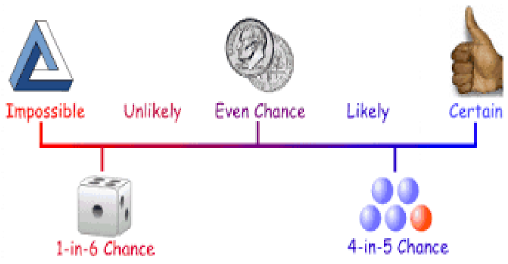
Week 1/2
Term 3
2022

HAWKER COLLEGE
Engage | Inspire | Achieve

Learning Brief
EM4

Goals



This week and next week we are going to:

- interpret commonly used probability statements including *possible*, *probable, likely, certain*
- describe ways of expressing probabilities formally using fractions, decimals, ratios, and percentages
- identify relative frequency as probability
- construct and use a sample space to determine outcomes for an experiment using tree diagrams and arrays

Theoretical Components

Resources:
PDF file: Week 1 and 2 Notes and Exercises

The clip below gives a good introduction to probability.

https://www.youtube.com/watch?v=_r-NpSzpcBY

This clip shows you how to do the calculations for a question on simple probability.

https://www.youtube.com/watch?v=yUaI0JriZtY

Knowledge Checklist
- The language of probability
- Outcome and sample space
- Calculating probabilities

Order
1. Work through the Week 1 and 2 booklet
2. Complete the Portfolio task
3. Complete the reflection at the end of the booklet
4. Show your teacher the completed booklet

Practical Components

Work through the exercises and show the completed tasks to your teacher.

Week 2 starts at the 'Relative Frequency' section.

Portfolio Task

See the last page of the booklet

Other

Make sure you have joined the Google Classroom. If you have not, see your teacher.

ESSENTIAL MATHEMATICS 4

WEEK 1 AND 2 – PROBABILITY

Probability

Probability is all around us:

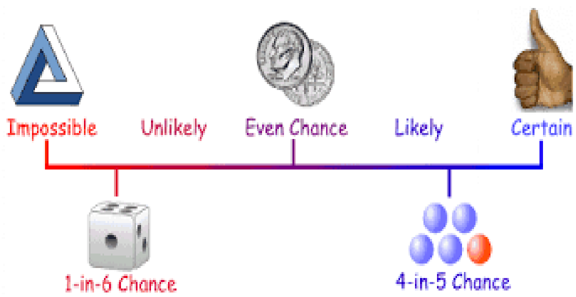
- Weather
- Safety
- Weekly Lotto and Powerball draws
- Poker machines in clubs
- Many games that use dice, coins, and cards

The results of all of these are determined by probability – also called **chance**.

In day-to-day life, there are also many other places where the language of probability is also used.

- The chance of rain today is 75%
- A medication has an effectiveness of 90%
- 4 in 5 people agree that a particular brand of toothpaste is more effective than another
- Most likely the supermarket would have the brand of milk I want to purchase

A probability continuum is one way to visualise the scale of likelihoods.



- **CERTAIN** – when something will definitely occur. We assign a value of 1, or 100% to the probability of certain. For example, if I were to roll a standard dice, it is certain that I will roll a number less than 10.
- **IMPOSSIBLE** – when something cannot occur. We assign a value of 0, or 0% to the probability of impossible. For example, if I were to draw a card from a standard pack of playing cards, it is impossible that I would pull out a card with a 27 on it.
- **EVEN CHANCE** or **EQUALLY LIKELY** – when something is exactly halfway along this continuum. We assign a value of 1/2 or 50%. For example, if I were to toss a fair coin, I could get a head or a tail. Both outcomes have an even chance of occurring.
- **UNLIKELY** or **NOT LIKELY** – when an outcome is between impossible and even chance (values between 0% and 50%)
- **LIKELY** – when an outcome is between even chance and certain (values between 50% and 100%)

Experiment or Trial

An experiment or trial are the words used to describe the even or action of doing something and recording results. For example, the act of drawing cards from a deck, tossing a coin, rolling some dice, watching the weather, asking questions in a survey, or counting cars in a carpark could all be examples of experiments or trials.

Sample Space

The sample space, sometimes called an **event space**, is a listing of all the possible outcomes that could arise from an experiment.

