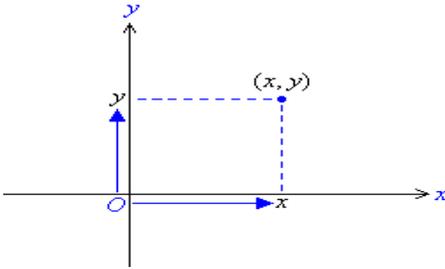


Goals

This week and next week we are going to:

- Plot Cartesian co-ordinates
- Generate tables for linear equations
- Graph linear equations
- Use linear modelling in a practical concept such as Break-even Analysis
- Apply further graphical concepts



Theoretical components

Resources:

PDF file: Week 13 and 14 Notes and Exercises

Knowledge Checklist:

- Plot points and straight lines
- Apply linear modelling
- Read travel graphs
- Apply further graphs

Order:

1. Work through the Week 13 and 14 notes and exercises
2. Complete the Portfolio task
3. Complete the reflection at the end of the booklet
4. Show your teacher your completed booklet.

Practical components

Work through the exercises and show the complete tasks to your teacher.

Be sure to ask for help as you need for the successful completion of all tasks.

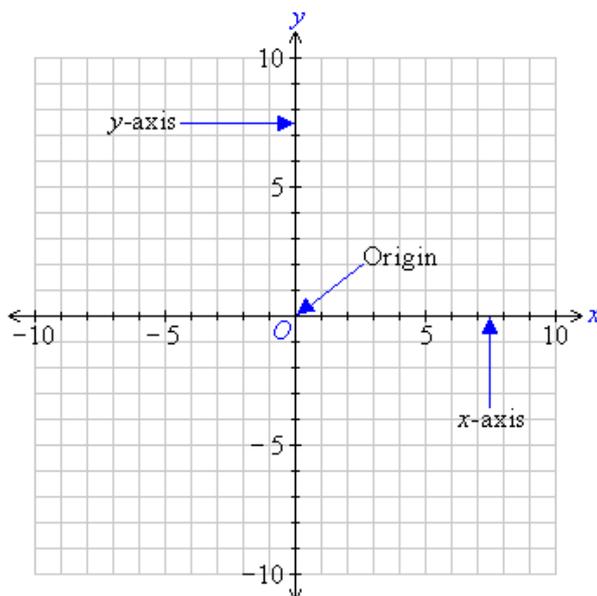
You will need some graph paper. There are some attached to the end of this booklet. If you need more, see your teacher.

Portfolio Task

See the last page of the booklet

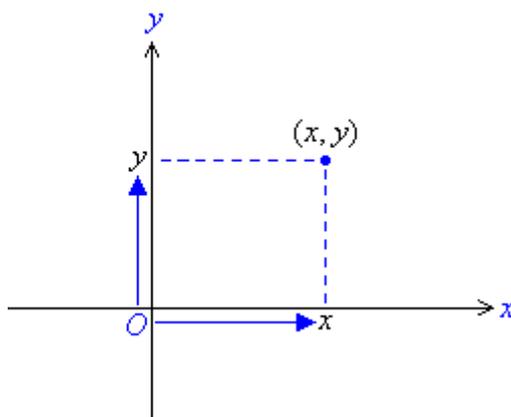
The Cartesian Plane

The **Cartesian plane** consists of two **directed lines** that **perpendicularly** intersect at a point called the origin.



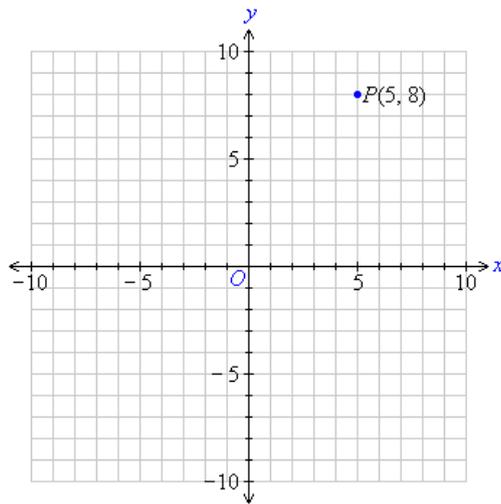
The **horizontal line** is called the **x-axis** and the **vertical line** is called the **y-axis**.

