## EM4

Loans and compound interest

## Goals

This fortnight we are going to:

- use technology to investigate the effect of the interest rate and the number of compounding periods on the future value of a loan or investment
- use technology and a recurrence relation to model a reducing balance loan
- investigate the effect of the interest rate and repayment amount on the time taken to repay a loan


## Theoretical Components

## Resources:

PDF file: Week 13/14 Loans
Watch these videos for an introduction to the two types of depreciation:

- https://www.youtube.com/watch?v=RHo 3 kWalJo
- https://www.youtube.com/watch?v=ziayge 17w6g


## Knowledge Checklist

- Straight line depreciation
- Reducing balance depreciation
- Saving for a deposit
- Using online calculators
- Repaying a mortgage - long term cost
- Repaying a mortgage - lump sum payment
- Repaying a mortgage - increasing payments
- Reading advertisements for property


## Order

1. Read through the notes and examples
2. Work through the exercises
3. Complete the Portfolio Task
4. Complete the reflection at the end of the booklet
5. Come and see your teacher and make sure you are up to date.

## Practical Components

Work through the exercises and show the completed tasks to your teacher.

Be sure to ask for help as you need for the successful completion of all tasks.

Remember to regularly check Google Classroom for messages.

Week 14 starts at the 'Percentage Review' section.

## Portfolio Task

Complete the task at the end of the booklet and submit your work for checking. (:)

## Other

## ESSENTIAL MATHEMATICS 4

## WEEK 13/14 NOTES AND EXERCISES

## DEPRECIATION

As mentioned in the previous brief, items which represent scarce resources such as land, collectables, paintings and antiques normally appreciate in value over time. Most of the goods that we consume, such as household items, cars, electrical and electronic applications, plant machinery and equipment, fixtures and furnishings lose (depreciate in) their value over time.

Depreciation means that items reduce in value from the time they are acquired. The price for which an item was originally bought is called the purchase price of the item. The portion of the purchase price an item loses in value due to depreciation is called its accumulated depreciation. The value of an item at any given time is called its book value. This information may be summarised by the following equation:

## Book value $=$ purchase price - accumulated depreciation

Items used for income producing purposes are called assets. Scrap value is the value that an asset is expected to have at the end of its useful life. The useful life of some assets is expected to be longer than others. An example of this is that we expect buildings to last 40 years and longer, whereas computers have a life expectancy of about 3 years. This is reflected in the depreciation rate, which varies for different assets. The depreciation rate normally is expressed as a percentage of the purchase price.

## Straight-line depreciation

The straight-line depreciation method, also known as flat rate depreciation or constant depreciation, allocates an equal amount of depreciation to each time period (normally one year) in the asset's useful life - that is, the asset's value loses the same amount each year. The depreciation is calculated in the same way as simple interest. Thus:

$$
D=\frac{P R T}{100}
$$

## Example

A company car purchased for $\$ 39,600$ depreciates at $12 \%$ per annum by straightline depreciation.
a. Calculate the book value of the car after 3 years.
b. Calculate the total depreciation over the first 6 years.

## Solution

a. Using the formula for $D$ with $P=\$ 39,600, R=12 \%, T=3$. This gives a result of $\$ 14,256$.
This means the car depreciates by an amount of $\$ 14,256$ over 3 years.
The book value is $39600-14256=\$ 25,344$
b. Using the formula for $D$ with $P=\$ 39,600, R=12 \%, T=6$. This gives a result of $\$ 28,512$.

We could also calculate how long it will take the car to reach its scrap value of $\$ 4,500$. A scrap value of $\$ 4,500$ means it has depreciated by $39600-4500=$ \$35,100.

To calculate time, we can rearrange the depreciation formula to:

$$
T=\frac{D \times 100}{P R}
$$

By substituting $D, P$ and $R$, we get $T=7.4$ years.
We can calculate this to the nearest month by multiplying the decimal part by 12 . We could then get 7 years and 5 months.

