Week 5/6
Term 3
2022

HAWKER COLLEGE

Engage | Inspire | Achieve

Learning Brief

EM4

Goals

This week and next week we are going to:

- perform simulations of experiments using technology
- recognise that the repetition of change events is likely to produce different results
- identify factors that could complicate the simulation of real-world events
- determine the probabilities associated with simple games and occurrence of simple traffic-light problems

Theoretical Components

Resources:
PDF file: Week 5 and 6 Notes and Exercises

This clip shows you about the Monty Hall problem.

https://www.youtube.com/watch?app=desktop&v=mhlc7peGlGg

Knowledge Checklist

- Applications of probability

Order

1. Work through the Week 5 and 6 booklet
2. Complete the Portfolio task
3. Complete the reflection at the end of the booklet
4. Show your teacher the completed booklet

Practical Components

Work through the exercises and show the completed tasks to your teacher.

Portfolio Task

See the last page of the booklet

Other

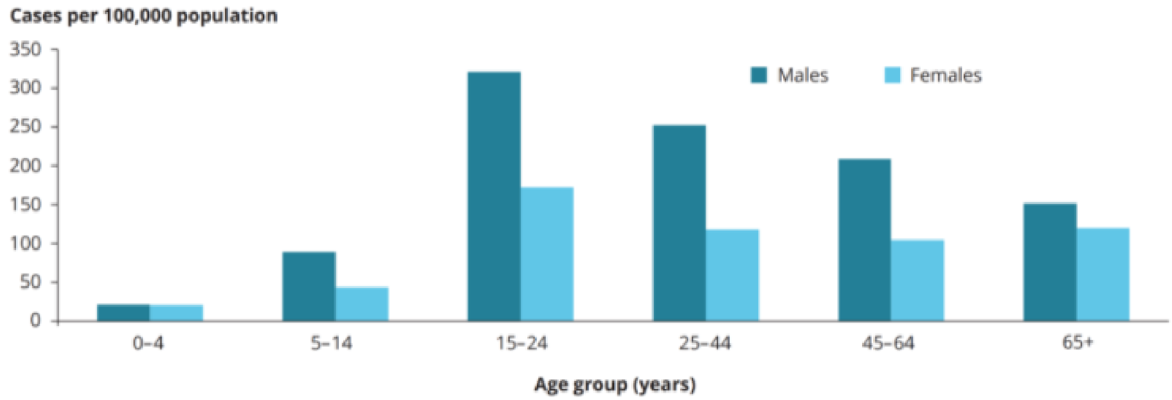
Do you have a calculator for the test?

Car Insurance Premiums

As a college student, you may already be driving, or thinking about buying a car. As such, you may well be aware how expensive car insurance can be for a young person.

Why is car insurance so expensive? Is there a difference between the cost for younger or older drivers? Males versus females?

Insurers know that young people are at a much higher risk than older adults of having a car accident. The graph below shows the rate of hospitalisations per 100,000 people from a report published by Australian Institute of Health and Welfare in 2018.



Age-specific rates for hospitalised injury cases due to on-road crashes by age and sex in 2014–15

In calculating your insurance premium (how much you pay for insurance), insurers want to ensure they are charging you the right price. If it is too expensive, you will end up getting insurance with another company instead. If it is too cheap, then the insurance company will end up losing money because the costs of car crashes will be more than the money they make from car insurance.

When calculating the “right price” for insurance, the insurance companies hire an actuary. Actuaries are part of the mathematical elite, using complex probability calculations to figure out the long-term risk of events like car crashes or natural disasters happening. Having such useful mathematical skills makes actuaries highly sought-after by a range of companies, and the profession been rated as the best job of 2013 based on its high income, good work environment and balanced lifestyle.

We can estimate our risk of a major crash by the following equation:

$$\text{Risk} = \frac{\text{number of crashes per age group}}{\text{number licence holders per age group}}$$

The first table below shows the number of crashes per age group and the second table shows the number of licence holders per age group.

Road user class	Sex	0-4	5-16	17-20	21-25	26-29
Car driver	M	0	97	4,018	4,483	2,940
	F	0	73	2,861	3,339	2,177
	Sub-total[1]	**0**	**170**	**6,881**	**7,827**	**5,122**

Age (years)	Male	Female
≤ 16	27,758	27,347
17 – 20	148,429	150,547
21 – 25	180,813	196,389
26 – 29	150,244	170,735

Exercise 1

1. Use an online insurance calculator (choose an insurer you know or ask your teacher) and calculate how much you would pay for a comprehensive car insurance per year to drive your own/parent's car? State both the premium and the company.

