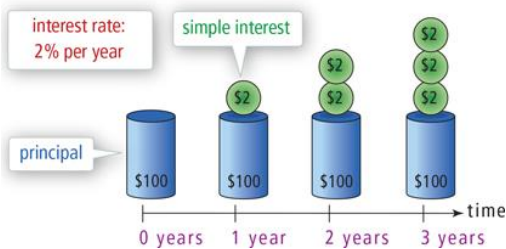


Goals

This week and next week we are going to:

- review the principles of simple interest
- understand the concept of compound interest as a recurrence relation
- consider similar problems involving compounding, for example, population growth
- use technology to calculate the future value of a compound interest loan or investment and the total interest paid or earned
- use technology to compare, numerically and graphically, the growth of simple interest and compound interest loans and investments



Theoretical Components

Resources:

PDF file: Week 12 and 13 Interest

Watch this video for a basic introduction to interest:

<https://www.youtube.com/watch?v=GtaoP0skPWc>

Knowledge Checklist

- Simple interest
- Using a simple interest calculator
- Graphing simple interest equations
- Compound interest
- Using a compound interest calculator
- Calculating time in compound interest
- Graphing compound interest equations

Order

1. Work through the Week 12 and 13 booklet
2. Complete the Portfolio task
3. Complete the reflection at the end of the booklet
4. Show your teacher the completed booklet

Practical Components

Work through the exercises and show the completed tasks to your teacher.

Week 13 starts at the 'Compound Interest' section.

Portfolio Task

See the last page of the booklet

Other

Financial Arithmetic

At various stages of our lives, we will be faced with making important and informed decisions regarding our finances. We may find ourselves asking:

- If I wish to purchase a car, should I borrow from the bank or accept a payment plan offered by the car dealer?
- If I wish to invest \$5,000, how much interest will I earn annually?
- If I wish to borrow a sum of \$100,000 from a bank or finance company, how much money will I need to repay in total?

To answer these questions, we need to have a basic understanding of 'money matters'. This fortnight will look at some of these financial issues, including the fundamental concept of simple and compound interest, appreciation and depreciation.

Simple Interest

When you lend money for a certain period of time (a term deposit) to a bank, building society, or other financial institution, you expect to be rewarded by eventually getting your money back, plus an extra amount commonly known as **interest (I)**.

Similarly, if you borrow money from any institution by taking out a loan or mortgage, you must pay back the original sum plus interest.

The following examples deal with **simple interest**, that is, interest which is paid only on the original sum of money invested or borrowed.

The formula used to calculate simple interest is given by:

$$I = \frac{PRT}{100}$$

where:

I = interest, \$

P = principle, \$ (the sum of money borrowed or invested)

R = rate of interest p. a., % (interest rate per annum, per year)

T = term of interest, years (the period of time for which the sum of money is to be borrowed or invested)

The sum of the principle, P , and the interest, I , is called the **total amount** and is denoted by the symbol **A**. The formula used to calculate the total amount is given by:

$$A = P + I$$

where:

A = total amount at the end of the term, \$

P = principle, \$

I = interest, \$

Example 1

Calculate the amount of simple interest, I , earned and the total amount, A , at the end of the term if:

- \$12,000 is invested for 5 years at 9.5% p.a.
- \$2,500 is invested for 3 months at 4.5% p.a.

Solution

- a. $P = \$12000, R = 9.5\%, T = 5$ years

$$I = \frac{PRT}{100}$$
$$I = \frac{12000 \times 9.5 \times 5}{100}$$

$$I = 5700$$

The amount of interest earned is \$5,700.

$$A = P + I$$

$$A = 12000 + 5700$$

$$A = 17700$$

The total amount at the end of the term is \$17,700.

- b. $P = \$2500, R = 4.5\%, T = 3$ months = $\frac{3}{12}$ years

$$I = \frac{PRT}{100}$$
$$I = \frac{2500 \times 4.5 \times \frac{3}{12}}{100}$$

$$I = 28.13$$

The amount of interest earned is \$28.13.

$$A = P + I$$

$$A = 2500 + 28.13$$

$$A = 2528.13$$

The total amount at the end of the term is \$2,258.13.

