NAME:______________________

Week 12
Term 2
2022

HAWKER COLLEGE
Engage | Inspire | Achieve

Learning Brief

MA3

Goals

This brief we are covering:

- further applications of sequences and series
- growth and decay
- revisit recursive/iterative sequences

Theoretical Components

Resources:
For this week the theory work is in the PDF file: *Week 12 Further Applications of Sequences*

Knowledge Checklist

- Understand which application is an arithmetic progression
- Understand which application is a geometric progression
- Write difference/recursive equations
- Use spreadsheets to solve recursive problems

Practical Components

There are questions to be answered in the booklet *Week 12 Further Applications of Sequences*

Investigation

See the end of the brief😊

Quiz

There is a quiz on Mathspace.

Applications of series and sequences

This section consists of a mixture of problems where the work covered in term and sum of arithmetic sequences, term and sum of geometric sequences and limiting sum (sum to infinity).

The following general guidelines can assist you in solving the problems.

1. Read the question carefully.
2. Decide whether the information suggests an arithmetic or geometric sequence. Check to see if there is a constant difference between successive terms or a constant ratio.
3. Write down the information from the problem using appropriate notation. For example, if you are told that the 5th term is 12, write $t_5 = 12$. If the sequence is arithmetic, you then have an equation to work with, namely: $a + 4d = 12$. If you know the sequence is geometric, then $ar^4 = 12$.
4. Define what you need to calculate and write an appropriate formula or formulas. For example, if you need to find the 10th number in a sequence which you know is geometric, you have an equation: $t_{10} = ar^9$. It can be calculated if a and r are known or can be established.
5. Use algebra to find what is required in the problem.

Example

A city produced 100 tonnes of rubbish in the year 2001. Forecasts suggest that this may increase by 2% each year. If these forecasts are true,

a) what will be the city's rubbish output in 2005?
b) in which year will the amount of rubbish reach 120 tonnes?
c) what was the total amount of rubbish produced by the city in the years 2001, 2002 and 2003?

This is an example of a geometric sequence where $a = 100$ and $r = 1.02$. Note that $r \neq 0.02$. If this was the case, then multiplying 100 by 0.02 would result in a lesser amount of rubbish in the second year and so on. We are told that the amount of rubbish increases by 2%. That is the original amount plus an extra 2%, or: original amount + 2% of original amount = original amount (1 + 2%) = original amount (1 + 0.02) = 1.02 × original amount.

a)

1 Find the first term, a.	**a** $a = 100$
2 Determine the common ratio, r.	Increase by 2% $1 + 2\% = 1 + 0.02$ $r = 1.02$
3 Determine which term is represented by the amount of rubbish for the year 2005.	Year 2001 is the first term, so $n = 1$. Year 2002 is the second term, so $n = 2$. Year 2005 is the fifth term, so $n = 5$.
4 Use $t_n = ar^{n-1}$ to find the amount of rubbish collected in the fifth year.	$t_5 = 100 \times 1.02^{5-1}$ $= 100 \times 1.0824$ $= 108.24$
5 Write your response.	The amount of rubbish produced in the fifth year, or 2005, will be 108.24 tonnes.

b) Solve by using trial and error:

1 Use $t_n = ar^{n-1}$ and $t_n = 120$.	$100(1.02)^{n-1} = 120$ $(1.02)^{n-1} = 1.2$
2 Try various values of n.	Let $n = 10$, $(1.02)^9 = 1.195$ Let $n = 11$, $(1.02)^{10} = 1.21$
3 Write your answer.	During the 11th year, that is, during 2011, the rubbish will have exceeded 120 tonnes.

c)

1 We need to find the sum of the first 3 years. Use $S_n = \frac{a(r^n - 1)}{r - 1}$ where $n = 3$.	**c** $S_3 = \frac{100(1.02^3 - 1)}{1.02 - 1}$ $= 306.04$

Exercise 1

Q1. A farmer harvests 4 tonnes of lucerne in his first year of production. In his business plan, he has estimated an annual increase of 6% on his lucerne harvest.

a) According to this plan, how many tonnes of lucerne should he harvest in his 7th year of production?
b) In which year will his harvest reach 10 tonnes?
c) How much will he expect to harvest in the first three years?

Q2. The population of a town is decreasing by 10% each year. The present population is 10000.

a) What will be the population in 5 years' time?
b) How long before the population will be zero?

Q3. A company exported $300 000 worth of manufactured goods in its first year of production. According to the business plan of the company, this amount should increase each year by 7.5%.

a) How much would the company be expected to export in its 5th year?
b) In which year would exports exceed $500 000?
c) What is the total amount exported by the company in its first 7 years of operation?

Q4. In 1970 the Smith family purchased a small house for $60 000.   With the years the value of their property rose steadily. In 1975 the value of the house was $69 000 and in 1980 it reached $79 350.

a) Assuming that the pattern continues through the years, find (to the nearest dollar) the value of the Smiths' house in i 1985, ii 1995.
b) By what factor will the value of the house have increased by the year 2000, compared to the original value?

