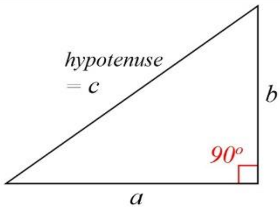
Week 10/11
Term 2
2022

HAWKER COLLEGE
Engage | Inspire | Achieve

Learning Brief

EM3

Goals



$$c^2 = a^2 + b^2$$

This week and next week we are going to:

- Apply and use Pythagoras' theorem to solve problems involving the length of the hypotenuse or one of the shorter sides
- Learn how to name the sides of a right-angled triangle in relation to an identified angle
- Apply the tangent ratio to find unknow angles and sides in right-angled triangles
- Apply the cosine and sine ratios to find unknown angles and sides in right-angled triangles
- Work with concepts of angle of elevation and depression

Theoretical Components

Resources:
PDF file: Week 11 and 12 Notes and Exercises

Here is a link for finding the short side in a right-angled triangle:
https://www.youtube.com/watch?app=desktop&v=izOnU4dCy_0

Here is a link for finding the angle in a right-angled triangle:
https://www.youtube.com/watch?app=desktop&v=LeIolWAuCEQ

Knowledge Checklist:

- Naming the sides of a triangle
- Pythagoras' Theorem
- Applications of Pythagoras' theorem
- Finding the length of the hypotenuse
- Finding the length of a shorter side
- Investigating the tangent ratio
- Find unknown angles and sides using tan ratio
- Angles of elevation and depression

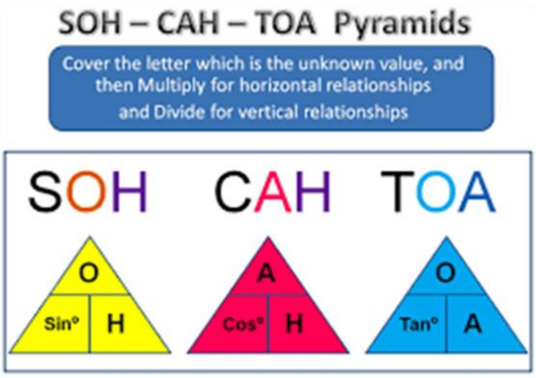
Order:

1. Work through the Week 11 and 12 notes and exercises
2. Complete the Portfolio task
3. Complete the reflection at the end of the booklet
4. Show your teacher your completed booklet.

Practical Components

Work through the exercises and show the complete tasks to your teacher.

Be sure to ask for help as you need for the successful completion of all tasks.



Portfolio Task

See the last page of the booklet

WEEK 10 AND 11 – TRIGONOMETRY

Trigonometry

The ancient Greeks, between the years 600 BC and 200 AD, laid the foundations of a new branch of mathematics, one that uses angles, triangles and circles to calculate lengths and distances that cannot be measured physically (as they are too large).

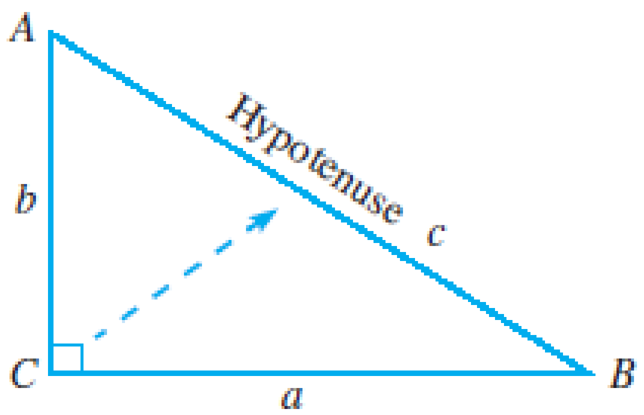
This new mathematics is now called **trigonometry**, from the Greek words *trigon* and *metron*, meaning ‘triangle’ and ‘measure’ respectively. Trigonometry is used widely today to calculate lengths and angles – in engineering, surveying, navigation, astronomy, electronics, and construction.