Exercise 1

1. Calculate the simple interest paid per year on the following investments:
 - a. \$3,690 at 11% p.a.
 - b. \$22,400 at 6.85% p.a.

 - c. \$620 at 14% p.a.
 - d. \$16,000 at 4.8% p.a.

2. For each of the following calculate the amount of simple interest, I , earned and the total amount, A , at the end of the term.
 - a. \$1,200 for 1 year at 10.5% p.a.
 - b. \$8,320 for 3 years at 6.45% p.a.

 - c. \$960 for 2 years at 9.2% p.a.
 - d. \$126,000 for 6 months at 8.3% p.a.

Simple Interest Calculator

The link below takes you to a Simple Interest Calculator. There are four variables associated with simple interest. The principle, P , the rate, r , the time, T , and interest earned, I . This calculator allows you to put any of the three variables in and it will calculate the fourth.

<https://www.easycalculation.com/simple-interest.php>

Example 2

How long will it take to earn \$500 (the interest) if \$5,000 is invested (the principle) at 3% p.a. (the rate)?

Solution

On the online calculator, set the input *I want to calculate* to **Time Period (T)**. Then enter **Principle or Sum (P)** as \$5,000, set **Rate per Annum (R)** as 3, set **Simple Interest (SI)** as \$500.

Note that **Time (T)** is highlighted in red – as this is what we want to calculate. Click on *Calculate* and the result is given as 3 years.

Exercise 2

Use the online calculator to help solve the following.

1. How long, to the nearest month, will it take to earn \$1,950 simple interest if \$16,325 is invested at 9.75% p.a.? Note: Make sure Time (T) is in months.

2. What sum, to the nearest dollar, must be invested for one year at 6% per annum simple interest, to earn \$1,200 interest?

3. Find the total value of an investment of \$3,500 after 2 years and 6 months if simple interest is paid at the rate of 5% per annum. Remember: Use $A = P + I$

4. How long must a principle of \$15,750 be invested at 9.8% p.a. for it to earn \$1,087 in simple interest?

5. What principle would earn \$3,729.60 in simple interest if it is invested for 3 years at 16.8% p.a.?

6. An amount of \$9,020 was invested for 2 years and earned \$1,731.84 in simple interest. Calculate the monthly interest rate.

7. How long will it take for a principle of \$3,760 to earn \$240 in simple interest at 2% p.a.?

Graphing Simple Interest

Example 3

Abdullah invests \$2,000 in an account that earns 6% p.a. simple interest. Construct a graph that shows the simple interest, I , earned in dollars over n years, for values of n from 0 to 8.