2. What do you think are the factors which make younger drivers so much more likely to have a car crash?

3. Using the table of crashes and licence holders per age group:
    a. What is the risk of a major car crash per year for you?

b. How much premium should you pay per year? Hint: To keep it simple, use $\text{Risk} \times \text{Cost of car}$

c. How does this compare with the figure you got from the actual insurance calculator? Can you think of why the two numbers would be different? What costs does your estimate not include?

Game shows

Many game shows are based on probability, especially game shows where contestants win prizes.

Exercise 2

1. Using the following website: https://zone.msn.com/gameplayer/gameplayerHTML.aspx?game=dealornodeal&instance=default

   Play a game of deal or no deal. Determine the probabilities of the different scenarios in the table below.

DEAL OR NO DEAL		
CHOOSE YOUR BRIEFCASE		
P(winning \$1,000,000) =	P(winning more than \$100,000) =	**Briefcase**
P(winning less than \$1,000) =	P(winning less than \$100) =	**# of unopened**

What offer did you take and how did you feel you went?

2. Create a game or game show that will require some probability. This may include rolling a die or choosing an item from many. Explain what the game is. You should include instructions, number of players and other vital information.

Medical Testing

Doctors use probability regularly in deciding whether it is worth “screening” patients for medical conditions. Screening refers to testing people who do not have any symptoms, just in case they have a disease which could be treated. Whilst screening can be lifesaving if used correctly, it can also go terribly wrong if used in the wrong circumstances.

A key example of medical testing is screening for cancer and a common example is the use of tumour marks to check if someone has cancer.

Imagine that around 1 in 10,000 asymptomatic people have pancreatic cancer, and there is a blood test which is 99% accurate in detecting pancreatic cancer.

Example 2

If a patient does this test and gets a positive result. What do you think the patient’s chance of having cancer is?

Solution

Most people would answer 99%. After all, the test is “99% accurate” right? The answer is only around 1%. This is due to probability.

To see how this works we need to construct a table.

		Disease	
		+	-
		1	9999
Test	+	1	99
	-	0	9900

If 10,000 people who get tested for pancreatic cancer, statistically only 1 person out of these 10,000 will have pancreatic cancer, so there will only be 1 true positive result. However, since the test has a 1% error rate, amongst the 9,999 people who get the test, there will be 99 false positives (people who do not have cancer but get a positive result). Therefore, there will be 1 true positive and 99 false positives. This means that anyone who gets a positive result has a 99% chance that it is a false positive and 1% chance that it is a true positive.

Exercise 3

1. The Pfizer-BioNTech and Oxford-AstraZeneca vaccine is reported to be 95% and 76% effective respectively. What do these percentages for the vaccines mean? You may need to do some research!

2. Imagine that you test positive for COVID-19 and you have already received the Pfizer vaccine, what does this mean?

3. Which vaccine would you prefer to get if they were available to you and why?

4. A COVID test is 96% accurate in detecting COVID-19. If 300 close contacts went to take the test, how many would receive a false positive result?

5. What other tests can produce a false positive result?

Traffic light problems

The important feature of probability questions involving time and physical space is that the sample space is continuous. There is an uncountable infinity of outcomes in the sample space if our focus is on instants of time or on individual points in physical space. This means that probabilities cannot be formed by counting the elements in the sets involved.

Instead, we measure **intervals** of time or space and we let the relative size of the interval compared with the measure of the whole space be the probability. Thus, if a light in a room is turned on for 45 seconds in every 5 minutes, we would say that the probability of a person who enters the room at a random moment encountering the light 'on' is $\frac{45}{5\times 60} = 0.15$.

Probabilities determined in this way behave in the same way as probabilities in a discrete sample space. This is the same if two events have non-overlapping intervals. There are two ways to calculate this:

$$\mathrm{P(A\ or\ B)} = \mathrm{P(A)} + \mathrm{P(B)}$$

$$\mathrm{P(A\ and\ B)} = \mathrm{P(A)} \times \mathrm{P(B)}$$

Example 1

Every 60 seconds, a traffic light remains green for 34 seconds, yellow for 3 seconds and red for 23 seconds.

a. What outcome is more likely to happen?
b. On his way to work, Iram passes through 3 sets of lights. What is the probability that none of the lights are green?
c. What is the probability that Iram passes through green at the first two sets of lights and red at the third set?