The position of any point on the **Cartesian plane** is described by using two numbers: (x, y) . The first number, x , is the horizontal position of the point from the **origin**. It is called the **x-coordinate**. The second number, y , is the vertical position of the point from the origin. It is called the **y-coordinate**. Since a specific order is used to represent the coordinates, they are called **ordered pairs**.



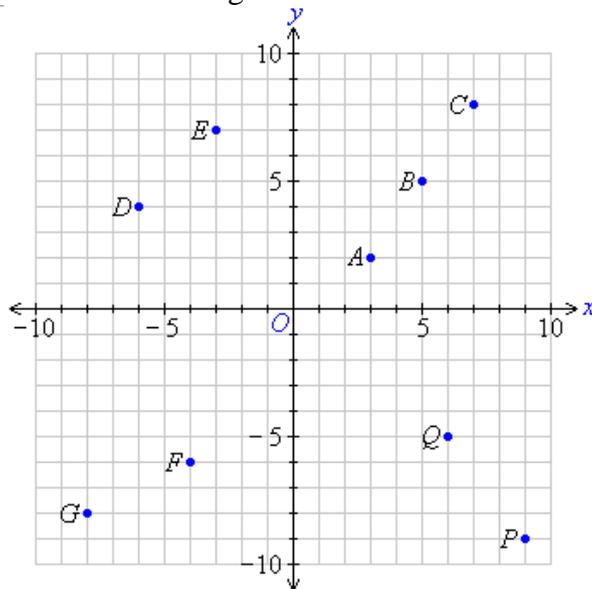
For example, the ordered pair $(5, 8)$ represents a point 5 units to the right of the origin in the direction of the x-axis and 8 units above the origin in the direction of the y-axis as shown in the diagram below.

The x-coordinate of point P is 5, and the y-coordinate of point P is 8. The coordinates of point P are $(5, 8)$.



Example 1

What are the co-ordinates of the letters in the diagram below?



Solution

A is 3 units to the right of and 2 units above the origin. So, point A is (3, 2).

D is 6 units to the left and 4 units above the origin. So, point D is (-6, 4).

F is 4 units to the left and 6 units below the origin. So, point F is (-4, -6).

Exercise 1

1. Write the co-ordinates of the other letters

B

C

E

G

P

Q

2. On the graph paper provided, draw a number plane with both axis (lines) extending from -6 to 6. On this Cartesian plane, mark the following points.

A (3, 1)

B (-4, 3)

C (-3, 4)

D (-2, -2)

E (0, -2)

F (1, -5)

G (4, -4)

H (-3, 0)

I (-6, 0)

Plotting Graphs from Ordered Pairs

We can use ordered pairs to draw straight lines. Such lines are called linear graphs. The ordered pairs are often arranged in tables.

Example 2

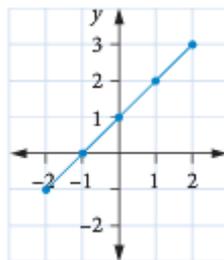
Draw a linear graph from the ordered pairs.

| | | | | | |
|----------|----|----|---|---|---|
| x | -2 | -1 | 0 | 1 | 2 |
| y | -1 | 0 | 1 | 2 | 3 |

Solution

The ordered pairs are (-2, -1), (-1, 0), (0, 1), (1, 2), (2, 3).

When the points are plotted on the Cartesian plane and joined by a line we get;



Exercise 2

1. From the following tables, write down the ordered pairs, plot the points on a Cartesian plane and join the points.

a.

| | | | | | |
|----------|----|----|---|---|---|
| x | 0 | 1 | 2 | 3 | 4 |
| y | -2 | -1 | 0 | 1 | 2 |

b.

| | | | | | |
|----------|----|----|---|---|---|
| x | -6 | -3 | 0 | 3 | 6 |
| y | -2 | -1 | 0 | 1 | 2 |

From Rules to Graphs

If we are given an algebraic rule (equation) we can complete a table of values and use this to draw a graph.

