HAWKER COLLEGE
Engage | Inspire | Achieve

Goals

Goals for this fortnight:
- review calculating a percentage of a given amount
- review one amount expressed as a percentage of another
- determine the overall change in a quantity following repeated percentage changes; for example, an increase of 10% followed by a decrease of 10%
- calculate simple interest for different rates and periods

Theoretical Components

STEP 1

Resources:
PDF file: Week 6 and 7 Notes and Exercises

This Fortnight:
We will be learning:
- Change fractions into percentages
- Express percentages as decimals
- Find a percentage of a given amount
- Converting a comparison of two quantities to percentages
- Simple interest formula
- Term deposits
- Calculating repayments

Practical Components

STEP 2
Read through Week 6 and 7 Notes and Exercises for instructions on what to do.

There are 6 Exercises in this booklet. Read any worked examples before you begin.

Remember to regularly check Google Classroom for messages.

Portfolio Task

STEP 3

Complete the task at the end of the brief and submit your weekly work. 😊

Other — **Remember to check Google Classroom or hawkermaths.com for each week's learning brief**

ESSENTIAL MATHEMATICS 2

WEEK 6 AND 7 – PERCENTAGES AND APPLICATIONS

How are we ever going to use this?

- To calculate interest earned or interest we will have to pay
- To make comparisons
- Whenever we need to calculate repeated percentages

PERCENTAGE REVIEW – CONVERTING BETWEEN FRACTIONS, DECIMALS AND PERCENTAGES
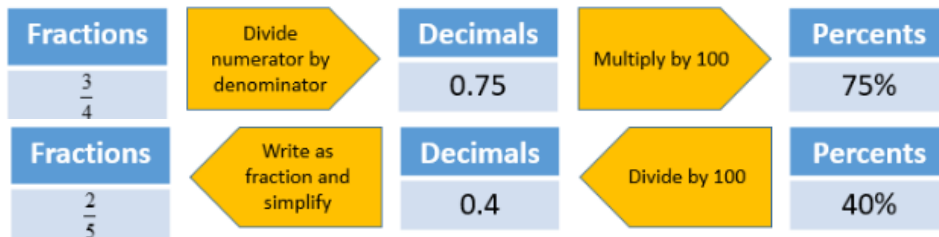
FRACTION	DECIMAL	PERCENT
$^3/_5$		
	0.64	
		55%
	0.16	
$^{17}/_{100}$		
	0.35	
		28%
		62.5%
	0.59	
$^4/_7$		
	0.44	
		32%
$^3/_2$		
	0.824	

EXPRESSING ONE QUANTITY AS A PERCENTAGE OF ANOTHER

We often change quantities into percentages so that we can compare them.

For example, we often calculate our marks in a test as percentages so we can compare our performances in each subject.

Example:
1. What percentage is 16 out of 20?
2. Twenty one of Patricia's 30 alpacas are white. What percentage of Patricia's alpacas are:
    a. white?
    b. not white?

Solution:
1. $16 \div 20 \times 100 = 80\%$

2.
    a. $21 \div 30 \times 100 = 70\%$
    b. $9 \div 30 \times 100 = 30\%$

$$\text{Percentage} = \frac{\text{amount}}{\text{whole amount}} \times 100\% \text{ OR amount} \div \text{whole amount} \times 100\%$$

Example:
1. Express 18 out of 25 as a percentage.
2. Express 36 minutes as a percentage of 2 hours.

Solution:
1. $18 \div 25 \times 100 = 72\%$

2. 2 hours = 120 minutes
   $36 \div 120 \times 100 = 30\%$

Exercise 1

1. Change these fractions to percentages.

   a. $\dfrac{3}{5}$

   b. $\dfrac{7}{20}$

   c. $\dfrac{9}{10}$

   d. $\dfrac{3}{4}$

2. Write the following as percentages.

   a. 32 out of 40   b. 60 out of 75

   c. 5 minutes of 1 hour   d. 3 months of 1 year

   e. 18 hours of 4 days   f. 50 days of 1 year

3. In Patricia's flock of 30 alpacas, three are black. What percentage of the flock is black?

4. As a result of a bushfire 170 out of 200 hectares in a national park were destroyed. What percentage of the national park was destroyed in the fire?