For example:

- tossing a coin would have a sample space of {Head, Tail} or {H, T}
- rolling a die would have a sample space of {1, 2, 3, 4, 5, 6}
- watching the weather could have a sample space of {sunny, cloudy, rainy} or {hot, cold}
- asking questions in a survey of favourite seasons could have a sample space of {Summer, Autumn, Winter, Spring}

Did you also notice how sample space is listed? Using curly brackets { }.

Event

An event is the word used to describe a single result of an experiment. It helps us to identify which of the sample space outcomes we might be interested in.

For example, these are all events:

- getting a tail when a coin is tossed
- rolling more than 3 when a die is rolled
- getting an ACE when a card is pulled from a deck

We use the notation, P(event) to describe the probability of particular events.

Adding up how many times an event occurred during an experiment gives us the **frequency** of that event. The **relative frequency** is the name given to the probability of that event happening.

Example 1

Identify the **experiment**, **sample space** and **event**.
A standard die is rolled 10 times and the results are recorded. James was interested in even numbers.

Solution

Experiment – rolling a standard die
Sample space – {1, 2, 3, 4, 5, 6}, we could get any of the numbers from 1 to 6
Event – P(even number), the probability of getting an even number.

Exercise 1

Write the answers to the probability questions as fractions unless you are told otherwise.

1. Which of these experiments have equally likely outcomes? (Choose 1)
   a. Spinning a tennis racquet to determine who serves
   b. Selecting the winner of a raffle
   c. Choosing a job applicant for an employment position from six people who have been interviewed

2. Describe the chance of each of these events happening, using the terms ***certain***, ***likely***, ***even chance***, ***unlikely*** and ***impossible***.
   a. It will snow here sometime during the next week
   b. We will have rain tomorrow
   c. There will be a holiday on 25th December
   d. I can walk from school to the Goulburn McDonald's without resting
   e. I will travel overseas before the end of the year
   f. I will live to the age of 90
   g. I will sit a Mathematics test during the next three months

3. Order these events in Question 2 from least likely to most likely.

4. Describe **two** events that have a probability of:
   a. Even chance

   b. Unlikely

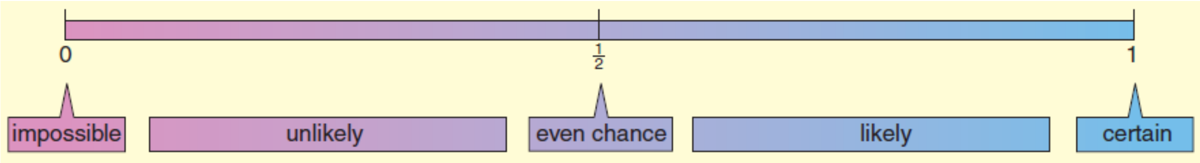
   c. Likely

5. Which probability term best describes an event that has a probability of:
    a. 0.5
    b. 2%
    c. $\frac{17}{20}$
    d. 0
    e. 1
    f. 50%

Theoretical Probability

The probability of an event is its chance of happening. This will be a number from 0 to 1.

0 ≤ probability of an event ≤ 1, 0 ≤ P(E) ≤ 1



The probability may be expressed as a fraction, a decimal or a percentage.

The **theoretical probability** of an event, E, is the number of times the event can occur, divided by the total number of possible outcomes (as long as each outcome is equally likely to occur).

$$P(E) = \frac{n(E)}{n(S)}$$

S is used to represent the sample space, which is the set of possible outcomes.

The sum of the probabilities of **all possible outcomes is always 1**. Two events are complementary if the sum of their probabilities is 1.

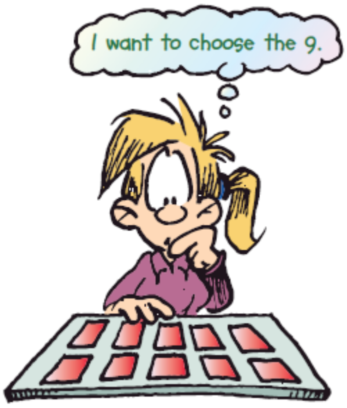
If P(E) is the probability that an event E will occur and P(E') is the probability that event E will not occur, then:

$$P(E') = 1 - P(E)$$

Example 2

A card is chosen at random from a group of ten cards numbered 0 to 9.

a. What is the probability of choosing the 9?
b. What is the probability of not choosing the 9?