A rule that is represented by a straight line when we graph it on a number plane is called a linear equation or a linear function.

Example 3

Construct a table of values for the rule: $y = x + 2$ and graph the linear function.

Solution

We draw a table and choose some x values.

| | | | | | | |
|----------|----|----|---|---|---|---|
| x | -2 | -1 | 0 | 1 | 2 | 3 |
| y | | | | | | |

Calculate the y values by substituting the x values into the rule.

$$y = -2 + 2 = 0$$

$$y = -1 + 2 = 1$$

$$y = 0 + 2 = 2$$

$$y = 1 + 2 = 3$$

$$y = 2 + 2 = 4$$

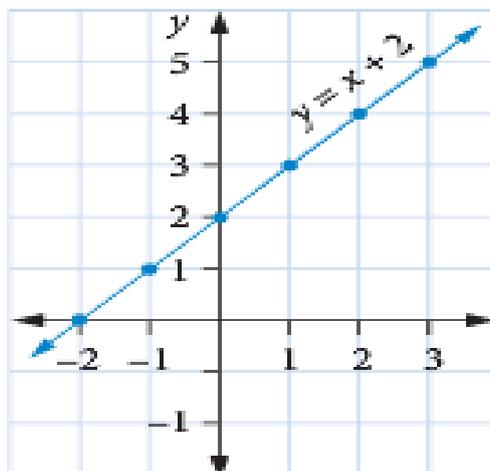
$$y = 3 + 2 = 5$$

Complete the table:

| | | | | | | |
|----------|----|----|---|---|---|---|
| x | -2 | -1 | 0 | 1 | 2 | 3 |
| y | 0 | 1 | 2 | 3 | 4 | 5 |

The ordered pairs are $(-2, 0)$, $(-1, 1)$, $(0, 2)$, $(1, 3)$, $(2, 4)$, $(3, 5)$.

The points are plotted on the number plane.



Exercise 3

1. Construct a table of values for each of the following rules. Use x values of -2 to 3.

a. $y = x - 4$

| | | | | | | |
|----------|----|----|---|---|---|---|
| x | -2 | -1 | 0 | 1 | 2 | 3 |
| y | | | | | | |

b. $y = x + 3$

| | | | | | | |
|----------|----|----|---|---|---|---|
| x | -2 | -1 | 0 | 1 | 2 | 3 |
| y | | | | | | |

2. Complete a table of values for the rule $y = 2x - 4$, and graph the linear function.

| | | | | | | |
|----------|----|----|---|---|---|---|
| x | -2 | -1 | 0 | 1 | 2 | 3 |
| y | | | | | | |

3. Draw a graph of the linear function $y = 3x$.

Linear Modelling

Many real-life applications, such as fees charged for services, cost of manufacturing or running a business, patterns in nature, sporting records and so on, follow linear relationships. These relationships may take the form of a linear equation; for example, $F = 50 + 30t$ may be used by a tradesperson to calculate her fee (in dollars) for t hours of work.

Here, F is the fee in dollars and t the time in hours. The 50 represents an initial fee for simply turning up, while the $30t$ is the amount charged for the time spent on the job.

For example, if $t = 2$ hours, $30t = 60$, so the total charge for the work would be $\$(50 + 60) = \110 . Equations like $F = 50 + 30t$ are sometimes referred to as 'linear models'.

These linear functions can always be graphed as straight lines.

Example 4

Nic is an electrician. He charges \$60 per hour. Construct a table of values.

Solution

We can use C for charge and h for the hours worked.

Thus, $C = 60h$. We can now draw up a table of values.

| | | | | | |
|----------|---|----|-----|-----|-----|
| h | 0 | 1 | 2 | 3 | 4 |
| C | 0 | 60 | 120 | 180 | 240 |

We could also plot this as a straight line graph.

Example 5

A generator company charges a \$200 delivery fee, and a rental fee of \$1500 per day. How much would a 4 day hire cost?