5. Of the 2500 people on a cruise ship, 75 are vegetarians. What percentage of the people on the cruise were vegetarians?

6. Arti has $50 in his wallet when he goes shopping. He has $5 left when he returns home.

    a. How much money does Arti spend?

    b. What percentage of his money does he spend?

7. In a class of 23 students, 8 walk to school and 11 come by bus. What percentage of students walks to school?

8. George and Sue are travelling to see their parents. The trip takes 7 hours. They take a break 2 hours and 40 minutes into the trip. What percentage of the trip have they completed when they take a break?

FINDING A PERCENTAGE OF A QUANTITY

$$\text{Percentage of a quantity} = \frac{\text{Percentage}}{100} \times \text{quantity} \quad \text{OR} \quad \text{Percentage} \div 100 \times \text{quantity}$$

Example:
1. Calculate 23% of $1650.
2. Find 32% of 4 m.

Solution:
1. $23 \div 100 \times 1650 = \$379.50$

2. Change 4 m to 400 cm
   $32 \div 100 \times 400 = 128$ cm

Exercise 2

1. Find:

   a. 11% of $400

   b. 60% of 140 kg

   c. 42% of 600 cm

   d. 85% of 700,000 people

   e. 12% of 8 m

   f. 60% of 1 year (answer in days)

   g. 87.5% of 16 weeks
      (answer in days)

   h. 40% of 1.5 tonnes
      (answer in kg)

2. Jane's laptop can be used for 6 hours without recharging. Her laptop shows she has 62% of her time left. How much time does she have left before her laptop needs recharging?

PERCENTAGE AFTER PERCENTAGE

In some situations, we may need to calculate one percentage after another; for example, buying something on sale with an additional discount for cash or for paying your bill in a certain time. In this situation, we need to remember to do each percentage calculation separately.

Example:

Dennis' Paints offers a 25% trade discount to builders and a further 5% discount if the account is paid within 10 days. Brian is a builder. He purchased $840 worth of paint and paid the account within 10 days.

a How much did he pay for the paint?

b What single discount is equivalent to a 25% discount followed by a 5% discount?

Solution:

a First, calculate the trade discount.

$$\text{Trade discount} = \frac{25}{100} \times 840$$
$$= 210$$

Then calculate the discounted price.

$$\text{Trade price} = 840 - 210$$
$$= \$630$$

Then calculate the further discount.

$$\text{Further discount} = \frac{5}{100} \times 630$$
$$= 31.5$$

Calculate the final price.

$$\text{Final price} = 630 - 31.5$$
$$= \$598.50$$

b Find the total discount.

$$\text{Total discount} = 210 + 31.5$$
$$= \$241.50$$

The single equivalent percentage discount is given by: $\frac{\text{total discount}}{\text{original price}} \times 100$

$$\text{Percentage discount} = \frac{\text{total discount}}{\text{original price}} \times 100$$
$$= \frac{241.50}{840} \times 100$$
$$= 28.75\%$$

1. The advertised price for a Luxura car is $85,999. When Angelique visits the car dealership, she is offered an end-of-financial-year deal of 30% discount. If she finalises the deal within 2 days, she is offered an additional 5% discount.

    a. What is the price of the Luxura car after the 30% discount?

    b. What price would Angelique pay if she finalises the deal in the two-day timeframe?

    c. Is this equal to 35% off the original price? Justify your answer with a calculation.

2. The Complete Discount Store is having an end-of-year sale, offering 25% of normal prices and a further 10% discount for cash.

    a. Simon and Maddy plan to pay cash for a bed that normally sells for $2300. How much will they pay during the sale?

    b. How much is their total saving?

    c. What percentage is this?

SIMPLE INTEREST

Interest is the money we earn when we invest with a bank, credit union or other financial institution. It is also the extra money we pay back when we take out a loan. When we take out a simple interest loan or make a simple interest investment, our interest payments are the same every year.

The simple interest formula is:
$$I = Prn$$

where:
I = interest earned or paid,
P = principal (what you invest or borrow)
r = rate of interest per time period, as a decimal
n = number of time periods

Example:

Jayne invested $700 at 5% p.a. simple interest for 3 years.
a. How much interest will she earn?
b. Calculate the total amount in her account at the end of 3 years.

