$$n = 14, \text{ this is 3 and a half years.}$$

$$3.5 + 6 = 9.5$$

Step 4: Write your answer.

The actual term of the loan will be 9 and a half years if Brad increases the repayment value after 6 years.

The total interest paid for both repayments

Step 1: Find the total interest paid with the change in repayments. Total interest = Total repayments – Amount borrowed

$$\begin{aligned} \text{Total Interest} &= 783.22 \times 24 + 879.59 \times 14 - 22000 \\ \text{Total Interest} &= 9111.54 \end{aligned}$$

Step 2: Find the total interest paid without the change in repayments. Total interest =

$$\begin{aligned} \text{Total Interest} &= 783.22 \times 40 - 22000 \\ \text{Total Interest} &= 9328.80 \end{aligned}$$

Total repayments – Amount borrowed

Step 3: Write your answer.

The total interest with the change in repayments is \$9,111.54 and the total interest without the change in repayments is \$9,328.80.

The interest savings achieved by increasing the repayment value

Step 1: Find the difference

$$9328.80 - 9111.54 = 217.26$$

Step 2: Write your answer.

The interest saved is \$217.26

By increasing his quarterly repayments by almost \$100, Brad managed to reduce the overall cost of his loan by \$217.26 and paid it off 6 months sooner. This saving may not seem significant, but the figure would increase with a longer original term and a greater amount borrowed.

Example 3

Carmen is repaying a reducing balance loan of \$50,000 over 15 years with monthly instalment of \$499.72 at a rate of 8.75% p.a. (debited monthly). She has already made 3 years' worth of payments but would like to repay the loan in full in the next 10 years. find the amount that she still owes, and the monthly repayment value needed to repay the loan in full in the next 10 years.

Solution

Step 1: Find P , Q , r and R .

$$\begin{aligned}P &= 50000, Q = 499.72 \\r &= \frac{8.75}{12} = 0.72917 \\R &= 1 + \frac{0.72917}{100} = 1.0072917\end{aligned}$$

Step 2: Using the spreadsheet in Week 5, find the amount still owing after 3 years.

$$A_{36} = \$44459.76$$

Step 3: Since Carmen wants to fully repay the loan in the next 10 years, we need to find Q . We need to find P , n , r and R

$$\begin{aligned}P &= 44459.76, n = 10 \times 12 = 120 \\r &= \frac{8.75}{12} = 0.7291 \\R &= 1 + \frac{0.72917}{100} = 1.0072917\end{aligned}$$

Step 4: Using the spreadsheet in Week 5, find Q .

$$Q = 557.20$$

Step 5: Write your answer.

In order to repay the loan in the next 10 years, Carmen will need to increase her monthly repayments to \$557.20.

Exercise 2

1. Megan wanted to borrow \$50,000 and was offered a reducing balance loan over 20 years at 6.9% p.a. (debited monthly) with monthly instalments.
 - a. What will be the monthly repayment value?
 - b. What would be the term of the loan if the repayments was:
 - i. Increased to \$577.97?
 - ii. Decreased to \$361.85?

2. Steve borrows \$30,000 and agrees to repay the reducing balance loan over 10 years with quarterly instalments of \$1,039.80 with interest debited quarterly at 6.8% p.a. However, after 2 years of the loan Steve decides to increase the repayments to \$1,152.20. Find the actual term of the loan.

3. Georgia borrowed \$40,000 and agreed to repay the loan over 15 years with quarterly instalments of \$1,235.80 and with interest debited quarterly at 9.2% p.a. However, after 8 years she decided to increase the repayment value. Calculate:
- The actual term of the loan
 - The total interest paid
 - The interest saving achieved by increasing the quarterly repayments to \$1,661.89.
4. Two years ago, Dimitri borrowed \$7,000 and has been repaying \$175.69 each month. Interest has been debited monthly at 9.45% p.a. and this rate has remained unchanged. His employer has recently given Dimitri a pay rise as well as a Christmas bonus of \$1,000 and he has decided to increase his repayment. However, he is unsure which of the options below would be best for him. By considering the total amount of interest paid and the term of the loan, advise Dimitri on which option he should take.
- Pay \$1,000 off the amount owed and increase the repayment to \$200.67 per month
 - Increase the repayment to \$228.97 per month

Effects of changing the rate

Of all the variables associated with reducing balance loans, the one that is most likely to change during the term of a loan is the interest rate. These rates rarely stay the same for the life of a loan. For most loans, the rate will change several times.