Solution

Use T for total charge, D for delivery fee and d for the number of days. This gives $T = D + 1500d$ or

$$T = 200 + 1500d$$

$$T = 200 + 1500 \times 4$$

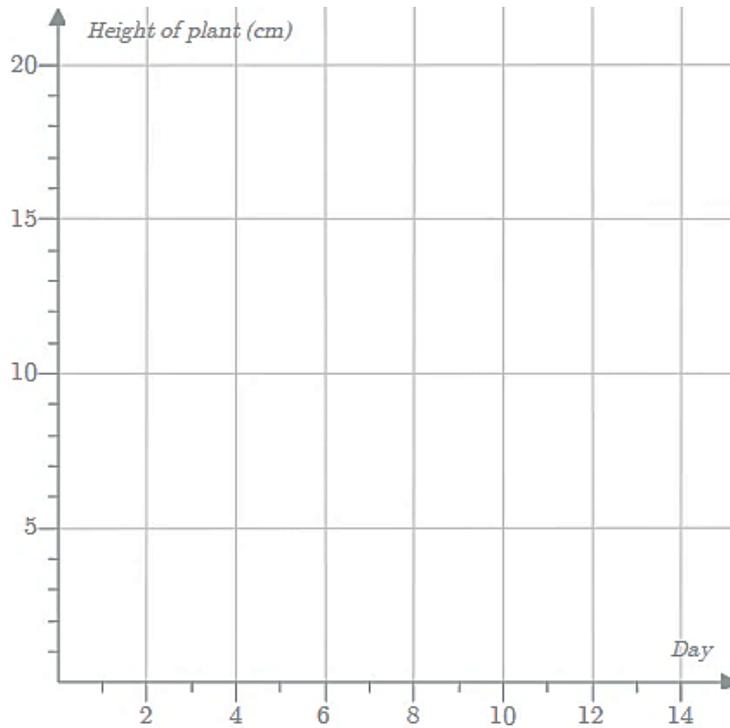
$$T = 200 + 6000$$

$$T = \$6200$$

3. Montana is collecting data on the growth of her plants. The table shows the data she has collected so far.

| | | | | | |
|---------------------------|---|---|---|----|----|
| Day (d) | 2 | 4 | 6 | 8 | 10 |
| Height of plant (h cm) | 5 | 7 | 9 | 11 | 13 |

a. Draw this linear function on the graph below.

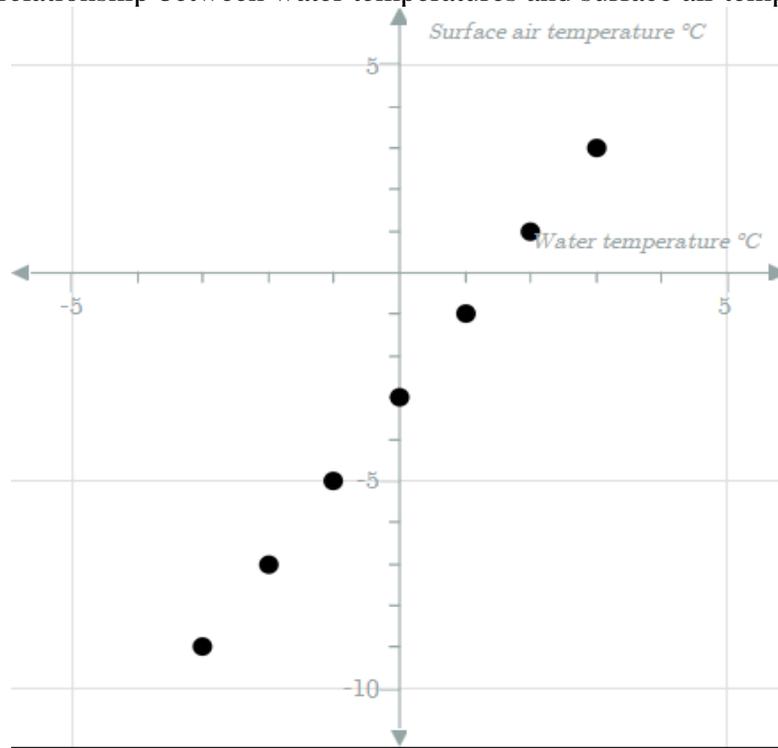


b. Write an equation that represents the height of the plant, h cm.

c. How tall is the plant when Montana started to collect data?

d. On which day will the plant be 10 cm tall?