Pythagoras’ Theorem

The Greek mathematician Pythagoras (580-496 BC) is credited with discovering the following rule about the sides of a right-angled triangle:

$$\boldsymbol{c^2 = a^2 + b^2}$$
$$(\text{hypotenuse})^2 = (\text{side})^2 + (\text{other side})^2$$

The square of the hypotenuse is equal to the sum of the squares of the other two sides.



The **hypotenuse** is the longest side of a right-angled triangle and is always opposite the right angle.

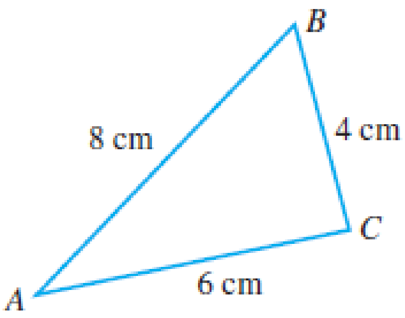
Labelling triangles

Note that in the diagram above, the angles of the triangle are labelled by *capital* letters *A*, *B*, *C,* while the sides are labelled by *lower* case letters, *a*, *b*, *c*. Also, the side with the lower case letter (e.g. *a*) is always *opposite* the angle with the corresponding capital letter (e.g. *A*). This is a **convention** (accepted rule or agreement) of triangle geometry.

Example 1

Determine where these triangles are right-angled.

a.



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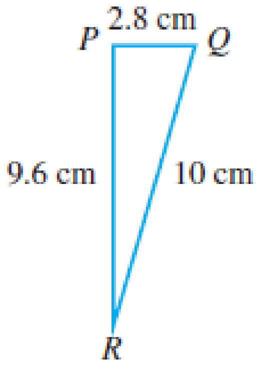


Solution

1. The longest side is 8 cm.
   $c^2 = 8^2 = 64$

   $a^2 + b^2 = 4^2 + 6^2$
   $a^2 + b^2 = 16 + 36$
   $a^2 + b^2 = 52$

   $c^2 \neq a^2 + b^2$

   Pythagoras' theorem does not work, therefore this triangle is not right-angled.

2. The longest side is 10 cm.
   $p^2 = 10^2 = 100$

   $q^2 + r^2 = 9.6^2 + 2.8^2$
   $q^2 + r^2 = 92.16 + 7.84$
   $q^2 + r^2 = 100$

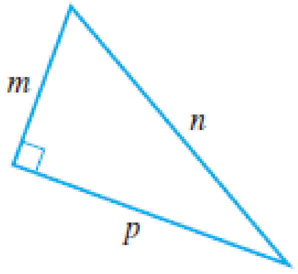
   $p^2 = q^2 + r^2$

   Pythagoras' theorem works, therefore this triangle is right-angled.

Exercise 1

1. Write Pythagoras' theorem for these right-angle triangles.

   a.



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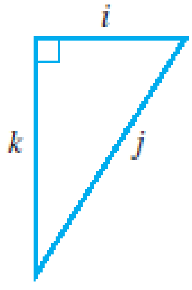
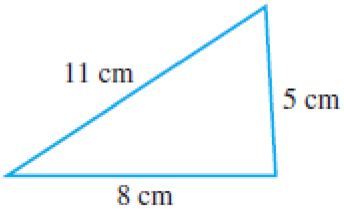


2. Determine whether each triangle is right-angled.

a.



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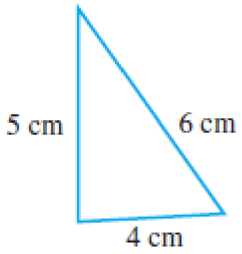
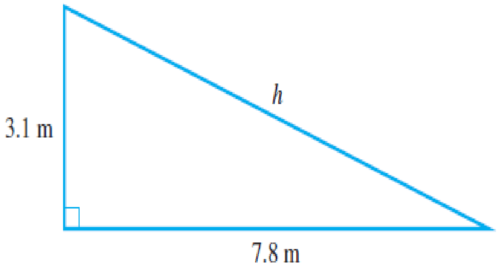


Finding the Hypotenuse

We can use Pythagoras' Theorem to find the length of the hypotenuse in a right-angled triangle.