## EXERCISE 1

1. A machine costing $\$ 286,000$ depreciates at $16.5 \%$ per annum, by straight-line depreciation.
a. Calculate the book value of the machine after the first 2 years of service.
b. Calculate the total depreciation over the first 5 years.
c. How long will it take until the machine reaches its scrap value of $\$ 10,000$ ?
2. Curtain fittings depreciate at $20 \%$ of the purchase price per annum. The purchase price of the fittings was $\$ 16,000$ and they will be discarded at the end of their useful life (when they reach a scrap value of zero).
a. Find their book value after 1 year.
b. Find their book value after 3 years.
c. After how many years will the fittings be discarded?
3. A building crane was purchased by a construction company for $\$ 450,000$ and is expected to have a useful life for 19 years. Calculate the crane's value at the end of 19 years of service if the recommended straight-line depreciation rate is $5.0 \%$ per annum.
4. George buys a used car for $\$ 10,000$ and the car depreciates at $15 \%$ per year.
a. Find the book value after 3 years.
b. If George plans to resell the car when it depreciates to $\$ 5,000$. What year should he resell his car?
5. Zoe buys a new TV for $\$ 3,000$ and she plans to trade in the TV after 4 years. The TV depreciates at $12 \%$ per year. How much will the TV be worth when she trades it in?
6. Noah buys a new phone for $\$ 2,500$ and the phone depreciates at $20 \%$ per year.
a. Calculate the total depreciation after 2 years.
b. Noah trades in his phone after 5 years. What is the book value of the phone?

## REDUCING-BALANCE DEPRECIATION

In the previous section, we observed the similarity in calculating straight-line depreciation and simple interest. Both are fixed percentages of a fixed amount. The difference is that interest is added to the principle and depreciation is subtracted from the principle.

The reducing-balance method, also known as diminishing value depreciation or reduced value, can be compared with compound interest. Both are characterised by applying a fixed percentage to an amount which changes at the beginning of each time period. As the book value of an asset declines from period to period, so does depreciation.

The formula for the book value of an asset which depreciates under the reducing balance method is:

$$
B . V .=P\left(1-\frac{R}{100}\right)^{n}
$$

where:
B.V. $=$ book value of depreciated item, $\$$ (current worth)
$P=$ purchase price, $\$$
$R=$ allowed annual rate of depreciation, \%
$n=$ number of depreciation periods, years
Note: the formula for calculating the book value is similar to the formula of compound interest (only difference is there is minus sign instead of a plus sign)

## Example

A construction company purchased a new scrapper for $\$ 86,000$. It depreciates at $12 \%$ per annual reducing-balance.
a. Calculate the book value of the scrapper after 3 years.
b. Calculate the total depreciation over the first 5 years of service.

## Solution

a. B.V. $=P\left(1-\frac{R}{100}\right)^{n}, P=\$ 86,000, R=12 \%, T=3$
B.V. $=86000\left(1-\frac{12}{100}\right)^{3}$
B.V. $=\$ 58,606.59$ The scrapper will be worth $\$ 58,606.59$ after 3 years.
b. B.V. $=P\left(1-\frac{R}{100}\right)^{n}, P=\$ 86,000, R=12 \%, T=5$
B.V. $=86000\left(1-\frac{12}{100}\right)^{5}$
B.V. $=\$ 45,384.94$
$D=P-B . V$.
$D=86000-45384.94$
$D=\$ 40,615.06$
The total depreciation over the first 5 years of service is $\$ 40,615.06$

## EXERCISE 2

1. A clothing factory purchased a new machine for $\$ 21,600$. It depreciates at $16 \%$ per annum reducing balance.
a. Calculate the book value of the machinery after 4 years.
b. Calculate the total depreciation over the first 6 years of use.
2. An industrial steam cleaner costs $\$ 3,200$ with diminishing value at $15 \%$ p.a. Calculate:
a. the amount of depreciation during the first year of service
b. the amount of depreciation during the fifth year and express this amount as a percentage of the cost. For this part, you will need to calculate the book value at the end of 4 years and the book value at the end of 5 years, then subtract to find the depreciation during the $5^{\text {th }}$ year. You can also find this answer using an online calculator:
https://www.calculatorweb.com/calculators/depcalc.shtml
3. A microwave costs $\$ 200$ with diminishing value at $19 \%$ p.a.
a. Calculate the amount of depreciation after 2 years.
b. Calculate the book value after 10 years.
4. Brae buys a motorbike valued at $\$ 8,950$ with a diminishing value of $10 \%$ p.a.
a. Calculate the book value after 2 years.
b. Calculate the amount of depreciation during the fourth year.
5. Jasmine buys robot vacuum for $\$ 1,300$ with a diminishing value of $7 \%$ p.a. She uses the robot vacuum for 5 years before upgrading to a newer model. What is the trade in value of the robot vacuum?

## BUYING YOUR FIRST PROPERTY

## Housing deposits

Real estate is expensive, and very few first-home buyers have sufficient money to pay for their first property. Just about everyone needs to borrow money when they buy their first home. While some financial institutions offer loans less than the full price, and the borrower must make up the rest of the price up-front as a deposit. Housing deposits are usually $10 \%$ to $20 \%$ of the price of a house.