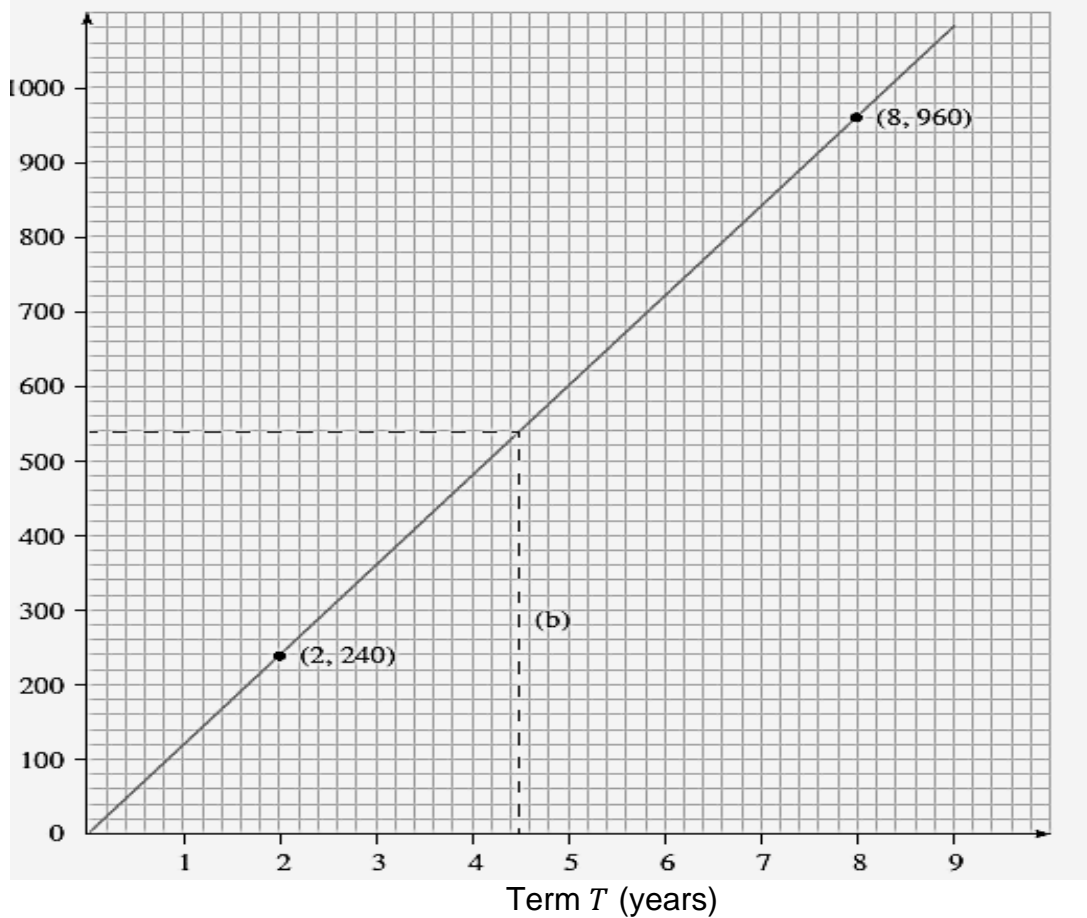
Solution

We use the formula $I = \frac{PRT}{100}$ and substitute $P = 2000$, $R = 6$, which gives $I = \frac{2000 \times 6 \times T}{100}$, which simplifies to $I = 120T$. This is a simple equation, and we can construct a table to help graph it.

T (years)	0	2	8
I (\$)	0	240	960

We can use this table to produce a graph.

Simple interest earned from an investment of \$2000 at 6% p.a.



The dotted line shows that after 4.5 years, the interest would be \$640.

Exercise 3

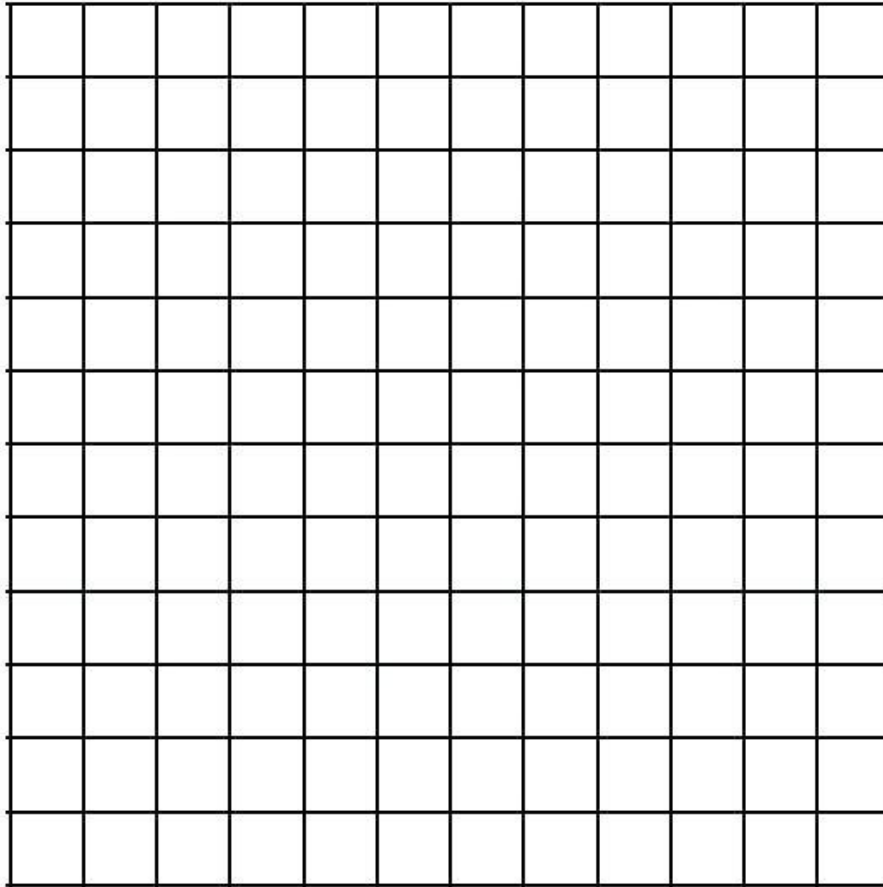
Use the simple interest graph above to answer Question 1 and 2.

