HAWKER COLLEGE
Engage | Inspire | Achieve

## Goals



This fortnight we are going to:

- construct a sample space for an experiment
- use a sample space to determine the probability of outcomes for an experiment
- use arrays or tree diagrams to determine the outcomes and the probabilities for experiments
- determine the probabilities associated with simple games
- determine the probabilities of occurrence of simple traffic-light problems


## Theoretical components

## Resources:

PDF file: Week 3/4 Notes and Exercises

More theory on probability and sample space.
https://www.mathsisfun.com/data/probability.ht ml

This clip shows you about the Monty Hall problem.
https://www.youtube.com/watch?app=desktop\& $\mathrm{v}=\mathrm{mhlc} 7 \mathrm{peGIGg}$

## Knowledge Checklist

- The language of probability
- Outcome and sample space
- Calculating probabilities
- Applications of probability


## Order

1. Read through the notes and examples
2. Work through the exercises
3. Complete the Portfolio Task
4. Complete the reflection at the end of the booklet
5. Come and see your teacher and make sure you are up to date.

## Practical Components

Work through the exercises and show the completed tasks to your teacher.

Be sure to ask for help as you need for the successful completion of all tasks.

## Remember to regularly check Google Classroom for messages.

## Portfolio Task

Complete the task at the end of the booklet and submit your work for checking. ©

## ESSENTIAL MATHEMATICS 4

WEEK 3/4 NOTES AND EXERCISES

## CALCULATING TOTAL NUMBER OF POSSIBILITIES

You can't rely on intuition in probability. There are several ways to systematically work out the total number of possibilities in a probability problem. Drawing a grid is often a good approach, particularly when a pair of dice are involved.

## Example

Rebecca tosses a die and a coin. What is the probability she tosses:
a. a 4 and a head?
b. a 4 or a head?


## Solution

It is important to know the shorthand notation $\mathrm{P}(4$ and H$)$ is often used instead of writing 'the probability of a 4 and a head'.

When Rebecca tosses the die, it can show any one of the numbers 1, 2, 3, 4, 5 or 6 . A head or a talk are the possibilities for flipping a coin.

First, we need to create a grid that shows all the possibilities for tossing a die and a coin.

|  | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Head | H and 1 | H and 2 | H and 3 | H and 4 | H and 5 | H and 6 |
| Tail | T and 1 | T and 2 | T and 3 | T and 4 | T and 5 | T and 6 |

This table shows that there are 12 possibilities.
a. $\mathrm{P}(4$ and H$)=\frac{1}{12}$ because there is only one way out of 12 possibilities that a 4 and a head can show.
b. $\mathrm{P}(4$ or H$)=\frac{7}{12}$ because there are seven ways this can happen. T and $4, \mathrm{H}$ and $1, \mathrm{H}$ and $2, \mathrm{H}$ and $3, \mathrm{H}$ and $4, \mathrm{H}$ and $5, \mathrm{H}$ and 6 .

In probability, questions including the word 'or' usually means 'one or the other or both'.

## EXERCISE 1

1. Jonah tosses a coin and an eight-sided die. The die has the numbers 1 to 8 on it.
a. Complete this table showing all the possibilities.

|  | 1 | 2 |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Head | H 1 |  |  |  |  |  |  |  |
| Tail |  |  |  |  |  |  |  |  |

b. What is the probability that Jonah tosses:
i. a 7 and a tail?
ii. a 7 or a tail?
iii. a number greater than 5 and a
iv. a head and an even number? head?
2. A debating team consists of two students from Year 11 and three students from Year 12. If one member of the team is picked at random to be the opening speaker. What is the probability that the student:
a. comes from Year 11?
b. does not come from Year 11?
3. When three coins are tossed together the probability of three heads showing is $\frac{1}{8}$. What is the probability of something other than three heads will show?
4. A pair of normal dice are used in a board game. Players add the two numbers showing on the dice to determine the score.
a. Complete this grid showing all the possible totals.

|  | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  |  |  |  |  |  |
| 2 |  |  | 5 |  |  |  |
| 3 |  |  |  |  |  |  |
| 4 |  |  | 7 | 8 |  |  |
| 5 |  |  |  |  |  |  |
| 6 |  | 8 |  |  | 11 |  |

b. What is the probability of scoring a total of:
i. 5 ?
ii. 9 ?
iii. 11?
iv. 12 ?
c. Why is 7 considered a lucky number?
d. Which is more likely: a sum of 8 or a sum of 10 ?