Solution

a. $\mathrm{P(green)} = \frac{34}{60}$, $\mathrm{P(yellow)} = \frac{3}{60}$ and $\mathrm{P(red)} = \frac{23}{60}$.
It is more likely we will arrive at the traffic light when it is green.

b. $\mathrm{P(not\ green)} = \mathrm{P(green')} = 1 - \mathrm{P(green)}$ This notation is from Week 1 and 2.
$\mathrm{P(green')} = 1 - \frac{34}{60} = \frac{26}{60}$
$\mathrm{P(green'\ and\ green'\ and\ green')} = \mathrm{P(green')} \times \mathrm{P(green')} \times \mathrm{P(green')}$
$\mathrm{P(green'and\ green'and\ green')} = \frac{26}{60} \times \frac{26}{60} \times \frac{26}{60} = \frac{2197}{27000} = 0.081$
The probability Iram will pass through no green lights is 0.081.

c. $\mathrm{P(green\ and\ green\ and\ red)} = \mathrm{P(green)} \times \mathrm{P(green)} \times \mathrm{P(red)}$
$\mathrm{P(green\ and\ green\ and\ red)} = \frac{34}{60} \times \frac{34}{60} \times \frac{23}{60} = \frac{6647}{54000} = 0.123$
The probability Iram will pass through two green lights first then a red light is 0.123.

Exercise 4

1. The probability that a set of traffic lights shows red, yellow, or green are equally likely. Monique is travelling down a road where there are two sets of traffic lights.
    a. Construct a tree diagram to indicate the possible pairs of traffic lights.

    b. What is the probability that both sets of traffic lights will be yellow?

2. A set of traffic lights is red for half the time, yellow for $\frac{1}{5}$ of the time and green for the rest of the time. Find the probability that the traffic light is green.

3. A set of traffic lights was found to staying green for 119 seconds, yellow for 5 seconds and red for 76 seconds in one direction. Find the probability, as a percentage, that the lights will be:
    a. green?

b. yellow?

c. red?

4. The time spent waiting for a green light at a set of traffic lights, in seconds, was recorded. The results are: 11, 119, 5, 74, 32, 90, 31, 66, 91, 33, 81, 37, 94, 17, 84, 101, 56, 41, 14, 52.
   a. Complete the following relative frequency table:

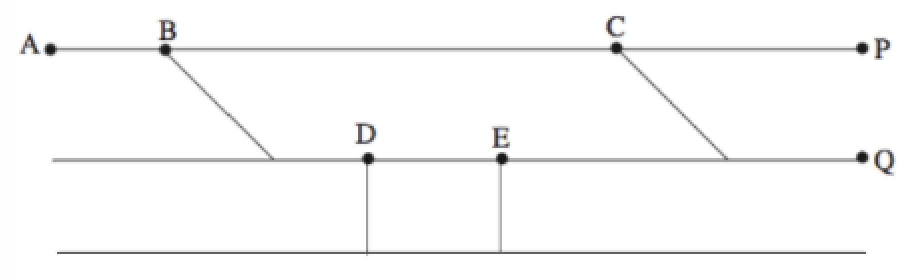
Time	**Tally**	**Frequency**	**Rel. Freq.**
$0 \leq t \leq 10$			
$10 < t \leq 20$			
$20 < t \leq 30$			
$30 < t \leq 40$			
$40 < t \leq 50$			
$50 < t \leq 60$			
$60 < t \leq 70$			
$70 < t \leq 80$			
$80 < t \leq 90$			
$90 < t \leq 100$			
$100 < t \leq 110$			
$110 < t \leq 120$			

b. Calculate the probability of a person waiting being between 20 and 29 seconds, include these times.

c. Calculate the probability of a person waiting less than 30 seconds.

Portfolio Task Week 5 and 6

Joel is at point **A** driving in the direction of point **B**. There are traffic lights at points **B**, **C**, **D** and **E**, and the probability of a green light at any one of these is 0.4. If Joel arrives at a traffic light when it is green, he will go straight. If it is not green, he will turn right. He does not turn or drive back in the direction of **A** at any time.



1. What is the probability that Joel drives to point **P**? Hint: What would be his route to travel to point **P**?

2. What is the probability that Joel drives to point **Q**? Hint: What would be his route(s) to travel to point **Q**?

MARKING RUBRIC

CRITERIA	EXPECTATIONS	POSS	MULT	GIVEN	TOTAL
Practical	Student completes practical work, including exercises of the brief to an acceptable standard set by the teacher.	**2**	**3**		**/6**
Portfolio Task	Student completes the portfolio task of the week to an acceptable standard set by the teacher.	**2**	**2**		**/4**
Reasoning and Communications	Student responses are accurate and appropriate in presentation of mathematical ideas, with clear and logical working out shown.	**4**	**-**		**/4**
Concepts and Techniques	Student submitted work selects and applies appropriate mathematical techniques to solve practical problems and demonstrates proficiency in the use of mathematical facts, techniques and formulae.	**4**	**-**		**/4**
	Submission Guidelines				
Timeliness	Student submits the exercises and portfolio tasks by the set deadline. See scoring guidelines for specific details.	**2**	**-**		**/2**
				FINAL	**/20**

Student Reflection:

How did you go with this week's work? What was interesting? What did you find easy? What do you need to work on?