1. Estimate the interest earned when \$2,000 is invested at 6% p.a. for:
 - a. 9 years
 - b. 2.5 years

2. What would the total value of the investments be after:
 - a. 3 years
 - b. 6 years

3. Graph on the same axes the simple interest, I , earned when \$4,000 is invested in an account earning:
- 4% p.a.
 - 12% p.a.
- over t years, for values of n from 0 to 10. Fill out the table below for each. Use the grid provided to draw the graph.

	0	1	2	3	4	5	6	7	8	9	10
4%											
12%											



Compound Interest

In reality, calculating interest is not so 'simple' and straightforward. Simple interest is used only when the interest earned is collected by the investor and not added to the investment, such as in a term deposit account. With most accounts, however, the balance plus the interest becomes the new balance on which the interest is calculated next time. In other words, the interest will increase because you also earn 'interest on your interest'. This is called **compound interest**. Compound means 'to combine'.

The effect of compounding (which oil billionaire, J.P. Getty called the 'eight wonder in the world' and theoretical physicist Albert Einstein described as the 'driving force of the Universe') is a secret of financial wealth creation.

Example 4

As mentioned previously, when we are dealing with simple interest, the interest is the same for each time period. The difference compounding makes can be seen in this example.

Consider an amount of \$1,000, to be invested for a period of 5 years at an interest rate of 10% p.a. Calculate the interest earned using:

- simple interest
- compound interest

Solution

- a. Simple interest:

$$P = \$1000, R = 10\%, T = 5 \text{ years}$$

$$I = \frac{PRT}{100}$$

$$I = \frac{1000 \times 10 \times 5}{100}$$

$$I = 500$$

The amount of interest earned is \$500.

- b. Compound interest

$$P = \$1000, r = 10\%, T = 5 \text{ years}$$

$$A = P \left(1 + \frac{r}{100}\right)^n$$

$$A = 1000 \times 1.1^5$$

$$A = 1610.51$$

$$I = A - P$$

$$I = 1610.51 - 1000$$

$$I = 610.51$$

The amount of interest earned is \$610.51.

The formula used to calculate compound interest is given by:

$$A = P \left(1 + \frac{r}{100} \right)^n$$

where:

A = the amount at the end of n compounding periods, \$

P = principle, \$ (the sum of money borrowed or invested)

r = rate of interest per period, % (interest rate per period)

n = number of compounding periods (the number of periods for which the sum of money is to be borrowed or invested)

Note: the formula for compounding interest, r is the rate of interest *per period*, not *per annum* and n is the number of *compounding periods*, not *years*. It reflects the fact that compounding occurs not only on an annual basis but can be more frequent. That is, semi-annually (half-yearly/every six months), quarterly (every three months), monthly, weekly or daily.