4. The graph shows the relationship between water temperatures and surface air temperatures.



a. Complete the table of values

| | | | | | | | |
|------------------------------|----|----|----|---|---|---|---|
| Water temperature (°C) | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| Surface air temperature (°C) | | | | | | | |

b. Which of the following equations accurately represents the surface air temperature (°C)?

i. $y = x - 3$

ii. $y = x - 2$

iii. $y = 2x - 3$

iv. $y = 3x - 3$

c. What is the surface air temperature when the water temperature is 15°C?

d. What is the water temperature when the surface air temperature is 25°C?

5. The table shows a diver's descent below the surface at a constant rate over a period of 5 minutes.

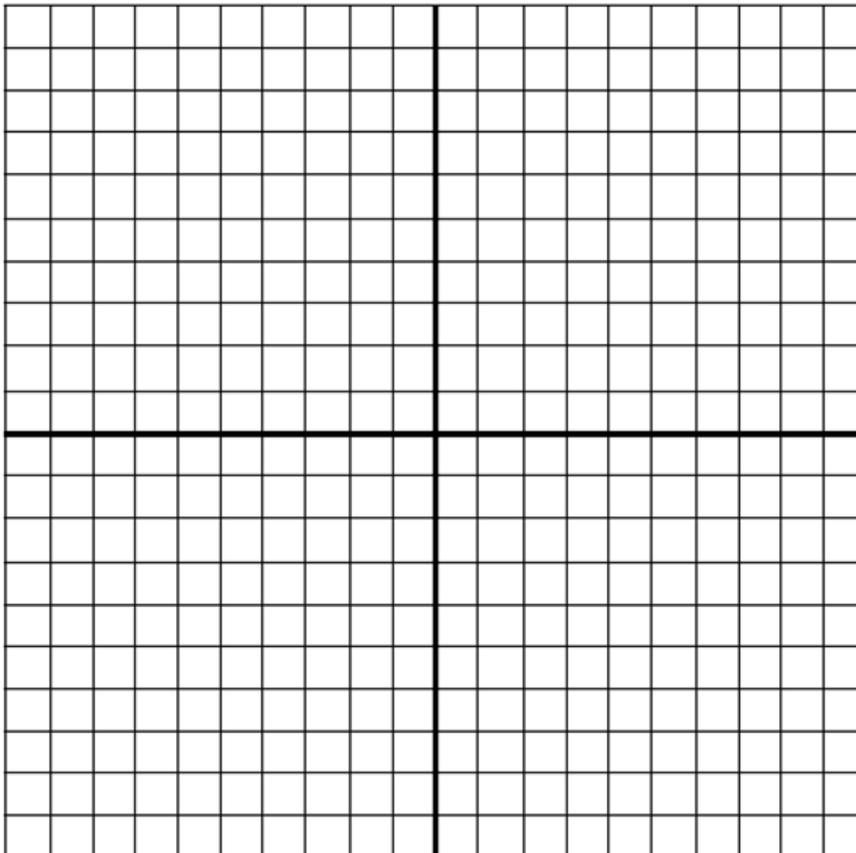
| | | | | | |
|--------------------------------------|---|-----|---|-----|---|
| Number of minutes passed (x) | 0 | 1 | 2 | 3 | 4 |
| Depth of the diver in metres (y) | 0 | 1.5 | 3 | 4.5 | 6 |

a. Write an equation representing the depth of the diver in metres.

b. What is the depth of the diver after 8 minutes?

c. How long will it take the diver to reach a depth of 13.5?

d. Draw this linear function on a graph.



