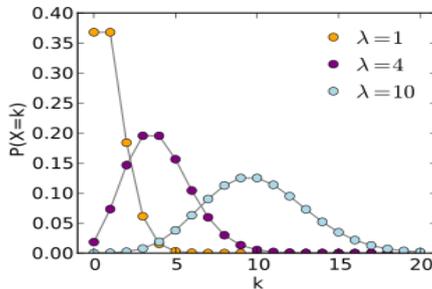


Goals



This fortnight we are going to:

- Recognise and use the formula for probabilities calculated from the Hypergeometric distribution (sampling without replacement)
- Recognise and use the formula for probabilities calculated from the Poisson model
- Appreciate the widespread applications in analysing traffic flow, in fault prediction on electric cables, in the prediction of randomly occurring accidents, etc
- Use the CAS to compute probabilities based on Hypergeometric and Poisson distribution

Theoretical Components

Make notes on the following chapters:

Reasoning and Data

- 5.1 - Hypergeometric distribution: sampling without replacement
- 5.2 Mean and variance of a hypergeometric distribution
- 5.4 Poisson distribution

Hypergeometric Distribution:

- <https://www.youtube.com/watch?v=BCeFgnh6A1U>

Poisson Distribution:

- <https://www.intmath.com/counting-probability/13-poisson-probability-distribution.php>
- <https://online.stat.psu.edu/stat414/lesson/12>

Practical Components

Do the following questions:

Organise your solutions neatly in your exercise book.

Chapter 5 of Reasoning and Data (pdf – Google Classroom)

- 5a: 1-15
- 5c: 1-6, even after 6

Mathspace

Investigation

See next page

Prepare a two-sided handwritten A4 summary sheet for your exam.

Other

Random fun fact: Did you know that it is possible to decompose a solid ball in three dimensions into a finite number of disjoint subsets, then reassemble the pieces in a different way to yield two identical copies of the original ball? This strange phenomenon is what the Banach-Tarski paradox, an important result in set-theoretic geometry, describes. While the construction is purely theoretical, it nevertheless illustrates that results in pure mathematics do not conform to commonly held intuitions.

Week 7 and 8 Investigation

A customer buys transistors in batches of eight and decides whether or not to accept a batch according to the following procedure. Two transistors are selected at random, without replacement. The batch is accepted if neither is defective. If one of the inspected transistors is defective the customer selects another at random from the remaining six. If this additional transistor is defective, or if both of the transistors in the first sample are defective, the batch is rejected. Otherwise the batch is accepted.

- a. Find the possibilities, to three decimal places, that batches containing 0, 1, 2, ... 8 defectives are accepted.
- b. If four per cent (4%) of transistors produced are defective, calculate the probabilities, to three decimal places, that a batch will contain 0, 1, 2, ... 8 defectives.
- c. Calculate the overall expected proportion of batches accepted.

Notes on Discrete and Continuous Random Variables

Discrete Random Variable: variables whose values can be counted.

Probabilities are listed in a probability distribution table. Binomial, Hypergeometric, Poisson

Binomial Probability Distribution: the probability distribution that gives the probability of x successes in n trials when the probability of success is p for each trial of the experiment. One repetition of a binomial experiment is called a Bernoulli trial (trials are independent) and each trial has only two possible outcomes (i.e. success or failure)

- The probability of x successes in n trials is given by: $P(x) = {}^n C_x p^x q^{n-x}$, where p = probability of success, q = probability of failure ($q = 1 - p$)
- A typical example: Five percent of all DVD players manufactured by a large electronics company are defective. A quality control inspector randomly selects three DVD players from the production line. What is the probability that exactly one of these three DVD players is defective?
- The mean and standard deviation of a binomial distributions are: $\mu = np$ and $\sigma = \sqrt{npq}$
- Exercise 4C in **Reasoning and Data** textbook

Hypergeometric Probability Distribution: the probability distribution that is applied to determine the probability of x successes in n trials when the trials are not independent (without replacement)

- The probability of x successes in n trials is given by: $P(x) = \frac{\binom{D}{x} \binom{N-D}{n-x}}{\binom{N}{n}}$
 - a. N = total number of elements in the population
 - b. D = number of successes in the population
 - c. $N - D$ = number of failures
 - d. n = number of trials
 - e. x = number of success in the trial
 - f. $n - x$ = number of failures in the trail
- A typical example: Dawn Corporation has 12 employees who hold managerial positions. Of them, seven are female and five are male. The company is planning to send three of these 12 managers to a conference. If three are randomly selected out of the 12.
 - a. Find the probability that all three of them are female
 - b. Find the probability that at most one of them is female
- The mean and standard deviation of a hypergeometric distribution are: $\mu =$ given or calculate using average and $\sigma =$ given
- Exercise 5a in **Reasoning and Data** textbook

Poisson Probability Distribution: the probability distribution that gives the probability of x occurrences in an interval when the average occurrences in that interval are λ .

- The probability of x occurrences in an interval is: $P(x) = \frac{\lambda^x e^{-\lambda}}{x!}$
- A typical example: On average a household receives 9.5 marketing phone calls per week. Find the probability that a randomly selected household receives exactly six marketing calls during a given week.
- The mean and standard deviation of a binomial distribution are: $\mu = \lambda$, $\sigma^2 = \lambda$ and $\sigma = \sqrt{\lambda}$
- Exercise 5c in **Reasoning and Data** textbook