## TREE DIAGRAMS

Tree diagrams are another way to systematically list all the possibilities.

## Example

Chloe places these three cards on the table to make a three-digit number.

a. How many three-digit numbers can she make?
b. What is the probability she will make 457 ?

## Solution

a. From the tree diagram, we have six different three-digit numbers possible: $457,475,547,574,745,754$

b. $\mathrm{P}(457)=\frac{1}{6}$

## EXERCISE 2

1. Three coins are tossed at the same time. Mathematically, this is the same as tossing one coin three times.
a. Construct a tree diagram to show all possible outcomes.
b. How many possibilities are there when the three coins are tossed?
c. Calculate the probability of these events when a coin is tossed three times.
i. $P(3$ heads $)$
ii. $P(2$ heads then a tail $)$
iii. $P(2$ heads and a tail in any order)
iv. $P$ (at least 1 head)
2. There are four coloured discs in a bag. Two of the discs are black, one is white and the other is red. Zachary is going to take a disc from the bag at random and then take another disc from the bag at random without putting the first disc back.
a. Draw a tree diagram to represent all possibilities. For the first 'branch' there should be four possibilities: black, black, white, red.
b. What is the probability that Zachary will take:
i. a black then a red disc
ii. at least one black disc
iii. a black and red disc in any order
3. Two students from Laura, Jack, Sara, and Will are going to be chosen at random to ask the teacher for help.
a. Construct a tree diagram to show that there are 12 possible outcomes of selecting two students.
b. What is the probability of Laura being the first student chosen and Jack the second student chosen?
c. Explain why the probability of Sara and Will being chosen is $\frac{1}{6}$.
4. Margaret and Joe are planning to have three children.
a. If the probability of a boy is equal to the probability of a girl, what is the probability that their first child will be a girl?
b. Draw a tree diagram to show all possible combinations of boys and girls. Each branch should represent the gender of the child (first branch = first child).
c. Use the tree diagram to help you calculate the probability that Margaret and Joe will have:
i. three sons
iii. at least one daughter
ii. two sons then a daughter
iv. three children of the same sex

## COUNTING THE NUMBER OF POSSIBILITIES

There are two common methods of calculating the number of ways items can be lined up (when the order is important).

The first method is to systematically list all the possibilities. The second method uses numbers in boxes.

## Example

How many ways can Dylan, Kai, and Noah line up in a row?

## Solution

## First Method:

They could line up: Dylan, Kai, Noah Dylan, Noah, Kai Kai, Dylan, Noah Kai, Noah, Dylan Noah, Dylan, Kai
Noah, Kai, Dylan
There are six different ways.

## Second Method:

Use three boxes to represent the three positions in the line.


Put three in the first box to show that there could be three people who could be first.


After one of the three people is in the first position, there are two people left to fill the second position. Put two in the second box.


There is now only one person left to fill the third position. Put one in the third box.


1

To calculate the total number of possibilities, multiply the numbers in the boxes together.