Break-even analysis

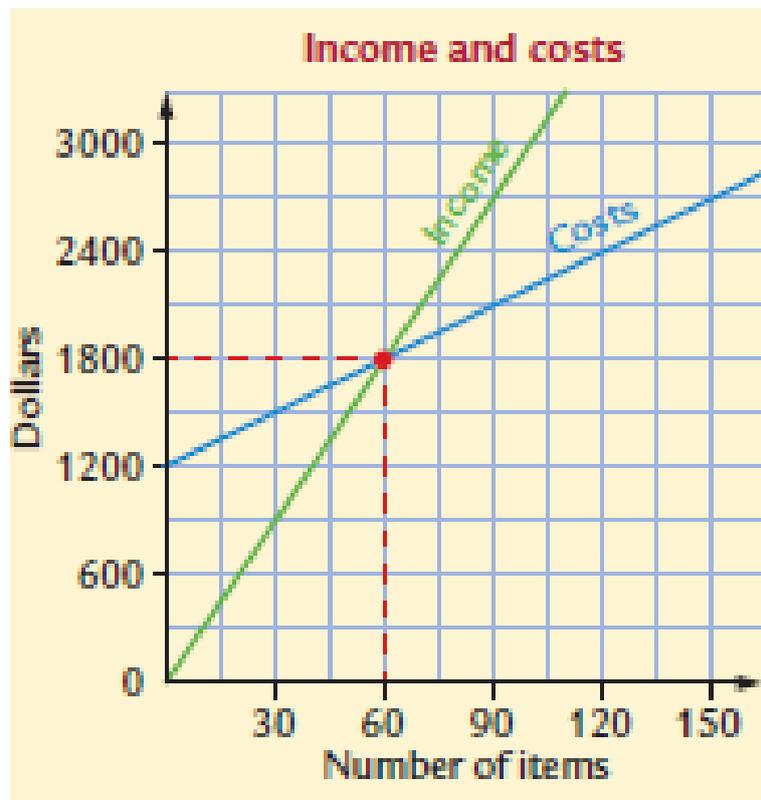
Break-even is the point where a business' costs are the same as the money they receive from sales. Knowing the break-even point is essential to making a profit. If a business is not making a profit, it won't last long!

Example 6

A small business's total fixed costs are \$1200 per week and its variable costs are \$10 per item it produces. Each item produced is sold for \$30. How many items does the business need to sell each week to breakeven?

Solution

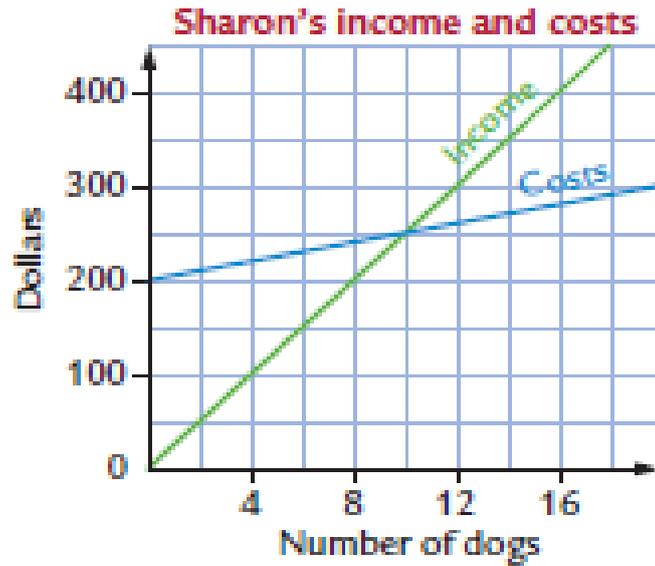
A graph is the easiest way to solve this problem. The blue line shows the business's cost of producing different number of items. The green line shows the income the business receives from selling different numbers of items. The red point, where the lines cross, is the business's break-even point.



When the business sells 60 items per week, the income and costs are equal. If the business sells more than 60 items, it will make a profit. If the business sells less than 60 items, it will make a loss.

Exercise 5

1. Sharon has a dog-washing business. The graph shows her weekly expenses and income.



- a. Explain how you know that Sharon's fixed costs are \$200 per week.

- b. How much does Sharon charge for washing dogs?

