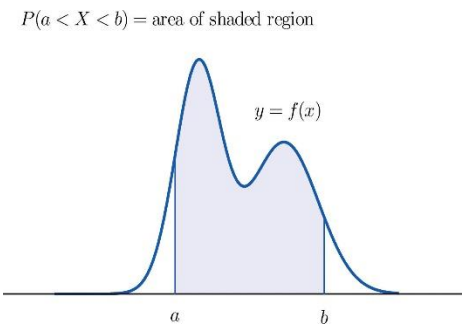
Week 11
Term 4
2022

HAWKER COLLEGE
Engage | Inspire | Achieve

Learning Brief
SMM4

Goals



This week we are going to:

- Understand the assumptions on which the normal model is based
- Understand the probability limits of *almost certainly* and *very probably*, that is, the three and two sigma limits

Theoretical Components

Make notes on the following chapters:

Maths Quest 12 Mathematical Methods

- 12E - The normal distribution
- 12F - The standard normal distribution
- 12G - The inverse cumulative normal distribution

Normal Curve:

- https://www.youtube.com/watch?v=McSFVzc8Swk

1. $f(x) \geq 0$ for all x
2. $\int_{-\infty}^{\infty} f(x)\,dx = 1$
3. $P(a \leq x \leq b) = \int_a^b f(x)\,dx$

Practical Components

Do the following questions:
Organise your solutions neatly in your exercise book.

Chapter 12 of Maths Quest 12 Mathematical Methods (pdf – Google Classroom)

- 12E: odd numbered questions
- 12F: odd numbered questions
- 12G: even numbered questions

Mathspace

Investigation

See next page

Other

Random fun fact: A twin prime is a prime number that is either two more or two less than another prime number - for example, 17 and 19. A famous result of Viggo Brun in 1915 is that the sum of the reciprocals of the twin primes is convergent, but one of the great open questions in all of mathematics is whether there are infinitely many twin primes. Brun's result shows that if there are infinitely many such primes, then they must be exceedingly rare. Breakthrough work by Yitang Zhang in 2013 demonstrated for the first time that there exists infinitely many pairs of primes that differ by a fixed finite number; in fact, Zhang gave an explicit value of 70 million for this difference. Subsequent work by an international group of mathematicians subsequently reduced this bound to 256 - much smaller than Zhang's original value, but still far off from the value of two required by the twin prime conjecture. For his pioneering work in this field, the British mathematician James Maynard was awarded the Fields Medal in 2022, regarded as the highest honour a mathematician can receive.

Week 11 Investigation

1. A random variable, X, has its frequency curve defined as:

$$f(x) = \begin{cases} \frac{1}{2}e^{-\frac{1}{2}x}, & x > 0 \\ 0, & \text{elsewhere} \end{cases}$$

   a. Draw the graph of $f(x)$
   b. Show that $f(x)$ is a probability density function.
   c. Find the probability, correct to 4 decimal places, that X is:
      i. smaller than 3
      ii. greater than 2.5
      iii. greater than 2.5, given that it is smaller than 3.

2. The wingspan of birds of a particular species has normal distribution with mean 50 cm and standard deviation 5 cm.
   a. Find the probability that a randomly selected bird has a wingspan greater than 60 cm.
   b. If the wingspan is measured to nearest cm, find the probability that a randomly selected bird has a wingspan measured as 50 cm.