Example 2

Find the length of the hypotenuse h (give the final answer to 1 decimal place).



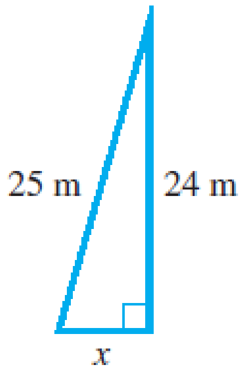
Solution

$h^2 = 3.1^2 + 7.8^2$
$h^2 = 9.61 + 60.84$
$h^2 = 70.45$
$h = \sqrt{70.45}$
$h = 8.393 \ldots$
$h \approx 8.4$ m

We can tell $h \approx 8.4$ m is a reasonable answer by observing the diagram, which is drawn to scale. Also 8.4 m is the longest side but not longer than the sum of 3.1 m and 7.8 m.

Example 3

Find the length of the side x.
Note: Find the length of one of the shorter sides, not the hypotenuse.



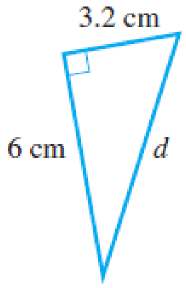
Solution

$25^2 = x^2 + 24^2$
$x^2 = 25^2 - 24^2$, it is easier to remember that you subtract when finding a shorter side
$x^2 = 625 - 576$
$x^2 = 49$
$x = \sqrt{49}$
$x = 7$ m

Exercise 2

1. Find the value of the pronumeral in each of these diagrams.

a.



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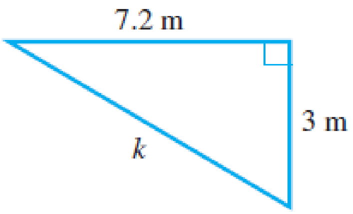

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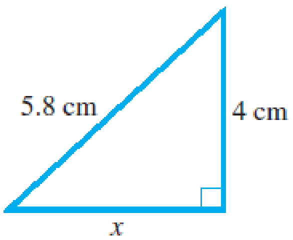

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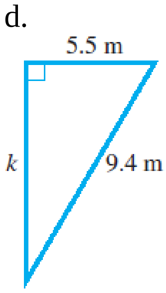
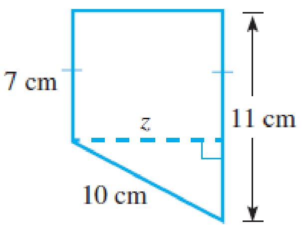


2. Find the value of the pronumeral in each diagram.

a.



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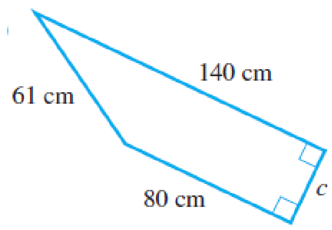
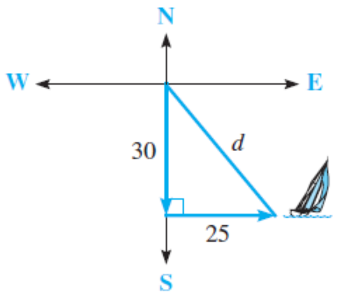


Applications of Pythagoras' Theorem

Pythagoras' Theorem can be used to solve problems that occur in the 'real' world.

Example 4

A boat sailed 30 nautical miles due south, then 25 nautical miles due east. How far is it from the starting point, correct to 2 decimal places?



Solution

Let d be the boat's distance from its starting point.