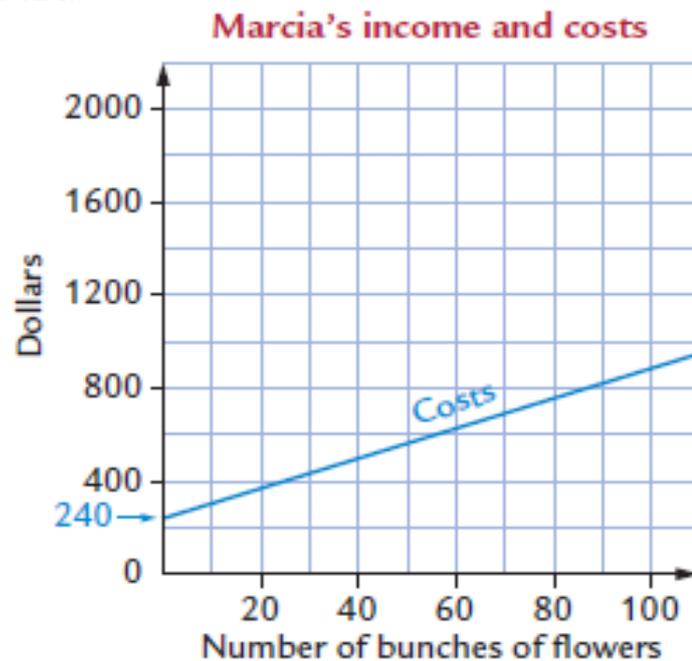
- c. How many dogs does Sharon have to wash each week to break even?

- d. How much profit does Sharon make in a week if she washes 16 dogs?

2. Marcia sells bunches of flowers from her street flower stall. She sells each bunch for \$15
- a. Complete this table of values to show the money she will receive from selling different numbers of bunches of flowers.

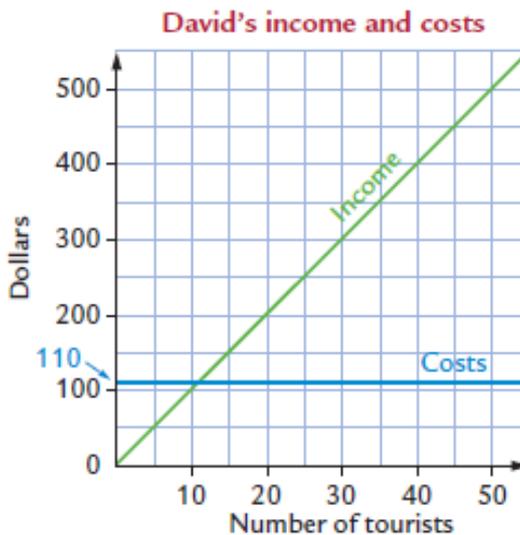
| | | | | | | |
|-------------------------|---|----|----|----|----|----|
| Bunches of flowers sold | 0 | 15 | 40 | 50 | 65 | 90 |
| Money received | | | | | | |

- b. The graph shows Marcia's weekly costs for selling flowers. Use the data from the table to show Marcia's income line on it.



- c. How many bunches of flowers does Marcia need to sell each week to break even?
- d. How much are Marcia's fixed costs per week?
- e. Calculate Marcia's profit when she sells 80 bunches of flowers.

3. During the summer tourist season, David shows tourists the sights of his area in a horse-drawn carriage. This graph shows his weekly costs and income. Use the graph to answer the following questions.



a. How much does David charge each tourist he takes in his carriage?

b. How many tourists does he need to drive each week to break even?

c. Suggest a reason why David's cost line is horizontal.