## Regular savings

Savings and investment accounts pay investors interest, and most bank websites contain savings calculators to help you plan your savings. You may be surprised at how quickly savings can accumulate and the dream of having a house deposit can eventuate.

Use this website to complete the Exercise 3:
https://www.infochoice.com.au/calculators/savings-calculator/
Most banks have their own calculators, and they all work in a similar fashion.

## Example

Charlize is saving for the deposit for a property. Her special purpose savings account pays $3.5 \%$ p.a. interest. At the moment, she has $\$ 500$ and is saving $\$ 120$ per week.
a. How much will Charlize have in her savings account in 4 years time?
b. How long will it take for Charlize to have $\$ 42,000$ in her account?

## Solution

In the online calculator provided, enter \$500 in Starting Deposit, \$120 in Regular Deposit, make sure the Interest and Deposit Frequency is set to Weekly, Interest Rate is changed to $3.5 \%$ and Savings Term is set at 4 years.
a. The calculator shows there is $\$ 27,357.12$ in the account. It also tells her the Amount Deposited is $\$ 25,460$ and Interest Earned is $\$ 1,897.12$.
b. By changing the Savings Term either by sliding the bar or entering a number, it can be determined that it takes just under 6 years to save $\$ 42,000$.

## EXERCISE 3

## Use this website to complete the questions in Exercise 3: <br> https://www.infochoice.com.au/calculators/savings-calculator/

1. Ella is saving for the deposit for a unit. She has $\$ 400$ and each week she saves another $\$ 90$. The account pays $2.5 \%$ p.a. interest.
a. How much will Ella have in 3 years time?
b. How long will it take for Ella to save $\$ 25,000$ ?
2. James has $\$ 725$ in his account, which pays $4 \%$ p.a. interest. Each month he saves another \$200.
a. How long will it take James to save $\$ 14,000$ ?
b. How much interest will be paid into his account?
3. Laura wants to buy a unit, but she doesn't have any savings to use as a deposit. She needs $\$ 24,000$. If she starts saving $\$ 135$ per fortnight into an account that pays $4.9 \%$ p.a. interest. How long will it take her to save the deposit?
4. Lara needs to save $\$ 20,000$ in 2 years for a deposit on an apartment. At present, she has only $\$ 200$ in her savings account. How much will she need to save per week, at $5 \%$ p.a. interest to reach $\$ 20,000$ in 2 years time?
5. Jack plans to save $\$ 250$ per fortnight, he already has $\$ 100$ in his account.
a. How long will it take him to save $\$ 40,000$ if the interest rate is $2 \%$ p.a.?
b. How much quicker will it take him to save $\$ 40,000$ if the interest rate is 4.5\% p.a.?
6. Will needs to save $\$ 25,000$ in 2 years.
a. How much does he need to save per week if the interest rate is $5.5 \%$ p.a.?
b. How much extra does he need to save per week if the interest rate falls to 3 \% p.a.?
7. Annastacia wants to purchase a house in 5 years. She needs to save $\$ 55,000$. She has $\$ 500$ in a savings account that pays $3.4 \%$ p.a. interest. How much should she need to save each week for the deposit?

## PERCENTAGE REVIEW

Calculations involving percentages are very common in real estate and finance. Real estate agents usually charge a percentage commission, and percentages are used to calculate the amount of deposit required. Percentages are even used to calculate the size of any fine you may be charged if you are late paying government charges or debt!

## Example

Find $5 \%$ of $\$ 56,000$.

## Solution

$5 \%$ of $56,000=\frac{5}{100} \times 56000=\$ 2,800$
You can enter it like this on the calculator: $5 \div 100 \times 56000$

## Example

Elka borrowed \$120,000 towards the cost of a $\$ 150,000$ property. What percentage of the value of the property did she borrow?

## Solution

Percentage borrowed $=\frac{120000}{150000} \times 100=80 \%$

## EXERCISE 4

1. Calculate the following percentage amounts:
a. $5 \%$ of $\$ 160,000$
b. $8 \%$ of $\$ 145,000$
c. $3 \%$ of $\$ 240,000$
d. $6 \%$ of $\$ 500,000$
2. This table shows the deposit paid and the full price of three properties. In each case, calculate what percentage the deposit is of the value of the property.

| Value of the property | Deposit paid | Percentage deposit |
| :--- | :--- | :--- |
| $\$ 180,000$ | $\$ 36,000$ |  |
| $\$ 160,000$ | $\$ 16,000$ |  |
| $\$ 240,000$ | $\$ 12,000$ |  |

3. Ethan is buying a unit. He needs a $15 \%$ deposit to obtain the loan he wants. The unit is valued at $\$ 254,000$. How much deposit does Ethan need?
4. Lachlan charges a commission of $15 \%$ for every house he sells. Calculate the commission Lachlan earns if he sells:
a. a unit worth $\$ 160,000$
b. a house worth $\$ 1,045,000$
c. a penthouse worth $\$ 1,540,000$
d. an apartment worth $\$ 650,000$
5. Isabel saves $10 \%$ of her fortnightly income. How much does she save if her fortnightly income is $\$ 2,324.62$ ?
6. Zachary receives $4 \%$ from shares each month. The shares are worth $\$ 5,300$. How much does Zachary receive each month?