The Reserve Bank of Australia is the main monetary authority of the Federal Government and, as such, is the overall guiding influence on monetary factors in the Australian economy. Consequently, it indirectly controls the lending interest rates of financial institutions.

There is usually some variation in rates between institutions. For example, a lower rate may be designed to attract more customers. Within each institution, there are rate variations as well for different types of reducing balance loans. Banks advertise their loan rates to attract customers.

Bank advertisements may show that a personal loan (to buy a car, for example) attracts interest at a rate of 9.5% p.a., yet a home loan (place of residence) attracts interest at a rate of 6.7% p.a. The rate for an investment property loan (property to be rented) is different as well at 6.9% p.a. All of these rates will vary with time.

In this next section, we investigate the effect that changing the interest rate has on the term of the loan and on the total interest paid. It should be remembered that as the interest rate increases so will the term (if Q remains constant) of the loan since more interest needs to be paid.

Example 1

A reducing balance loan of \$18,000 has a term of 5 years and it is to be repaid by monthly instalments of \$364.98 at a rate of 8% p.a. (debited monthly).

- What is the total interest paid?
- If the rate was 9% p.a. (debited monthly) instead and the repayments remained the same, what would be the term of the loan and the total amount of interest paid?

Solution

What is the total interest paid?

Step 1: Total interest = Total repayments – Amount borrowed for 8% p.a.

$$\begin{aligned}\text{Total Interest} &= 364.98 \times 60 - 18000 \\ \text{Total Interest} &= 3898.80\end{aligned}$$

Step 2: Write your answer.

The total interest over 5 years will be \$3,898.80

What is the term of the loan for 9% p.a.?

Step 1: Find P , Q , r and R .

$$\begin{aligned}P &= 18000, Q = 364.98 \\ r &= \frac{9}{12} = 0.75 \\ R &= 1 + \frac{0.75}{100} = 1.0075\end{aligned}$$

Step 2: Using trial and error, find n .

$n = 62$ months, which is 5 years and 2 months

What is the total interest paid for 9% p.a.?

Step 1: Total interest = Total repayments – Amount borrowed. However, the final payment is usually not the same as the regular repayments. This means that we need to find A_{61} first.

$A_{61} = 293.88$, this may be different due to rounding. Make sure you round at the very last step.

$$\begin{aligned}\text{Total Interest} &= 364.98 \times 61 + 293.88 - 18000 \\ \text{Total Interest} &= 4557.66\end{aligned}$$

Step 2: Write your answer.

The total interest over 62 months will be \$4,557.66

In Example 1, the rate was increased by only 1% p.a. on \$18,000 for 5 years, yet the amount of interest paid has increased from \$3,898.80 to \$4,559.86, a difference of \$661.06. This difference takes on even more significant proportions over a longer period of time and with a larger principal.

Let us now consider varying the rate during the term of the loan.

Example 2

Sam and Amy take out a loan for home improvements. The loan of \$42,000 was due to run for 10 years and attract interest at 7% p.a., debited quarterly on the outstanding balance. Repayments of \$1,468.83 were made each quarter. After 4 years the rate changed to 8% p.a. (debited quarterly). The repayment value didn't change.

- Find the amount outstanding when the rate changed.
- Find the actual term of the loan.
- Compare the total interest paid to what it would have been if the rate had remained at 7% p.a. for the 10 years.

Solution

Find the amount outstanding when the rate changed.

Step 1: Find P , Q , n , r and R .

$$P = 42000, n = 4 \times 4 = 16, Q = 1468.83$$

$$r = \frac{7}{4} = 1.75$$

$$R = 1 + \frac{1.75}{100} = 1.0175$$

Step 2: Using the formula for A in the spreadsheet or calculate it manually to find the amount still owing after 4 years.

$$A_{16} = 28584.36$$

Step 3: Write your answer.

The amount still owing after 4 years will be \$28,584.36.

Find the actual term of the loan.

Step 1: Find n to repay \$28,584.36 in full at the new rate of 8% p.a. Use trial and error. $P = 28584.36$, $Q = 1468.83$, $r = \frac{8}{4} = 2$, $R = 1 + \frac{2}{100} = 1.02$ and A is as close to zero.

$n = 25$ terms which is 6 years and 4 months.

Total term = 4 years + 6 years and 4 months = 10 years and 4 months or 10 and a quarter years.

Step 2: Write your answer.