Q5. An accountant has been working with the same company for 15 years. She commenced on a salary of $28 000 dollars and has received a $2500 increase each year.

a) What type of sequence of numbers does her annual income follow?
b) How much did she earn in her 15th year of employment?
c) How much has she earned from the company altogether?

Q6. Recurring decimals can be expressed as rational numbers ie fractions. The technique for doing this is as follows.

0.22222222……. = 0.2 + 0.02 + 0.002 + 0.0002 + ………

This is a geometric progression with *a* = 0.2 and *r* = $\frac{0.02}{0.2}$ = 0.1

Using the sum to infinity $S_\infty = \frac{a}{1-r}$ gives $\frac{0.2}{1-0.1}$ gives $\frac{0.2}{0.9}$ which simplifies to $\frac{2}{9}$ (multiply numerator and denominator by 10).

Thus $\frac{2}{9}$ = 0.222222………

Use this technique to convert the recurring decimal to a fraction.

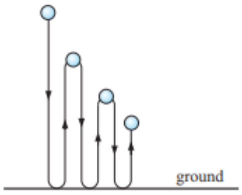
a) 0.6666666...

b) 0.777777...

c) 0.27272727...

d) 0.42857142857...

Q7. A ball is dropped from a height of 1 metre. On each bounce it reaches a height of 80% of the previous bounce. What is the total distance the ball travels before it comes to rest?



FURTHER APPLICATIONS OF SERIES AND SEQUENCES

1. A diving vessel descends below the surface of the water at a constant rate so that the depth of the vessel after 1 minute, 2 minutes and 3 minutes is 50 meters, 100 meters and 150 meters.
    a. How much is the depth increasing each minute?
    b. What will the depth of the vessel be after 4 minutes?
    c. Continuing at this rate, what will be the depth of the vessel after 10 minutes?
    d. If n is the number of minutes it takes to reach a depth of 120 meters, solve for n.

2. A worker at a factory is stacking cylindrical-shaped pipes which are stacked in layers. Each layer contains one pipe less than the layer below it. There are 6 pipes in the topmost layer, 7 pipes in the next layer, and so on.
    a. Form an equation for the number of pipes in the nth layer.
    b. How many pipes are in the 10^{th} layer?

3. A rare figurine was purchased for $60 and ten years later it is worth $460.
    a. The figurine appreciated in value by a constant amount each year. How much did it appreciate each year?
    b. What will the value of the figurine be in another 10 years' time?

4. A telecommunications company sells 1600 mobile phones in the first month of its operation. The company plans to increase its sales by 200 mobile phones each month.
    a. How many phones does the company plan to sell in the last month of the 5^{th} year of its operation?
    b. How many phones does the company plan to sell in the entire 5-year period?
    c. How long will it take the company to sell 8600 mobile phones?

5. A ball is dropped from a height of 12 meters and will bounce back off the ground to 35% of the height of the previous bounce (or the height from which it is dropped when considering the first bounce)
    a. Write a function, *y*, to represent the height of the *n*th bounce.
    b. Find the height of the fourth bounce. Give your answer correct to two decimal places.

6. Sam starts his career with a monthly wage of $3500. At the beginning of each year that follows he receives a raise and his monthly wage for that year will be $140 greater than the previous year.
    a. What will be his yearly salary in the second year of his service?
    b. Write down an expression for the total amount earned in *m* years. Hint: which formula would you use, T_n or S_n? Do we need to convert any units?
    c. How many years, *y*, would it take for him to earn a total of $495600?

7. Jade is learning to drive. Her first lesson is 24 minutes long, and each subsequent lesson is 2 minutes longer than the lesson before.
    a. How long will her 12th lesson be?
    b. If Jade reaches 35.4 total *hours* of driving in her *n*th lesson, solve for *n*. Hint: which formula would you use, T_n or S_n? Do we need to convert any units?

8. Suppose you save $1 the first day of a month, $2 the second day, $4 the third day, $8 the fourth day, and so on. That is, each day you save twice as much as you did the day before.
    a. What will you put aside for savings on the 17th day of the month?
    b. What will your total savings be for the first 17 days?
    c. What will you put aside for savings on the 29th day of the month?
    d. What will your total savings be for the first 29 days?

9. This year, 600 people are expected to enter the workforce as registered nurses. This number is expected to increase by 4% next year and increase by the same percentage every year after that.
    a. Calculate the number of nurses expected to enter the workforce between six and seven years from now. Round your answer to the nearest whole number.
    b. Calculate the total number of nurses expected to enter the workforce over the next six years. Round your answer to the nearest whole number.

Task 1

A mobile phone depreciates in value by a constant amount per month and its value is given by the explicit rule:

$T_n = 1200 - 20n$, where T_n is the balance (in dollars) after *n* months

a) How much does the value of the phone depreciate each month?

b) What is the purchase price of the phone?

c) Write a recursive rule for V_n in terms of V_{n-1}, and an initial condition V_0. Write both parts of the rule on the same line, separated by a comma.