## TYPES OF LOANS

You've probably heard the expression 'Wise shoppers shop around to get the best deal'. This common expression is very relevant to purchasing a mortgage for your home. Even slight differences in interest rates and fees can make a huge difference to the final amount you repay on the loan.

## Repaying a mortgage: long term cost

Use this website to complete the Exercise 5: https://www.westpac.com.au/personal-banking/home-loans/calculator/mortgage-repayment/

## Example

Abdullah borrowed \$200,000 over 30 years at $8 \%$ p.a. interest to buy a house and land package.
a. How much will Abdullah repay the bank during the 30 years?
b. How much interest will Abdullah pay the bank?

## Solution

Using the website, enter How much you would like to borrow? \$200,000, Repayment type Principal \& Interest, Interest rate type Fixed 2 years, Interest rate 8\% p.a., Loan term 30 years. Leave the other options as they are.

The monthly repayments should be calculated. Scroll down the website until you see a summary of your loan.
a. Total repayments are $\$ 528,310$
b. Total interest is $\$ 328,310$

## EXERCISE 5

Use this website to complete the questions in Exercise 5:
https://www.westpac.com.au/personal-banking/home-loans/calculator/mortgagerepayment/

1. Chloe borrowed $\$ 175,000$ at $7.25 \%$ p.a. interest over 30 years.
a. How much will her monthly repayments be?
b. How much interest will Chloe pay the bank?
c. Find the total amount Chloe will repay the bank.
2. Erin borrowed $\$ 395,000$ at $5.75 \%$ p.a. interest over 25 years. Calculate the total amount she will repay the bank.
3. Gemma borrowed $\$ 450,000$ at $6.25 \%$ p.a. interest for 15 years.
a. How much will she have to repay the bank each month?
b. How much extra will she have to repay the bank each month if interest rates go up to $7.25 \%$ p.a.?
4. Rhianna borrowed $\$ 210,000$ at $5.5 \%$ p.a. interest for 30 years. Explain how you know Rhianna will repay the bank more than 2 times the amount she borrowed.
5. Complete the table of values.

| Amount <br> borrowed | Interest rate | Loan term | Monthly <br> repayments | Total amount <br> repaid |
| :--- | :--- | :--- | :--- | :--- |
| $\$ 350,000$ | $6.5 \%$ | 10 years |  |  |
| $\$ 350,000$ | $6.5 \%$ | 15 years |  |  |
| $\$ 350,000$ | $6.5 \%$ | 20 years |  |  |
| $\$ 350,000$ | $6.5 \%$ | 25 years |  |  |
| $\$ 350,000$ | $6.5 \%$ | 30 years |  |  |

6. Three years ago, Alif borrowed $\$ 180,000$ at $8 \%$ p.a. interest over 15 years. Alif has received a $\$ 12,000$ work bonus. If Alif uses the bonus to make an additional lump sum payment on the loan, what effect will the payment have on the term of the loan and on the total amount of interest he will pay? Use this website: https://www.infochoice.com.au/calculators/lump-sum-calculator/
7. When Shaye borrowed $\$ 250,000$ at $7.25 \%$ p.a. interest for 15 years, her monthly repayments were $\$ 2,283$. Two years after she started repaying the loan, Shaye got a pay rise and she decided that she could afford to increase her repayments by $\$ 700$ per month. Use this website to answer the following questions: https://www.infochoice.com.au/calculators/extra-loan-repayments-calculator/
a. How many years quicker will she repay her loan by increasing her repayments by $\$ 700$ per month?
b. How much interest will she save?

## FINDING A PROPERTY

The best way to make a good deal in the property market is to buy below market value. Before you start looking for the right property to buy, there are a few things you can do that will save you money and worry in the future.

- Save as big a deposit as you can. A big deposit makes it easier to get finance and the finance can be cheaper. With a big deposit, you won't have to pay mortgage insurance.
- Work out how much you can afford to spend and, if possible, arrange to have 'pre-approved finance'. When you have pre-approved finance, it is easier to get the person selling the property you want to accept your offer. Preapproved finance will also help your finance institution to process your final approval more quickly than an approval starting from scratch.
- Take your time and look at a lot of properties. After you know the market, choose about four you like and make a cheap offer on each of them, you never know which vendor needs to sell quickly and you could get a bargain.