## EXERCISE 3

1. Four friends are going to sit in a row at the movies.
a. Complete the boxes to calculate the ways they can sit.

b. How many ways can they sit?
c. What is the probability of correctly predicting the order in which they end up sitting?
2. Sixty-five people have entered the Mt Ainslie Run Up and Power Walk fun run. Use the box method to calculate the number of ways first and second place can be filled.
3. Rihanna is going to put six new books on a shelf on her bookcase. Use the box method to calculate the number of ways she can order the books.
4. How many different ways can 10 people be arranged on 10 seats in a row? (One person per seat)
5. Eight swimmers compete for first, second and third place in the Olympics. How many ways can these three positions be filled?
6. When two friends go to their favourite Thai restaurant, they always choose three dishes to share. They choose one of the four different types of rice and noodles, one of the eight different beef dishes and one of the six vegetable dishes. How many different meals can they choose?
7. Standard ACT number plates have six characters. It is the letter $Y$ followed by two letters, two digits and then a letter.

Answer the following questions. Make sure you show your working.
a. Things to know before you begin. You may need to have a look at the carparks.
i. Can letters and digits be repeated?
ii. Which letters in the alphabet are allowed?
iii. Which digits are allowed?
b. How many different car number plates are there?

| Y |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |

c. How many number plates would be possible if the system was changed to six letters and no numbers?

| Y |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |

d. What is the probability of being given the number plate YYY 999 for a standard plate?
e. What is the probability of being given a number plate with three Ys ? (YYY ???)

## TRAFFIC LIGHT PROBLEMS

The important feature of probability questions involving time and physical space is that the sample space is continuous. There is an uncountable infinity of outcomes in the sample space if our focus is on instants of time or on individual points in physical space. This means that probabilities cannot be formed by counting the elements in the sets involved.

Instead, we measure intervals of time or space and we let the relative size of the interval compared with the measure of the whole space be the probability. Thus, if a light in a room is turned on for 45 seconds in every 5 minutes, we would say that the probability of a person who enters the room at a random moment encountering the light 'on' is $\frac{45}{5 \times 60}=0.15$.

Probabilities determined in this way behave in the same way as probabilities in a discrete sample space. This is the same if two events have non-overlapping intervals. There are two ways to calculate this:

$$
\begin{gathered}
\mathrm{P}(\mathrm{~A} \text { or } \mathrm{B})=\mathrm{P}(\mathrm{~A})+\mathrm{P}(\mathrm{~B}) \\
\mathrm{P}(\mathrm{~A} \text { and } \mathrm{B})=\mathrm{P}(\mathrm{~A}) \times \mathrm{P}(\mathrm{~B})
\end{gathered}
$$

## Example

Every 60 seconds, a traffic light remains green for 34 seconds, yellow for 3 seconds and red for 23 seconds.
a. What outcome is more likely to happen?
b. On his way to work, Iram passes through 3 sets of lights. What is the probability that none of the lights are green?
c. What is the probability that Iram passes through green at the first two sets of lights and red at the third set?

## Solution

a. $\mathrm{P}($ green $)=\frac{34}{60}, \mathrm{P}($ yellow $)=\frac{3}{60}$ and $\mathrm{P}($ red $)=\frac{23}{60}$.

It is more likely we will arrive at the traffic light when it is green.
b. $\mathrm{P}($ not green $)=\mathrm{P}\left(\right.$ green $\left.^{\prime}\right)=1-\mathrm{P}($ green $)$ This notation is from Week 1 and 2 .
$P\left(\right.$ green $\left.^{\prime}\right)=1-\frac{34}{60}=\frac{26}{60}$
$\mathrm{P}\left(\right.$ green' $^{\prime}$ and green' and green' $)=\mathrm{P}\left(\right.$ green $\left.^{\prime}\right) \times \mathrm{P}\left(\right.$ green $\left.^{\prime}\right) \times \mathrm{P}\left(\right.$ green $\left.^{\prime}\right)$
$P($ green'and green'and green' $)=\frac{26}{60} \times \frac{26}{60} \times \frac{26}{60}=\frac{2197}{27000}=0.081$
The probability Iram will pass through no green lights is 0.081 .
c. $\quad \mathrm{P}($ green and green and red $)=\mathrm{P}($ green $) \times \mathrm{P}($ green $) \times \mathrm{P}($ red $)$
$P($ green and green and red $)=\frac{34}{60} \times \frac{34}{60} \times \frac{23}{60}=\frac{6647}{54000}=0.123$
The probability Iram will pass through two green lights first then a red light is 0.123 .