Example 5

Hayden has \$15,000 to invest for 3 years. He considers the following options:

- a term deposit at 5.25% p.a. compounded annually
- shares, paying a dividend rate of 5.08% p.a. compounded quarterly
- a building society, paying a return of 5.4% p.a. compounded monthly
- a business venture with guaranteed return of 7.3% p.a. compounded daily

All the investments are equally secure. Advise Hayden which option to take.

Solution

- a. a term deposit at 5.25% p.a. compounded annually

$$P = \$15,000, r = 5.25\%, T = 3 \text{ years}$$

$$A = P \left(1 + \frac{r}{100} \right)^n$$

$$A = 15000 \left(1 + \frac{5.25}{100} \right)^3$$

$$A = 17488.7018$$

The compounded amount is \$17,488.70.

- b. shares, paying a dividend rate of 5.08% p.a. compounded quarterly

$$P = \$15,000, r = \frac{5.08}{4} = 1.27\%, T = 3 \text{ years} \times 4 = 12$$

$$A = P \left(1 + \frac{r}{100} \right)^n$$

$$A = 15000 \left(1 + \frac{1.27}{100} \right)^{12}$$

$$A = 17452.63391$$

The compounded amount is \$17,452.63.

- c. a building society, paying a return of 5.4% p.a. compounded monthly

$$P = \$15,000, r = \frac{5.4}{12} = 0.45\%, T = 3 \text{ years} \times 12 = 36$$

$$A = P \left(1 + \frac{r}{100}\right)^n$$

$$A = 15000 \left(1 + \frac{0.45}{100}\right)^{36}$$

$$A = 17631.49499$$

The compounded amount is \$17,631.49.

- d. a business venture with guaranteed return of 7.3% p.a. compounded daily

$$P = \$15,000, r = \frac{7.3}{365} = 0.02\%, T = 3 \text{ years} \times 365 = 1095$$

$$A = P \left(1 + \frac{r}{100}\right)^n$$

$$A = 15000 \left(1 + \frac{0.02}{100}\right)^{1095}$$

$$A = 18672.06028$$

The compounded amount is \$18,672.06.

The best option for Hayden is option d, as he receives the greatest amount of interest in the same period of time.

Exercise 4

1. Rebecca wishes to invest \$10,000. The following investment alternatives are suggested to her. All investments are for 7 years. Calculate the following:

a. Simple interest at 9% p.a.

b. Compound interest at 8% p.a. compounded annually.

c. Compound interest at 7.5% p.a. adjusted quarterly. **Note:** Remember to adjust the interest rate and time period.

d. Compound interest at 7.5% p.a. adjusted daily. **Note:** Remember to adjust the interest rate and time period.

e. Which investment alternative will produce the greatest return on her money?

2. India has \$25,000 to invest for 5 years. She considers the following options:

a. A term deposit at 6.75% p.a. compounded annually.

b. Shares, paying a dividend rate of 5.15% p.a. compounded quarterly. **Note:** Remember to adjust the interest rate and time period.

c. A building society, paying a return of 5.3% p.a. compounding monthly. **Note:** Remember to adjust the interest rate and time period.

- d. A business venture with guaranteed return of 6.4% p.a. compounded daily.
Note: Assume there is only one leap year in the given 5-year period and remember to adjust the interest rate and time period.
- e. All investments are equally secure. Advise India which option to take and why.
3. Over the last 3 years a comprehensive hospital cover from 'TakeCare' private medical insurance rose at an average of 9.5% and currently costs \$1,980 per year. If this rate of increase continued, what would be the insurance premium after another 3 years? **Note:** The increase in premium compounds each year.

Appreciation

Items which represent scarce or valuable resources such as land, collectables, paintings and antiques normally increase or **appreciate** in value over time. They become more valuable as time passes because they become rarer and scarcer. This is called **appreciation**. Some people like to invest their money by buying and selling such items.