## Reading advertisements and looking at property

Real estate agents advertise in many different places. Newspapers and local papers regularly include a 'Real Estate Guide', and letterbox ads are common. However, the Internet is rapidly becoming the most popular way to advertise real estate. Using the Internet to advertise real estate means that even in lockdown real estate agents can still sell property and provide updates to buyers.

When you read an advertisement for property, then look at the property, often you wonder whether the advertisement was for the property you saw! Very few real estate advertisements will list, or mention, any poor or undesirable features. Most highlight any good features and try to make the property sound as attractive as possible. Real estate agents have a language of their own. They talk about 'WCs' when they mean toilets or advertise 'lake glimpses' even if you can only see the lake when standing on your toes on the upstairs balcony.

## Example

What does this advertisement mean?
'This is a property that boasts potential. Only 150 m from the freeway makes it ideal for commuting to work in the city. Features include 3bdr, 2bths, large lounge, renovated kitchen, and plenty of off-street parking. Great first home.'


## Solution

Boasts potential usually means the house needs to be renovated.
150 m from the freeway probably means that there may be a problem with traffic noise.
To work in the city might mean there could be high unemployment in the area. 3 bdr, 2 bths means 3 bedrooms and 2 bathrooms.
Renovated kitchen means the kitchen has been renovated but everything else is probably old.
Great first home means it is probably considered cheap for property.

## EXERCISE 6

1. Match each of the abbreviations in $\mathbf{a}$ to $\mathbf{g}$ with its meaning in $\mathbf{A}$ to $\mathbf{G}$.
a. WC
A. built-in wardrobe
b. WIR
B. reverse cycle air-conditioning
c. BIW
C. ensuite (a small bathroom off a bedroom
d. B/ins
D. 5 minutes to the central business district
e. R/C air con
E. built-ins (e.g. cupboards, bar)
f. 5 mins to $C B D$
F. toilet
g. ens
G. walk-in wardrobe
2. Write a sentence to explain what each of these expressions used in real estate advertisements really mean. You may need to research the terms
a. In need of TLC
b. Renovate or detonate
3. Explain what the following advertisement really means.

This immaculate three bdr plus ens home is on the market for the first time since its completion in 2010. Totalling 170sqm, this spectacular family home offers a wonderful lifestyle opportunity.

This welcoming, light filled terrace home is located across the road from the CBD offering a number of restaurants and general amenities. Ovals, medical centre and childcare are also within walking distance - perfect for young families, downsizers and investors alike.

A separate formal lounge or dining room to the right of the entry leads into a large, bright and open meals/living and kitchen area with a R/C air con. 2.7 metre ceilings and square-set cornices show just some of the quality features of this desirable residence. Stainless steel appliances, stone bench tops and a quality well-designed kitchen will appeal to the home chef and provide practical living.
Upstairs encompasses three well-sized bdr, all with BIW and the master with a generous ens.

The low maintenance and spotless backyard has plenty of room for pets or kids and is secure and private. The double lock-up carport is at the rear of the yard, completing what is a fantastic residence.

## WEEK 13/14 PORTFOLIO TASK

You own a car worth \$10,000 and it depreciates at a rate of $20 \%$ p.a. Would you prefer this depreciation be by straight line depreciation or diminishing value depreciation?

Show and explain which depreciation you prefer.

## MARKING RUBRIC

| CRITERIA | EXPECTATIONS | POSS | MULT | GIVEN | TOTAL |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Practical | Student completes practical work, including exercises of the brief to an acceptable standard set by the teacher. | 2 | 3 |  | /6 |
| Portfolio Task | Student completes the portfolio task of the week to an acceptable standard set by the teacher. | 2 | 2 |  | 14 |
| Reasoning and Communications | Student responses are accurate and appropriate in presentation of mathematical ideas in different contexts, with clear and logical working out shown. | 4 | - |  | 14 |
| Concepts and Techniques | Student submitted work selects and applies appropriate mathematical modelling and problem solving techniques to solve practical problems and demonstrates proficiency in the use of mathematical facts, techniques and formulae. | 4 | - |  | 14 |
|  | Submission Guidelines |  |  |  |  |
| Timeliness | Student submits the exercises and portfolio tasks by the set deadline. See scoring guidelines for specific details. | 2 | - |  | /2 |
|  |  | FINAL |  | 120 |  |

## Student Reflection:

How did you go with this week's work?

What was interesting?

What did you find easy?

What do you need to work on?