The actual loan will be 10 and a quarter years.

Compare the total interest paid to what it would have been if the rate had remained at 7% p.a. for the 10 years.

Step 1: Total interest = Total repayments – Amount borrowed. However, the final payment is usually not the same as the regular repayments. This means that we need to find A_{24} first.

$A_{24} = 1291.61$, this may be different due to rounding. Make sure you round at the very last step.

$$\begin{aligned}\text{Total Interest} &= 1468.83 \times 40 + 1291.61 - 42000 \\ \text{Total Interest} &= 18070.64\end{aligned}$$

Step 2: Find the total interest paid without the change in interest. Total interest = Total repayments – Amount borrowed

$$\begin{aligned}\text{Total Interest} &= 1468.83 \times 40 - 42000 \\ \text{Total Interest} &= 16753.20\end{aligned}$$

Step 3: Find the difference

$$18070.64 - 16753.20 = 1317.44$$

Step 4: Write your answer.

The extra interest paid is \$1,317.44 due to the increase in interest rate.

In Example 2, the repayment value, Q , remained the same, even though the rate varied. In practice, this is what happens if the rate decreases and so the term of the loan decreases. However, when the rate increases, financial institutions will generally increase the repayment value to maintain the original term of the loan. If this is not done, the term of the loan can increase quite dramatically. This may occur to such an extent that the repayments are insufficient to cover the interest added, so that the amount outstanding increases.

Exercise 3

1. A reducing balance loan of \$25,000 has been taken out over 5 years at 8% p.a. (debited monthly) with monthly repayments of \$506.91.
 - a. What is the total interest paid?
 - b. If the rate was increased 9% p.a. (debited monthly) with the same repayments maintained, what would be:
 - i. The term of the loan?
 - ii. The total interest paid?
 - c. If the rate was increased 10% p.a. (debited monthly) with the same repayments maintained, what would be:
 - i. The term of the loan?
 - ii. The total interest paid?

2. Eric borrowed \$56,000 at 9.5% p.a. of the amount owing (debited monthly) and he was repaying the loan over 10 years with instalments of \$724.63 per month. Two years into the loan, the rate decreased to 8.5% p.a. (debited monthly) but he maintained the same repayment. After another 4 years, the rate fell again, this time to 7.5% p.a. (debited monthly) and the repayments still remained the same. If the rate didn't change again, calculate:
- a. The actual term of the loan
 - b. The total amount of interest charged

2023 MA4 Week 6/7 Investigation

Find a house for sale in Canberra (<https://www.allhomes.com.au/>, <https://zango.com.au/sale/> or <https://www.realestate.com.au/> might be helpful).

1. Home loans require a deposit to be paid to secure the rest of the loan, usually 20% of the total value of the house must be paid as a deposit. Remove the 20% deposit from the price of the house to calculate the size of the loan that you will need to buy this house.
2. After removing the 20% deposit, find a home loan with a good 'comparison rate' (<https://www.canstar.com.au/home-loans/>). Calculate the monthly repayment needed to pay off this loan in 30 years if repayments and interest is compounded monthly.
3. Calculate the amount of interest that will be paid over the full term of this loan.
4. If your home loan is on a 'variable' rate, the interest rate can change at any time. If the interest rate increases by 1% one year into your loan repayments, how much will that increase the total amount of interest the bank charges on this loan?

MARKING RUBRIC**Week 6 and 7****NAME:**

CRITERIA	EXPECTATIONS	POSS	MULT	GIVEN	TOTAL
Practical	Student completes practical work, including exercises and any Mathspace and/or other tasks, of the brief to an acceptable standard set by the teacher.	2	3		/6
Investigation Task	Student completes the investigation task of the week to an acceptable standard set by the teacher.	2	2		/4
Communication and Reasoning	Student responses are accurate and appropriate in presentation of mathematical ideas in different contexts, with clear and logical working out shown.	4	-		/4
Knowledge and Application	Student submitted work selects and applies appropriate mathematical modelling and problem-solving techniques to solve practical problems and demonstrates proficiency in the use of mathematical facts, techniques, and formulae.	4	-		/4
	Submission Guidelines				
Timeliness	Student submits the practical work, including exercises and any Mathspace and/or other tasks, and investigation by the set deadline. See scoring guidelines for specific details.	2	-		/2
				FINAL	/20

Student Reflection: How did you go with this week's work? What was interesting?
 What did you find easy? What do you need to work on?