4. A painting that Charlie bought for \$240 from an art exhibition appreciates on average by 14% p.a. What would the value of the painting be after 25 years?

Compound Interest Calculator

The link below is for a compound interest calculator. The calculator allows you to calculate A , P , r , or t .

<https://www.calculatorsoup.com/calculators/financial/compound-interest-calculator.php>

Exercise 5

1. An original painting is currently valued at \$4,500 and is known to appreciate an average of 8% per year. After how many years would the painting have a value of at least \$25,000?

In this question $P = \$4,500$, $r = 8\%$, $A = \$25,000$ and we want to find t . Set 'Calculate' to Time (t).

2. According to the last census, Whitehorse Marsh has a population of 23,600. The population increases at 6% p.a. How long will it take for the population of Whitehorse Marsh to reach 40,000?
3. What is the amount, rounded to the nearest \$100, to be invested for 6 years and compounded semi-annually at 8% p.a. for the value at the end of the investment to be \$15,000? Set the 'Compound 'n' to Semi-annually.
4. Seth bought an apartment for \$175,000 in an area where apartment houses appreciate an average of 3% per year. He decides to hold the apartment until its value is at least \$250,000. How many years should he wait until he sells the apartment?

Graphing Compound Interest

Example 4 continued

Using the data from Example 4 in Compound Interest.

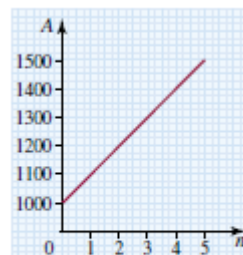
Draw two graphs to represent an amount of \$1,000, to be invested for a period of 5 years at an interest rate of 10% p.a. for:

- simple interest
- compound interest

Solution

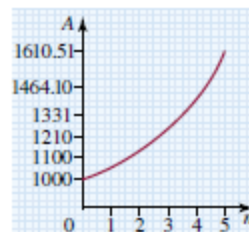
- Simple interest investments are represented by a straight line.

n	A
0	1000
1	1100
2	1200
3	1300
4	1400
5	1500



- Compound interest investments are represented by an exponential graph.

n	A
0	1000
1	1100
2	1210
3	1331
4	1464.10
5	1610.51



The graph of any simple interest scenario is always a straight line (linear), while the graph of any compound interest scenario is always represented by an exponential curve.

The total amount, A , in compounding interest always grows at a much faster rate than in simple interest.

PART B:

In this part of the investigation, you will be required to draw three graphs, all on the same axis. Each graph will represent compound interest with different compounding periods.

The graphs will show an investment of \$5,000 invested over 3 years at 6% p.a., compounding:

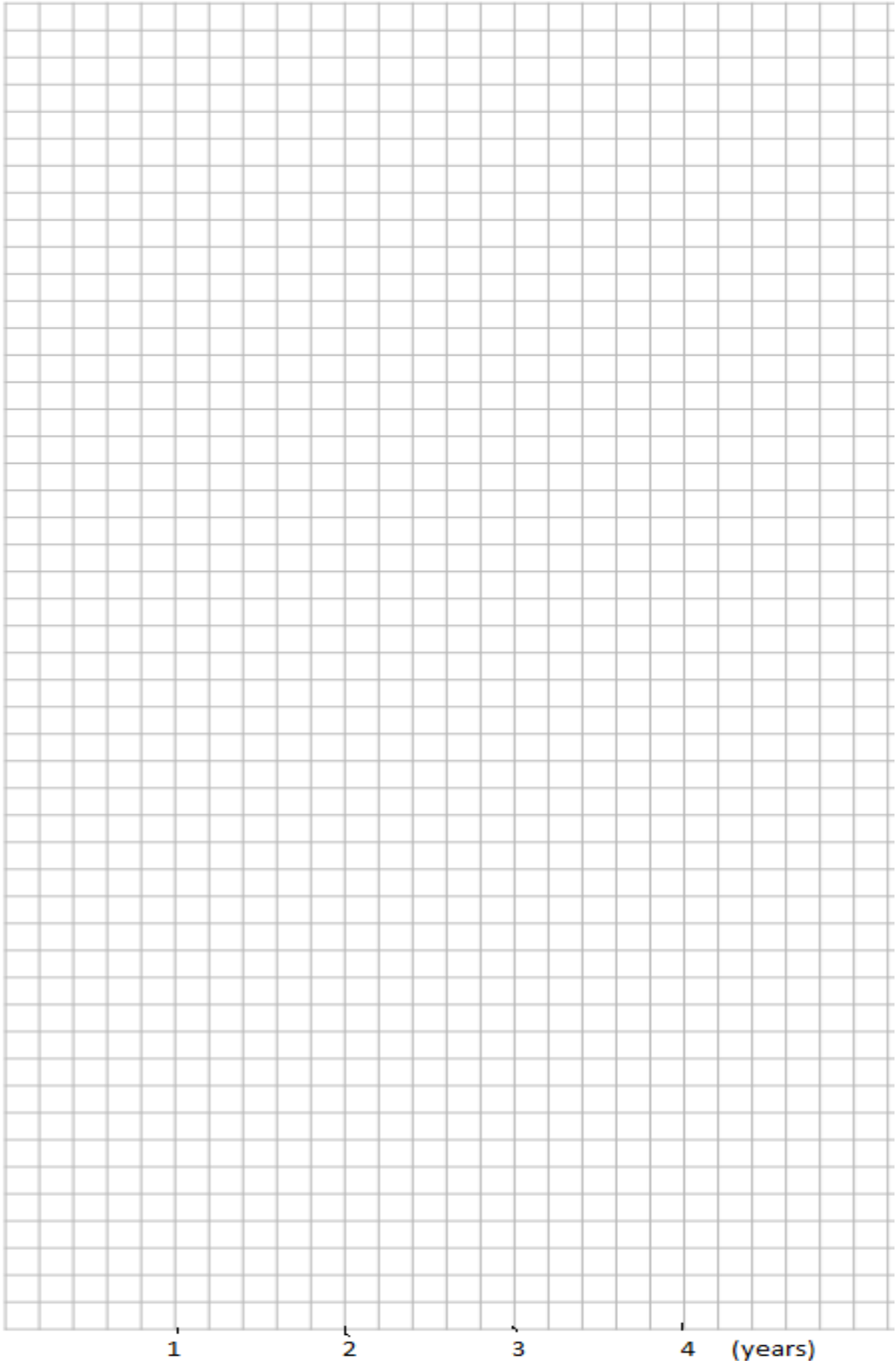
- a. yearly
- b. half-yearly (semi-annually)
- c. monthly

Fill in the tables below and use the data to draw the graphs. Use the **compound interest calculator** to determine the figures.

Intervals (years)	Amount (Principle + Interest)
1	
2	
3	

Intervals (half-years)	Amount (Principle + Interest)
2	
4	
6	

Intervals (months)	Amount (Principle + Interest)
12	
24	
36	



MARKING RUBRIC

CRITERIA	EXPECTATIONS	POSS	MULT	GIVEN	TOTAL
Practical	Student completes practical work, including exercises of the brief to an acceptable standard set by the teacher.	2	3		/6
Portfolio Task	Student completes the portfolio task of the week to an acceptable standard set by the teacher.	2	2		/4
Reasoning and Communications	Student responses are accurate and appropriate in presentation of mathematical ideas, with clear and logical working out shown.	4	-		/4
Concepts and Techniques	Student submitted work selects and applies appropriate mathematical techniques to solve practical problems and demonstrates proficiency in the use of mathematical facts, techniques and formulae.	4	-		/4
	Submission Guidelines				
Timeliness	Student submits the exercises and portfolio tasks by the set deadline. See scoring guidelines for specific details.	2	-		/2
				FINAL	/20

Student Reflection:

How did you go with this week's work? What was interesting? What did you find easy? What do you need to work on?