$d^2 = 30^2 + 25^2$

$d^2 = 900 + 625$

$d^2 = 1525$

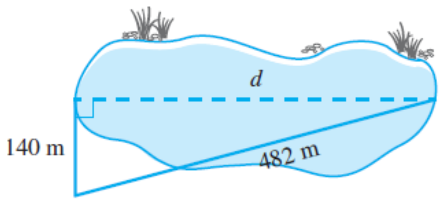
$d = \sqrt{1525}$

$d = 39.0512 \ldots$

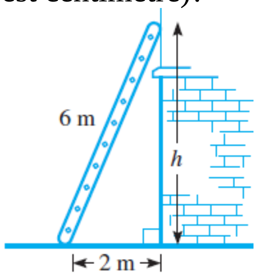
$h \approx 39.05$ nautical miles

Exercise 3

1. Chloe calculated the distance across the lake by taking measurements show. What was the distanced (to the nearest metre)?

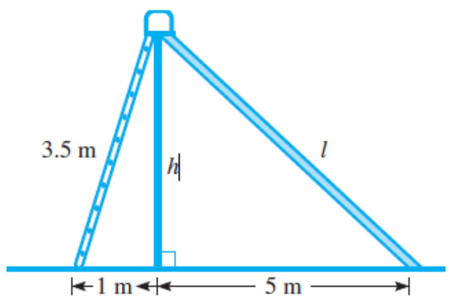


2. A 6 m ladder leans against a house so that its base is 2 m out from the bottom of the house. How far up the house does the ladder reach (to the nearest centimetre)?

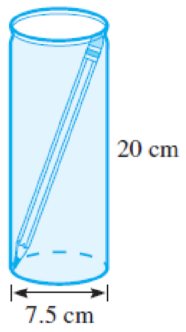


3. A yacht leaves Newcastle and sails 160 nautical miles due north. It turns and sails due east until it is directly 200 nautical miles from Newcastle. How far east did it sail? (Draw a diagram).

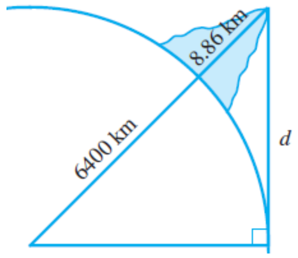
4. A playground slide is made up of two right-angled triangles. Find, correct to the nearest centimetre:
   a. *h*, the height of the slide
   b. *l*, the length of the slide



5. Erin wants to use an old tennis-ball can as a pencil case. If the can has a diameter of 7.5 cm and a height of 20 cm, what is the length of the longest pencil that will fit inside the can (to the nearest millimetre)?



6. Mount Everest, the highest mountain in the world, is 8.86 km above sea level.
(a) If the Earth has a radius of 6400 km, what is the distance *d* (to the nearest kilometre) to the visible horizon from the top of the mountain?



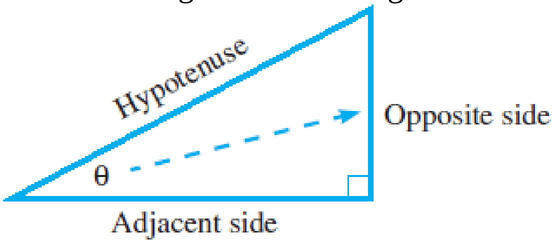
b) The formula $d = 8\sqrt{\frac{h}{5}}$ also gives the distance to the horizon in kilometres, where *h* is the height in metres.
Use this formula to calculate the distance *d* that can be seen from the top of Mount Everest (to the nearest kilometre) and compare it with your answer from part (a).

The tangent ratio

Trigonometry is the mathematics of using angles and triangles to calculate lengths and distances that are either difficult or impossible to measure. It uses the three sides of a right-angled triangle, each of which has a special name related to a particular angle in the triangle:

- **Opposite side:** the side directly opposite the angle, not joined to the angle
- **Adjacent side:** the side leading to the right angle ('adjacent' means next to)
- **Hypotenuse:** the longest side (this has already been introduced with Pythagoras' theorem).