Solution

The cards are: 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9.

a. The probability of choosing the 9 is:

$$\boldsymbol{P(E) = \frac{n(E)}{n(S)}}$$

$$P(9) = \frac{1}{10} = 0.1 = 10\%$$

b. $\boldsymbol{P(E') = 1 - P(E)}$

$$P(9') = 1 - P(9)$$

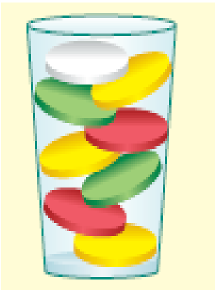
$$P(9') = 1 - \frac{1}{10}$$

$$P(9') = \frac{9}{10} = 0.9 = 90\%$$

Exercise 2

1. A glass contains eight counters. One is white, three are yellow, two are green and two are red. A counter is drawn at random. What is the probability that the counter is:

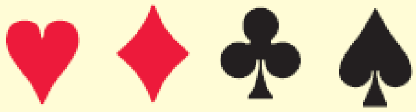
   a. white?
   b. red?
   c. white or yellow?
   d. not white?
   e. not red?
   f. black or blue?



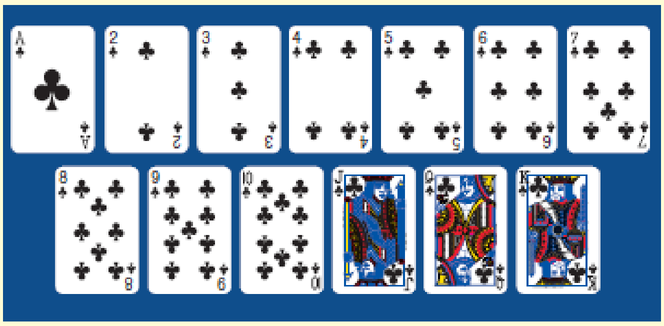
2. A die is thrown once. What is the probability of throwing:
   a. a 2? b. a 6? c. anything but a 6?
   d. 2 or less? e. a 0? f. a number less than 10?

3. From a standard pack of cards, a card is chosen at random. What is the probability that the card is:
   a. black? b. not black? c. yellow?
   d. a 5? e. not a 5? f. a court card?
   g. a club? h. not a club? i. an Ace or a King?

A standard pack of cards has four suits: hearts, diamonds, clubs and spades. Hearts and diamonds are red. Clubs and spades are black.



In each suit there are thirteen cards: Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen and King. All the clubs are shown below. The Jack, Queen and King are called *court cards*.

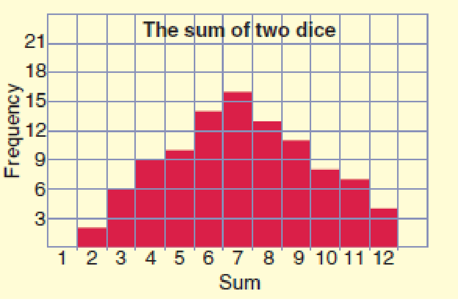


Since there are 4 suits with 13 cards in each suit, the number of cards in a standard pack is 52. In some games, a Joker is also used.

4. A factory tested a sample of 500 DVDs and found 5 to be faulty. Use these results to estimate the probability that a DVD produced by this factory would be:
    a. faulty?

    b. not faulty?

5. Two dice were thrown 100 times, and each time the sum of the upper faces was recorded. The results are shown on this graph. Use these results to find the experimental probability of throwing a total:



    a. of 7?
    b. of 6?

    c. greater than 7?
    d. other than 6?

    e. of 6 or 7?
    f. that is even?

6. High school students were asked to vote on the predominant colour to be used on their school uniforms for the sports carnival. The results are shown in the table on the right. If one of these students are chosen at random, what is the probability (as a percentage) that the person:

	Male	*Female*
Green	38	40
Gold	44	32
Blue	7	8
Orange	2	14
Fawn	7	3
Other	2	3
Totals	100	100

Number of students = 200

    a. is female?

    b. chose gold?

    c. is a male who chose orange?

    d. is a female who chose green?

    e. is not a female who chose green? (i.e. a male who chose green)

    f. choose either green or gold?

    g. chose neither green nor gold?

7. Into a barrel are placed 100 blue tickets numbered 1 to 100, 50 red tickets numbered 1 to 50, and 50 green tickets numbered 1 to 50. If one ticket is drawn at random from the barrel, what is the probability that the ticket:

a. is green?

b. is green or red?

c. is a 36?

d. is a 72?

e. is 50 or less?

f. is 60 or less?

g. is not a 50?

h. is not 60 or less?

i. is either a 36 or a 72?