Solution:

a. P = $700, r = 0·05 and n = 3 years.
   Interest = $700 × 0·05 × 3 = $105

b. At the end of 3 years, she will have $700 + $105 = $805 in her account.

Example:

Daniel invested $5,600 at 8.2% p.a. simple interest for 11 months. How much interest will he earn?

Solution:

$P = \$5600, r = 0.082$ and $n = 11/12$ years
Interest $= \$5600 \times 0.082 \times \dfrac{11}{12}$
$\qquad = \$420.93$

1. Express the following percentages as decimals.

    a. 11%   b. 15%

    c. 9%   d. 3%

    e. 0.5%   f. 2.3%

2. Calculate the simple interest on each of these investments using the formula $I = Prn$. Express your answer in dollars and cents.

    a. $8,000 at 4% p.a. for 5 years   b. $6,200 at 6% p.a. for 3.5 years

    c. $12,000 for 7 years at 5.3% p.a.   d. $1,500 at 4.3% p.a. for 2 years

3. Determine whether the following statement is true or false. Use some calculations to justify your answer.

        'If the interest rate doubles, you earn twice as much interest.'

4. Spot the errors in Fred's calculation below and give the correct solution. Lindsay invested $5,000 for 3 months at 8.3% p.a. simple interest. Calculate the interest Lindsay will receive. This is Fred's calculation:

$I = Prn$
$I = \$5000 \times 8.3 \times 3$
$I = \$124\,500$

5. Nicole borrowed $2,400 from a finance company for 2 years at 18% interest p.a.
   a. How much interest did she have to pay?

   b. How much, including interest, did Nicole have to repay the finance company?

6. James invested $1,500 in an account paying 4.5% p.a. for 3 years.
   a. How much interest did he earn?

   b. How much, including interest, did James, have in his account at the end of three years?

If you have money to invest and you know that you won't need any of the money for some time, you can invest it in a term deposit. The term, or length of the investment, can be as short as a month or for many years. Usually, a term deposit has a fixed rate of interest for the whole term.

Example:
Ian has $75 000 to invest and he knows he won't need the money for several years. For what term and interest rate can he invest the money?

Solution:

This table shows the annual term deposit rates offered by a building society.

Term	$10 000 to $20 000	More than $20 000 to $50 000	More than $50 000 to $100 000	More than $100 000
1 month	4·00%	4·05%	4·05%	4·20%
2 months	4·25%	4·25%	4·30%	4·34%
3 months	4·25%	5·10%	5·60%	5·65%
6 months	4·75%	5·20%	5·80%	4·50%
12 months	5·25%	5·60%	5·95%	6·05%
2 years	5·30%	5·50%	5·95%	6·20%

The value of Ian's investment is in the range 'More than $50 000 to $100 000'. He should look at the rates in the second column from the right. The highest rate he can get is 5.95% for 12 months or 2 years. If he thinks that interest rates will go down, he should choose the 2-year option.

Example:
Ian is going to invest his $75,000 for the short term of 6 months. On the accounts in the table above, interest is paid at maturity. How much interest will he earn from this investment?

Solution:

'Interest paid at maturity' means that the investment will earn simple interest. From the table, the interest rate on Ian's investment will be 5·80% p.a.

In the formula $I = Prn$, $P = \$75,000$, $r = 0{\cdot}058$ and $n = \frac{1}{2}$ year.

Interest $= \$75,000 \times 0{\cdot}058 \times \frac{1}{2} = \$2,175$

When his investment matures, Ian will receive $2,175 in interest. The total he will receive back from the bank will be $77,175.

Use the table on the previous page to answer the following questions.

1. Find the interest rate that applies to each of these term investments.

    a. $65,000 for 12 months   b. $24,000 for 3 months

    c. $12,000 for 2 years   d. $125,000 for 1 month

2. Alex has $8,000 to invest for 12 months. Why can't he invest in a term deposit?

3. Megan plans to invest $140,000 for 6 months. Explain how she could invest the money and earn more than the 4.50%.

4. Calculate the interest on each of these term deposits.

    a. $24,000 invested for 6 months   b. $31,500 invested for 2 months

    c. $83,000 invested for 1 year   d. $103,000 invested for 3 months

Simple interest is also known as flat-rate interest. This is because the same amount of interest is paid or earned each year.