The diagram shows the three sides related to angle θ in the triangle.



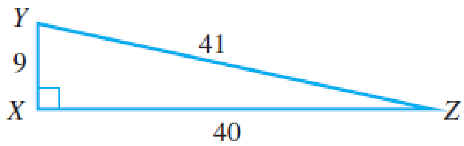
In trigonometry, three ratios are used: **sine**, **cosine**, and **tangent**. We will first investigate the **tangent** ratio, abbreviated to **tan θ**.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

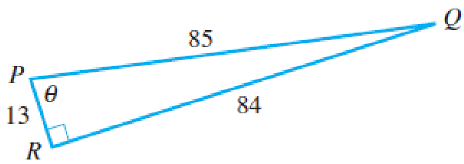
The tangent of an angle is the ratio of the length of the opposite side to the length of the adjacent side.

Example 5

a. For ΔXYZ, write $\tan Z$ and then express this ratio as a decimal.



b. Calculate $\tan\theta$ as a decimal correct to 4 decimal places.



Solution

a. $\tan Z = \frac{\text{opposite}}{\text{adjacent}} = \frac{9}{40} = 0.225$

b. $\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{84}{13} = 6.4615$

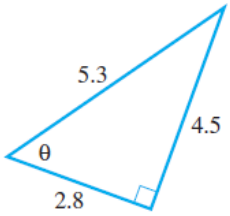
The sine and cosine ratio

The **sine** ratio is abbreviated to **sin θ** and the **cosine** ratio is abbreviated to **cos θ**. The ratios are: $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$ and $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$. Make sure you label the three sides first before deciding which trigonometry ratio to use.

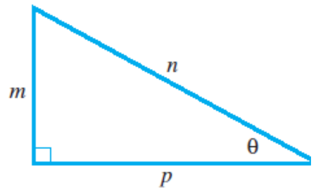
Exercise 4

6. Write tan θ as a ratio (or fraction) for each of these triangles.

a.

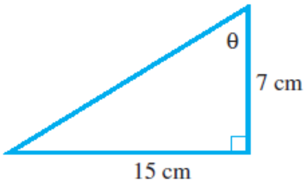


b



7. Calculate tan θ correct to 4 decimal places for each of these triangles.

a.



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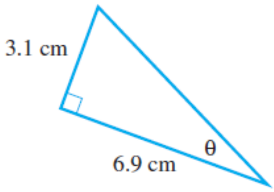
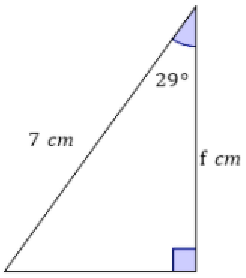


8. Find the value of the pronumeral in each of these diagrams.

a.



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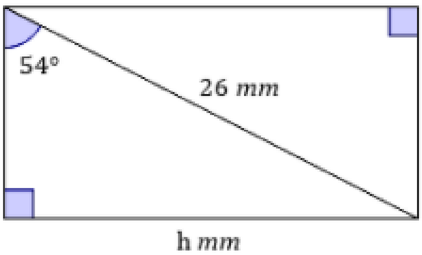
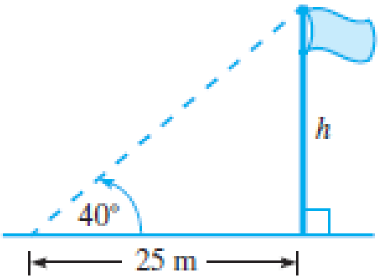


Using the tangent ratio to find an unknown side

The tangent ratio is a constant value for a given angle, regardless of the size of the triangle. We can use this fact to calculate unknown lengths and distances involving right-angled triangles.

Example 6

Find h, the height of the flagpole, correct to 1 decimal place.