## EXERCISE 4

1. The probability that a set of traffic lights shows red, yellow, or green are equally likely. Monique is travelling down a road where there are two sets of traffic lights.
a. Construct a tree diagram to indicate the possible pairs of traffic lights.
b. What is the probability that both sets of traffic lights will be yellow?
2. A set of traffic lights was found to staying green for 119 seconds, yellow for 5 seconds and red for 76 seconds in one direction. Find the probability, as a percentage, that the lights will be:
a. green?
b. yellow?

c. red?
3. A set of traffic lights is red for half the time, yellow for $\frac{1}{5}$ of the time and green for the rest of the time. Find the probability that the traffic light is green.
4. The time spent waiting for a green light at a set of traffic lights, in seconds, was recorded. The results are: $11,119,5,74,32,90,31,66,91,33,81,37,94,17$, 84, 101, 56, 41, 14, 52.
a. Complete the following relative frequency table:

| Time | Tally | Frequency | Rel. Freq. |
| :---: | :---: | :---: | :---: |
| $0 \leq t \leq 10$ |  |  |  |
| $10<t \leq 20$ |  |  |  |
| $20<t \leq 30$ |  |  |  |
| $30<t \leq 40$ |  |  |  |
| $40<t \leq 50$ |  |  |  |
| $50<t \leq 60$ |  |  |  |
| $60<t \leq 70$ |  |  |  |
| $70<t \leq 80$ |  |  |  |
| $80<t \leq 90$ |  |  |  |
| $90<t \leq 100$ |  |  |  |
| $100<t \leq 110$ |  |  |  |
| $110<t \leq 120$ |  |  |  |

b. Calculate the probability of a person waiting being between 21 and 30 seconds, inclusive.
c. Calculate the probability of a person waiting at most 30 seconds.

## WEEK 3/4 PORTFOLIO TASK

Joel is at point $\mathbf{A}$ driving in the direction of point $\mathbf{B}$. There are traffic lights at points $\mathbf{B}$, $\mathbf{C}, \mathbf{D}$ and $\mathbf{E}$, and the probability of a green light at any one of these is 0.4 . If Joel arrives at a traffic light when it is green, he will go straight. If it is not green, he will turn right. He does not turn or drive back in the direction of $\mathbf{A}$ at any time.


1. What is the probability that Joel drives to point $\mathbf{P}$ ? Hint: What would be his route to travel to point $\mathbf{P}$ ?
2. What is the probability that Joel drives to point $\mathbf{Q}$ ? Hint: What would be his route(s) to travel to point $\mathbf{Q}$ ?

## MARKING RUBRIC

| CRITERIA | EXPECTATIONS | POSS | MULT | GIVEN | TOTAL |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Practical | Student completes practical work of the brief to an acceptable standard set by the teacher. | 2 | 3 |  | /6 |
| Portfolio Task | Student completes the portfolio task of the week to an acceptable standard set by the teacher. | 2 | 2 |  | 14 |
| Reasoning and Communications | Student responses are accurate and appropriate in presentation of mathematical ideas in different contexts, with clear and logical working out shown. | 4 | - |  | 14 |
| Concepts and Techniques | Student submitted work selects and applies appropriate mathematical modelling and problem solving techniques to solve practical problems, and demonstrates proficiency in the use of mathematical facts, techniques and formulae. | 4 | - |  | 14 |
|  | Submission Guidelines |  |  |  |  |
| Timeliness | Student submits the exercises and portfolio tasks by the set deadline. See scoring guidelines for specific details. | 2 | - |  | /2 |
|  |  |  |  | FINAL | /20 |

## Student Reflection:

How did you go with this week's work?

What was interesting?

What did you find easy?

What do you need to work on?