8. Three students, A, B and C, are placed in a line in random order. The possible orders are ABC, ACB, BAC, BCA, CAB, CBA. Use this list to find the probability that:

a. A is placed before B in the order?

b. A is placed before both B and C?

c. A is placed between B and C?

d. A is placed before either B or C?

e. A is not placed before either B or C?

9. Answer the following questions in either fractions, decimals or percentages.
   a. The probability that two cards are drawn from a standard pack of cards will both be hearts is $\frac{1}{17}$. What is the probability that the two cards drawn are *not* both hearts?

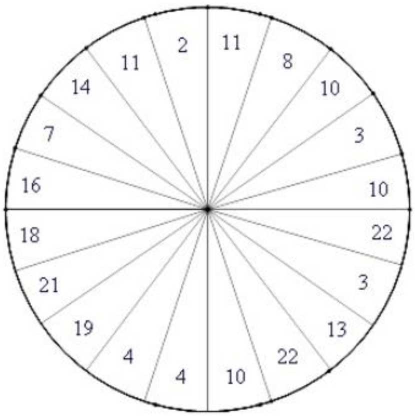
   b. The probability of throwing a sum of either 5 or 6 using two dice is 0.25. What is the probability that the sum is *neither* 5 *nor* 6?

   c. There is a 37% chance that I will be able to travel overseas next year. What is the chance that I will not be travelling overseas?

   d. If 3 coins are tossed, the probability of getting 3 heads is 0.125. What is the probability of getting fewer than 3 heads?

   e. The probability of my dying this year is 0.5%. What is the probability that I will not die this year?

10. A spinner is placed on the board below and spun to land on one of the numbers. Find each of the following probabilities, where P(A) is the probability that the spinner lands on A.



a. P(multiple of 2)?

b. P(multiple of 5)?

c. P(multiple of 13)?

d. P(odd)?

11. To investigate the cause of road crashes in which a fatality occurred, Ben studied a number of crashes chosen at random from those that occurred during the years 2019 and 2020. The details of fatal crashes occurring in Australia during that time are shown below. Give your answers correct to the nearest tenth of a percent (1 decimal place).

Fatalities by state/territory and road user, 2019 and 2020									
	ACT	NSW	NT	QLD	SA	TAS	VIC	WA	Australia
2020	7	297	31	106	93	36	109	67	746
2019	6	352	35	92	114	29	144	90	862
Percentage change 2019-2020	16.7	-15.6	-11.4	15.2	-18.4	24.1	-24.3	-25.6	-13.5

a. If Ben chose a fatal crash at random **from 2020**, give the probability (as a percentage) that the crash was:

i. from NSW? ii. from the ACT?

iii. from Queensland?

iv. not from Tasmania?

v. from neither NSW nor the ACT?

b. If Ben chose a fatal crash at random **from 2019**, give the probability (as a percentage) that the crash was:

i. from NSW?

ii. from the ACT?

iii. from Queensland?

iv. not from Tasmania?

v. from neither NSW nor the ACT?

c. Even though the percentage change in fatal crashes in the ACT is 16.7%, the probability of choosing a crash from the ACT had changed only 0.2% Explain why the change in probability is so small.

d. From the data collected, what could be a possible reason for a decrease in fatalities in 2020?

Relative Frequency

Sometimes it is useful to know not just how many times a score occurred, but what fraction of the total this represents.

For instance, of the eight scores: 6, 3, 6, 9, 4, 6, 3, 6
Four scores are sixes, as a fraction, this is $\frac{1}{2}$ of the total number of scores.
Likewise, two scores are threes, this is $\frac{1}{4}$ of the total.

This fraction is called the **relative frequency** of the score.

To obtain the relative frequency, we simply divide the frequency of any particular score by the total number of scores. Often this fraction is expressed as a decimal.

$$\textbf{Relative frequency} = \frac{\textbf{frequency of score}}{\textbf{total frequency}}$$

The total of the relative frequency of all scores should be 1.

Example 3

1. Determine the relative frequency of the score 7 in the following.
   a. 7, 3, 8, 7, 6

   b. 1, 3, 7, 5, 7, 7, 5

2. Calculate the relative frequency of each score in the tables.

a.