Solution

$\tan\theta = \frac{\text{opp}}{\text{adj}}$ which gives

$\tan 40 = \dfrac{h}{25}$

By cross multiplying: $25 \times \tan 40 = h$

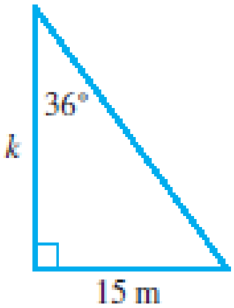
$20.977\ldots = h$

Therefore, $h \approx 21$ m

Note: When the angle is less than 45° the opposite side is shorter than the adjacent side, but when the angle is greater than 45° the opposite side is longer.

Example 7

Find k correct to 2 decimal places.



Solution

$\tan 36 = \frac{15}{k}$, the problem here is that the pronumeral, k, is the denominator (bottom part of the fraction). To solve this, we must use two steps.

First cross multiply to get: $k \times \tan 36 = 15$

Then divide to find k: $k = \frac{15}{\tan 36}$

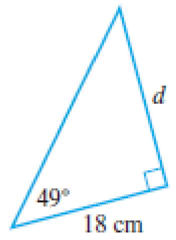
$k = 20.6457\ldots$

Therefore, $k \approx 20.65$ m

Exercise 5

1. Find the value of the pronumeral in these triangles (to 1 decimal place)

a.



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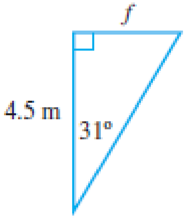

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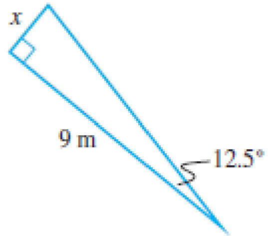

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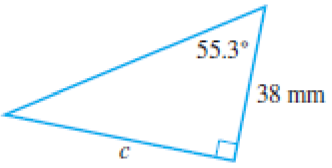
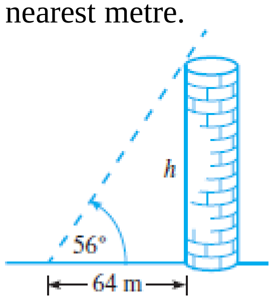
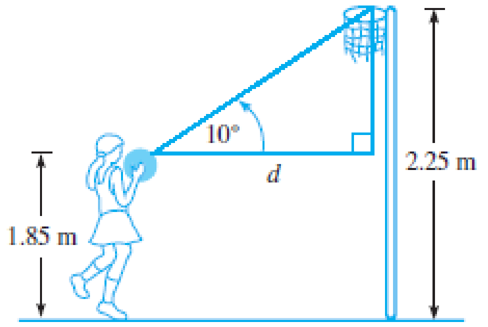


2. Calculate the height of the chimney to the nearest metre.



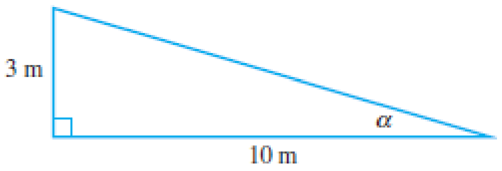
3. What is the distance between the netballer and the post if her shooting angle is 10°?



Using tangent ratio to find a missing angle

Example 8

Find the size of the angle α to the nearest degree.



Solution

We can use a calculator to solve this problem. A calculator has a reverse button, $\mathbf{tan^{-1}}$, which gives the angle once the tangent (tan) ratio is entered. The correct name for this feature is **inverse tan**.

$$\tan \alpha = \frac{3}{10}$$

Using the $\mathbf{tan^{-1}}$ button gives: $\alpha = 16.6992 \ldots$

Therefore $\alpha \approx 17°$ (see your teacher if you are unsure how to use this feature as it works differently on different calculators).