Example:

Melinda borrowed $24 500 for 3 years from a finance company to buy a car. The interest rate is 9.5% p.a. flat.
a How much interest will the finance company charge on the loan?
b How much will Melinda pay the finance company to repay the loan?
c Melinda is going to repay the loan over 3 years, making a payment to the finance company every month. How much will she have to pay each month?

Solution:

a Flat-rate interest is the same as simple interest, so we use the simple interest formula.
$P = 24\ 500$, $r = 0.095$, $n = 3$

$I = 24\ 500 \times 0.095 \times 3$
$= 6982.5$
Melinda will pay $6982.50 interest.

b Melinda has to repay the amount she borrowed plus the interest.

$24\ 500 + 6982.5 = 31\ 482.5$
Melinda will pay $31 482.50 to the finance company to repay the $24 500 loan.

c We need to calculate the number of months in 3 years by multiplying by 12. The total amount must be repaid over 36 payments.

$3 \times 12 = 36$ months

$31\ 482.50 \div 36$
$= \$874.51$
Melinda has to pay $874.51 each month.

1. Vanessa borrowed $30,000 from a finance company to set up a catering business. She agreed to repay the money in monthly instalments over 5 years at 12% flat-rate interest.

   a. How much interest will Vanessa have to pay?

   b. Calculate the total amount she must repay.

   c. How much will Vanessa repay each month?

2. Steven has just finished his apprenticeship and he wants to borrow $1500 to buy some additional tools. The finance company offers him the money over 2 years at 8.7% flat-rate interest provided he makes monthly repayments.

   a. How much interest will Steven be charged?

   b. How much will Steven owe the finance company altogether?

   c. Calculate the value of his monthly repayments.

3. Denise wants to buy a new ride-on mower for her lawn-mowing business. The mower costs $9495 and she is offered a $700 trade-in on her old mower. The store offers her terms over 3 years at 7.4% flat-rate interest after a deposit of $2500.

    a. How much will Denise owe after she trades in in her old mower?

    b. How much will Denise owe after she pays a deposit?

    c. How much interest will be charged?

    d. Calculate the total amount Denise must pay in instalments.

    e. What is the amount of Denise's monthly repayments?

Messi plays for Sydney Football Club and earns an annual taxable income of $57,600. His employer has deducted $230 per week in tax instalments.

Taxable income	Tax on this income
0 – $18,200	Nil
$18,201 – $45,000	19 cents for each $1 over $18,200
$45,001 – $120,000	$5,092 plus 32.5 cents for each $1 over $45,000
$120,001 – $180,000	$29,467 plus 37 cents for each $1 over $120,000
$180,001 and over	$51,667 plus 45 cents for each $1 over $180,000

1. Use the table above to calculate the tax on his income correct to two decimal places.

2. *What percentage of Messi's taxable income is this? This is Messi's average rate of tax.*

3. Medicare Levy is calculated at 2% of taxable income. Calculate the Medicare Levy payable on Messi's income correct to the nearest cent.

4. What percentage of Messi's income has his employer withheld in tax instalments?

5. Has Messi's employer deducted enough in tax instalments to cover Messi's tax and Medicare Levy? Is Messi entitled to a tax refund or does he need to pay the Tax Office? How much?

MARKING RUBRIC

CRITERIA	EXPECTATIONS	POSS	MULT	GIVEN	TOTAL
Practical	Student completes practical work, including exercises of the brief to an acceptable standard set by the teacher.	2	3		/6
Portfolio Task	Student completes the portfolio task of the week to an acceptable standard set by the teacher.	2	2		/4
Reasoning and Communications	Student responses are **accurate** and appropriate in presentation of mathematical ideas, with **clear** and logical **working out** shown.	4	-		/4
Concepts and Techniques	Student submitted work selects and applies **appropriate** mathematical modelling and **problem solving techniques** to solve practical problems, and demonstrates **proficiency** in the use of **mathematical facts, techniques** and **formulae**	4	-		/4
	Submission Guidelines				
Timeliness	Student submits the exercises and portfolio tasks by the set deadline. See scoring guidelines for specific details.	2	-		/2
		FINAL			/20

Student Reflection: 😊

How did you go with this week's work?

What was interesting?

What did you find easy?

What do you need to work on?