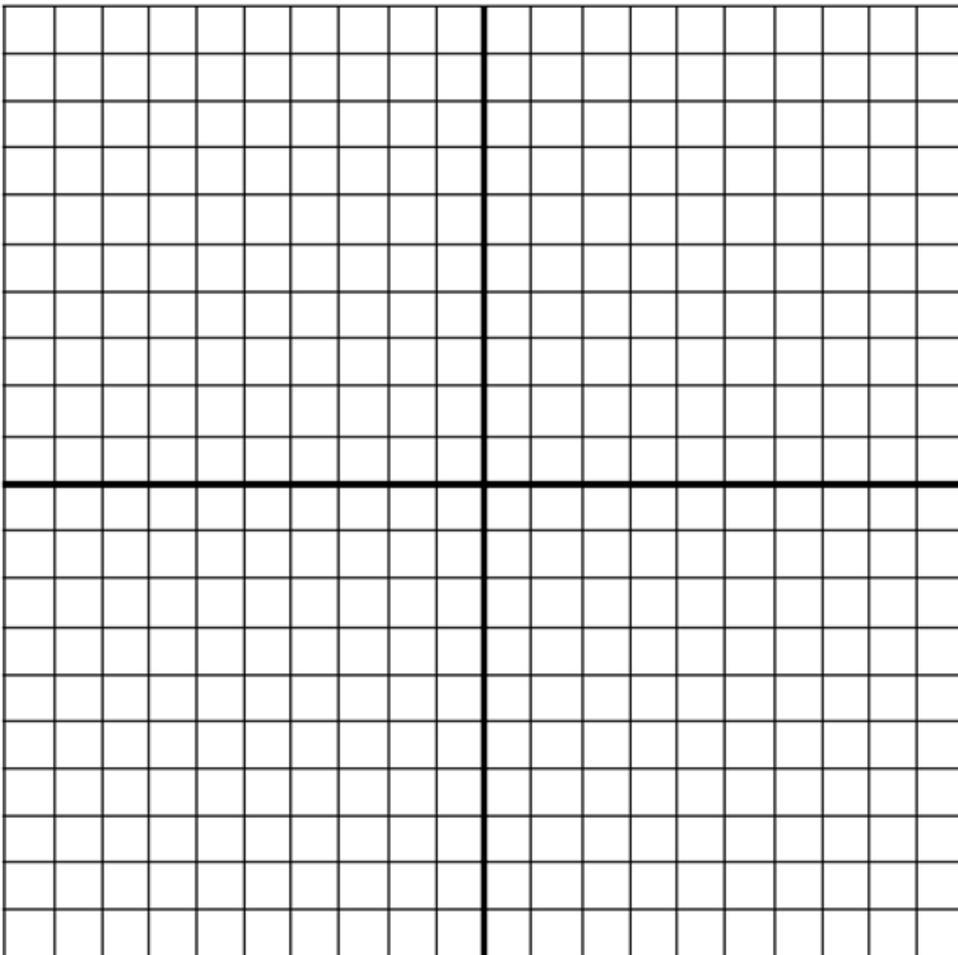
Portfolio Task Week 13/14

The cost for a furniture manufacturer to make a dining table is \$450 per dining table plus a fixed setup cost of \$6000. The dining tables will sell for \$700 each.

- a. Complete a table of values for the cost of manufacturing the table and write an equation to represent this information.

- b. Complete a table of values to show the money received for each table sold and write an equation to represent this information.

- c. Draw these functions on the same graph.



d. How many dining tables will they need to sell to break even?

e. Calculate manufacturer's profit when they sell 25 dining tables.

MARKING RUBRIC

| CRITERIA | EXPECTATIONS | POSS | MULT | GIVEN | TOTAL |
|-------------------------------------|--|------|------|--------------|------------|
| | | | | | |
| Practical | Student completes practical work, including exercises of the brief to an acceptable standard set by the teacher. | 2 | 3 | | /6 |
| Portfolio Task | Student completes the investigation task of the week to an acceptable standard set by the teacher. | 2 | 2 | | /4 |
| | | | | | |
| Reasoning and Communications | Student responses are accurate and appropriate in presentation of mathematical ideas, with clear and logical working out shown. | 4 | - | | /4 |
| Concepts and Techniques | Student submitted work selects and applies appropriate mathematical techniques to solve practical problems and demonstrates proficiency in the use of mathematical facts, techniques and formulae. | 4 | - | | /4 |
| | Submission Guidelines | | | | |
| Timeliness | Student submits the exercises and investigation by the set deadline. See scoring guidelines for specific details. | 2 | - | | /2 |
| | | | | FINAL | /20 |

Student Reflection:

How did you go with this week's work? What was interesting? What did you find easy? What do you need to work on?