Exercise 6

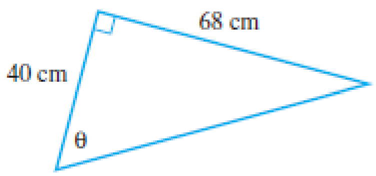
1. Find θ to the nearest whole degree.

   a. $\tan\theta = 0.7508$

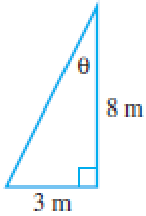
   b. $\tan\theta = \frac{1}{4}$

2. For each of these triangles, find θ to the nearest whole degree.

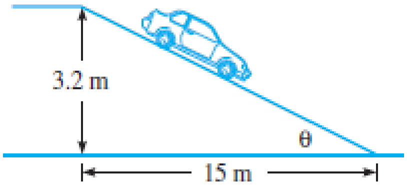
   a.



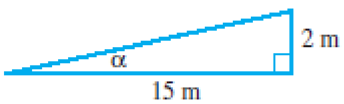
   b.



3. Find the angle of inclination (θ) of this ramp in a multistorey car park, to the nearest whole degree.

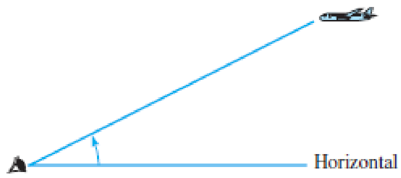


4. A hill has a gradient of $\frac{2}{15}$, which means that it rises 2 m for every 15 m horizontally. Find the angle the hill makes with the horizontal, to the nearest degree.

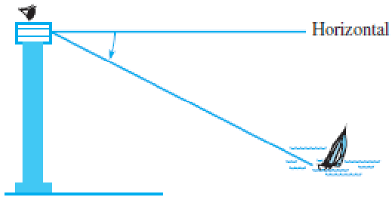


Angles of Elevation and Depression

The **angle of elevation** is the angle of looking up, measured from the horizontal.

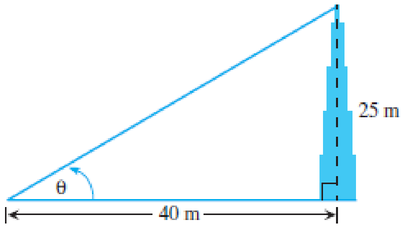


The **angle of depression** is the angle of looking down, measured from the horizontal.



Example 9

Rebecca stands 40 m away from the base of a 25 m tower. What is her angle of elevation to the top of the tower, to the nearest degree?

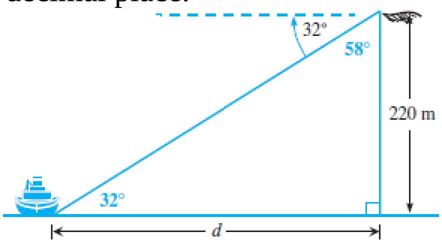


Solution

$\tan\theta = \frac{25}{40}$
$\theta = 32.0053 \ldots$
$\theta \approx 32°$

Example 10

The angle of depression of a ship from the top of a 220 m vertical cliff is 32°. Find the distance of the ship from the base of the cliff correct to 1 decimal place.



Solution

Angle of depression = 32°, so the adjacent complementary angle is 90° - 32° = 58°.

$\tan 58° = \frac{d}{220}$
$d = 220 \times \tan 58°$
$d = 352.1 \text{ m}$

Exercise 7

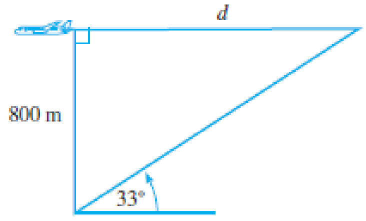
In each of the questions draw a diagram if one is not given.

1. From the ground, Laura sees the top of a 180 m tower at an angle of elevation of 49°. How far is she from the base of the tower, to the nearest metre?