Score	Frequency
16	3
17	5
18	7
19	4
20	1
Total	20

b.

Score	Frequency
10	9
20	13
30	19
40	11
50	8
Total	60

Solution

1.
   a. 7 occurs two times out of five scores, so the relative frequency is $\frac{2}{5}$ or 0.4

   b. 7 occurs three times out of seven, so the relative frequency is $\frac{3}{7}$ or 0.429

2. For frequency distribution tables, a column titled ‘relative frequency’ may be added as shown below. Remember the frequency of each score has been divided by the total frequency. The answers have been given correct to three decimal places for table b. **Don’t forget that the total relative frequency should be 1.**

a.

Score	Frequency	Rel. Freq.
16	3	0.15
17	5	0.25
18	7	0.35
19	4	0.20
20	1	0.05
Total	20	1

b.

Score	Frequency	Rel. Freq.
10	9	0.150
20	13	0.217
30	19	0.317
40	11	0.183
50	8	0.133
Total	60	1

Experimental Probability

When carrying out an experiment to find the **experimental probability** of an event occurring, we are actually finding the **relative frequency** of that event.

Example 4 and Solution

Two dice were thrown 80 times. In each case, the numbers showing were added, and this result was recorded in the table below. Find the experimental probability of each of the possible results.

Result	Tally	Freq.	Experimental Probability
2	||	2	$\frac{2}{80}$ or 0.025 or 2.5%
3	𝍸	5	$\frac{5}{80}$ or 0.0625 or 6.25%
4	𝍸 |	6	$\frac{6}{80}$ or 0.075 or 7.5%
5	𝍸 |||	8	$\frac{8}{80}$ or 0.1 or 10%
6	𝍸 𝍸 |	11	$\frac{11}{80}$ or 0.1375 or 13.75%
7	𝍸 𝍸 ||||	14	$\frac{14}{80}$ or 0.175 or 17.5%
8	𝍸 𝍸 ||	12	$\frac{12}{80}$ or 0.15 or 15%
9	𝍸 ||||	9	$\frac{9}{80}$ or 0.1125 or 11.25%
10	𝍸 |	6	$\frac{6}{80}$ or 0.075 or 7.5%
11	||||	4	$\frac{4}{80}$ or 0.05 or 5%
12	|||	3	$\frac{3}{80}$ or 0.0375 or 3.75%
Total		80	$\frac{80}{80}$ or 1.000 or 100%

Exercise 3

1. Express as a fraction the relative frequency of the score **5** in the following:
   a. 6, 5, 1, 7, 5
   b. 0, 5, 6, 3, 4, 5, 1, 9
   c. 5, 3, 5, 5, 6, 8
   d. 5, 4, 6, 5, 7, 5, 4
   e. 2, 5, 9, 8, 6, 1
   f. 5, 4, 5, 5, 6, 9, 5, 5
   g. 3, 5, 8, 8, 4, 5
   h. 10, 6, 5, 5, 4, 2, 5, 3, 5, 8

2. Write the answers to question 1 as a decimal correct to two decimal places.
   a.
   b.
   c.
   d.
   e.
   f.
   g.
   h.

3. Complete the relative frequency column for each table below. Give answers correct to three decimal places.

a.

Score	Freq.	Rel. Freq.
1	1	
2	2	
3	3	
4	2	

Total =

b.

Score	Freq.	Rel. Freq.
6	2	
7	4	
8	7	
9	5	
10	2	

Total =

c.

Score	Freq.	Rel. Freq.
28	2	
29	5	
30	6	
31	7	
32	4	

Total =

d.

Score	Freq.	Rel. Freq.
6	2	
8	7	
10	3	
12	8	
14	10	
16	3	
18	6	
20	1	

Total =

e.

Score	Freq.	Rel. Freq.
10	4	
20	9	
30	11	
40	11	
50	15	
60	7	
70	3	
80	1	

Total =

f.

Score	Freq.	Rel. Freq.
1	14	
2	18	
3	25	
4	31	
5	29	
6	40	
7	33	
8	21	
9	11	
10	8	

Total =

4. Three coins were tossed 50 times and the number of heads was recorded each time. The results are: 1, 1, 2, 1, 2, 2, 3, 1, 1, 1, 2, 1, 2, 2, 0, 1, 2, 2, 2, 0, 3, 3, 1, 1, 2, 0, 3, 2, 3, 2, 0, 2, 2, 1, 3, 1, 2, 1, 1, 0, 2, 1, 1, 1, 2, 0, 3, 2, 2, 1.

   a. Use this data to complete an experimental probability table. Write the experimental probability as a fraction and as a percentage.