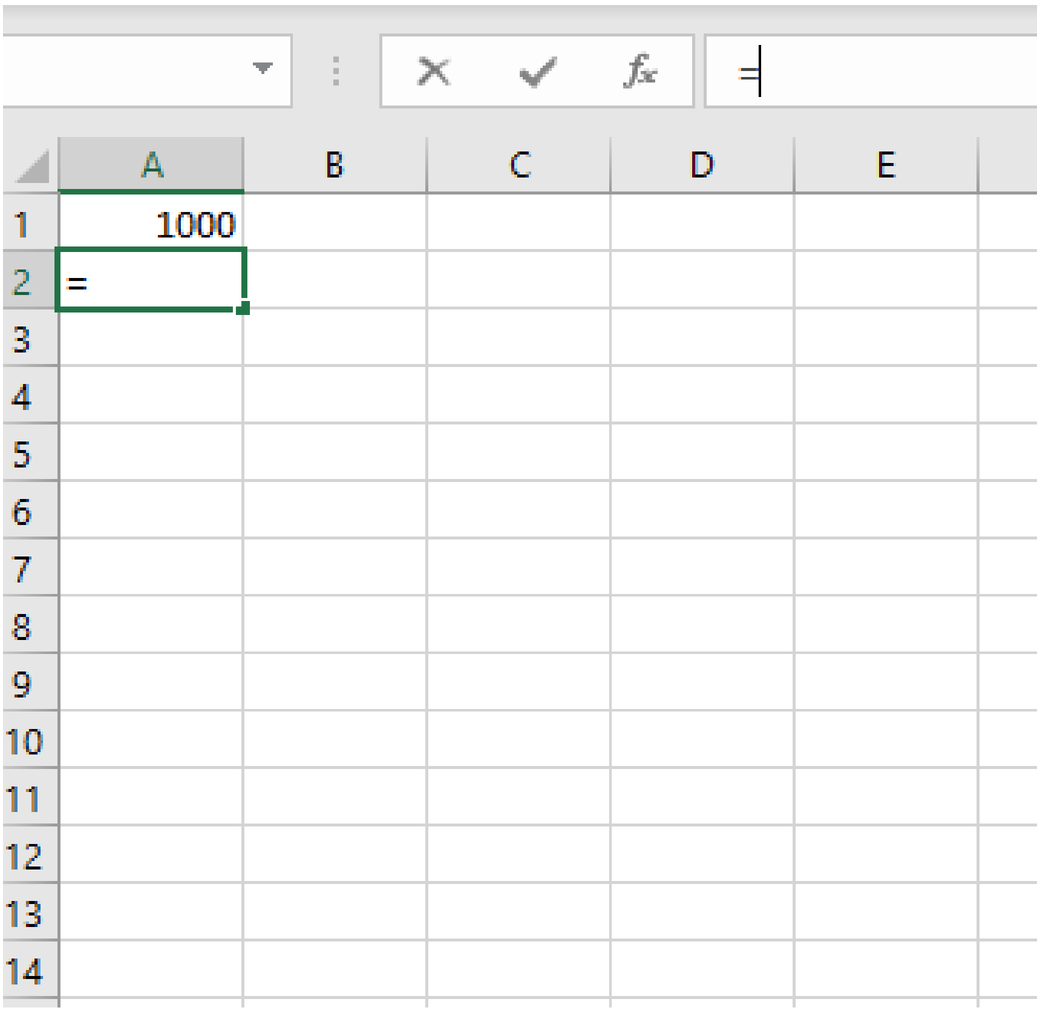
Hint: you may need to revisit recursive/difference sequences from the previous brief.

Task 2

Each year, 1000 salmon are stocked in a creek and the salmon have a 30% chance of surviving and returning to the creek the next year. How many salmon will be in the creek each year and what will be population in the future? That is, what is the equilibrium population for the salmon

Write a difference equation which may be used to calculate the size of the salmon population at the end of each year.

Construct a spreadsheet to find the equilibrium population for the salmon. Fill in your solution and the required equation below.



Marking Rubric **Week 12** **Name:**

CRITERIA	EXPECTATIONS	POSS	MULT	GIVEN	TOTAL
Practical	Student completes practical work, including exercises and Mathspace task, of the brief to an acceptable standard set by the teacher.	**2**	**3**		**/6**
Investigation Task	Student completes the investigation task of the week to an acceptable standard set by the teacher.	**2**	**2**		**/4**
Reasoning and Communications	Student responses are accurate and appropriate in presentation of mathematical ideas, with clear and logical working out shown.	**4**	**-**		**/4**
Concepts and Techniques	Student submitted work selects and applies appropriate mathematical techniques to solve practical problems and demonstrates proficiency in the use of mathematical facts, techniques, and formulae.	**4**	**-**		**/4**
	Submission Guidelines				
Timeliness	Student submits the exercises, Mathspace/online task, and investigation by the set deadline. See scoring guidelines for specific details.	**2**	**-**		**/2**
				FINAL	**/20**

Student Reflection: How did you go with this week's work? What did you learn? What did you find easy?
What do you need to work on?