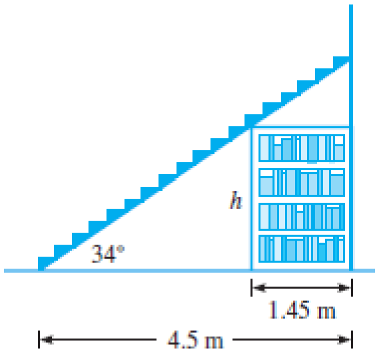
2. A ship's captain observes a lighthouse on top of a vertical cliff at an angle of elevation of 16.3°. If the lighthouse is 40 m above sea level, what is the distance between the ship and the bottom of the cliff?

3. A ladder leaning against a wall makes an angle of 68° with the ground. If it stands 1.2 m away from the base of the wall, calculate (correct to 2 decimal places):
   a. The distance the ladder reaches up the wall
   b. The length of the ladder

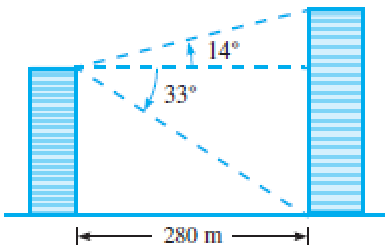
4. Jack observed a plane flying at a constant height of 800 m. At first it appeared at an angle of elevation of 33°, then it flew directly overhead. What was the distance travelled by the plane (to the nearest metre) during the period of observation?



5. A stairwell is inclined at 34° to the ground floor and has a horizontal length of 4.5 m. Abbey wants to place a bookcase of length 1.45 m underneath the stairwell. What is the height *h* of the tallest bookcase that can fit under the stairwell? Set your working out carefully and write your answer correct to 2 decimal places.

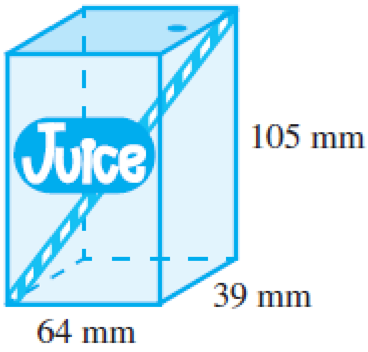


6. Emily stands at the top of her city apartment block and observes a taller office tower 280 m away. The angle of elevation of the top of the tower is 14° and the angle of depression is 33°. Calculate the height of the tower to the nearest metre. Set your working out carefully and write your answer correct to 2 decimal places.



Portfolio Task Week 10/11

This investigation involves using Pythagoras' Theorem in 3-dimensions.
(You need to show all working!)



You are required to show that the length of the longest straw that can fit into the pack is approximately 129 mm.

Step 1

Find the length of the diagonal on the base of the fruit-juice pack. We assume that the angle between the 64 mm and 39 mm sides is 90˚.

Step 2

Use this length to form a right-angled triangle from the diagonal on the base of the fruit-juice pack, the 105 mm side and the straw. Use Pythagoras' Theorem to find the length of the straw.

MARKING RUBRIC

CRITERIA	EXPECTATIONS	POSS	MULT	GIVEN	TOTAL
Practical	Student completes practical work, including exercises and Mathspace task, of the brief to an acceptable standard set by the teacher.	**2**	**3**		**/6**
Portfolio Task	Student completes the investigation task of the week to an acceptable standard set by the teacher.	**2**	**2**		**/4**
Reasoning and Communications	Student responses are accurate and appropriate in presentation of mathematical ideas, with clear and logical working out shown.	**4**	**-**		**/4**
Concepts and Techniques	Student submitted work selects and applies appropriate mathematical techniques to solve practical problems and demonstrates proficiency in the use of mathematical facts, techniques and formulae.	**4**	**-**		**/4**
	Submission Guidelines				
Timeliness	Student submits the exercises, Mathspace task and investigation by the set deadline. See scoring guidelines for specific details.	**2**	**-**		**/2**
				FINAL	**/20**

Student Reflection:
How did you go with this week’s work? What was interesting? What did you find easy? What do you need to work on?