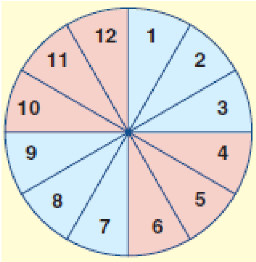
Number of heads	Tally	Freq.	Experimental Probability

b. Draw a column graph to show the experimental probability of getting 0, 1, 2, and 3 heads.

c. According to these results, what would be the most likely outcome if I were to toss three coins?

d. According to these results, what would be the least likely outcome if I were to toss three coins?

5. Gemma threw 100 darts at the target on the right and recorded the score for each dart in the table below. Give your answers in decimals.



Score	1	2	3	4	5	6	7	8	9	10	11	12
Freq.	7	9	11	11	12	12	10	8	5	4	5	6

a. What is the experiment probability of Gemma hitting a:
   i. 6? ii. 1? iii. 10?

b. What is the experimental probability of Gemma hitting the target:
   i. on the right side?

   ii. on the left side?

c. What is the experimental probability of Gemma hitting the target:
   i. on the upper half?

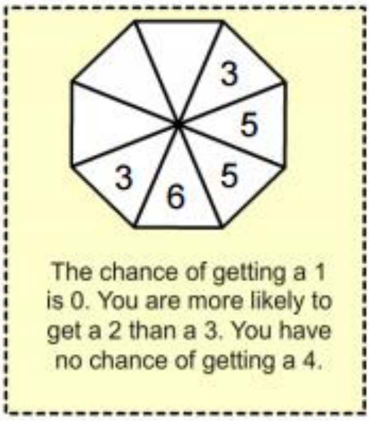
   ii. on the lower half?

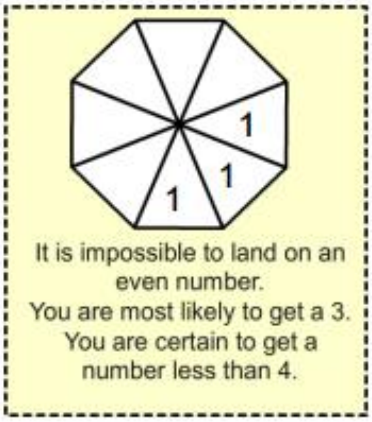
d. Using the results above, describe how Gemma tends to throw her darts.

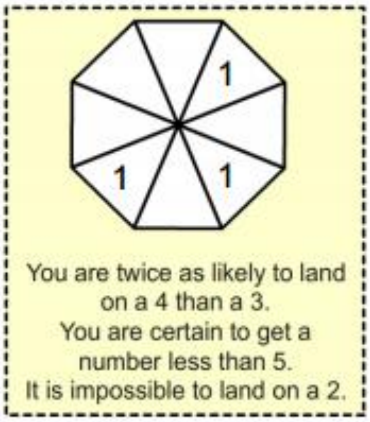
Portfolio Task Week 1 and 2

1. Design a spinner!

   Using only the numbers 1 to 6, complete these spinners so that they match the probability statements that describe them.







2. Design a four coloured spinner that would give one colour a chance of occurring that is twice that of any of the other colours.

MARKING RUBRIC

CRITERIA	EXPECTATIONS	POSS	MULT	GIVEN	TOTAL
Practical	Student completes practical work, including exercises of the brief to an acceptable standard set by the teacher.	**2**	**3**		**/6**
Portfolio Task	Student completes the portfolio task of the week to an acceptable standard set by the teacher.	**2**	**2**		**/4**
Reasoning and Communications	Student responses are accurate and appropriate in presentation of mathematical ideas, with clear and logical working out shown.	**4**	**-**		**/4**
Concepts and Techniques	Student submitted work selects and applies appropriate mathematical techniques to solve practical problems and demonstrates proficiency in the use of mathematical facts, techniques and formulae.	**4**	**-**		**/4**
	Submission Guidelines				
Timeliness	Student submits the exercises and portfolio tasks by the set deadline. See scoring guidelines for specific details.	**2**	**-**		**/2**
				FINAL	**/20**

Student Reflection:

How did you go with this week's work? What was interesting? What did you find easy? What do you need to work on?