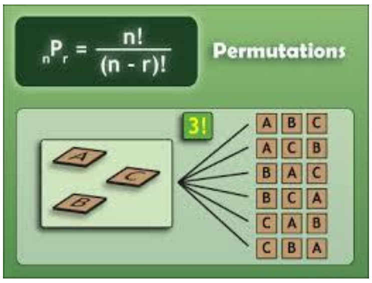
Week 3
Term 1
2021

HAWKER COLLEGE
Engage | Inspire | Achieve

Learning Brief

SMM1

Goals



This week:

- Apply basic probability rules
- Use ^nP_r to count number of possible arrangements (permutations)
- Compute number of ways of arranging *n* objects which include *p* identical objects of one type, *q* identical objects of another type, *r* identical objects of yet another type…
- Compute number of arrangements when *n* objects divided into *m* groups
- Compute number of arrangements when distinguishable objects are arranged in a circle
- Use combinations to count selections of objects where order is not important; use the ^nC_r notations to represent selections where order is not important; use CAS to compute ^nC_r for a given n and a given r
- Investigate patterns in Pascal's triangle and the relationship to combinations, establish counting principles and use them to solve simple problems involving numerical values for *n* and *r*

Theoretical Components

Read through the following chapters and make notes:

- 12E Permutations involving restrictions
- 12F Arrangements in a circle
- 12G Combinations using nCr

View the following websites and make notes:

Permutations and Combinations:

- https://www.tutors4you.com/circularpermutations.htm
- https://www.mathsisfun.com/combinatorics/combinations-permutations.html
- https://prezi.com/vjvjclbxn8zc/combinations/

Set Notation handout (to be completed over the next three weeks

Practical Components

Do the following questions:
Organise your solutions neatly in your exercise book.

You will require Chapter 12 of Maths Quest 11 Mathematical Methods (pdf – Google Classroom)

- Ex 12E: All odd numbered questions
- Ex 12F: All odd numbered questions
- Ex 12G: All odd numbered questions

Mathspace Task

Investigation

See next page

QFO
Quiz/Forum/Other

Complete the Mathspace task.

Week 3 Investigation

Pascal's triangle is a triangular array where each number is the sum of the two numbers above it (except for the edges, which are all "1"). It is named after the 17th century French mathematician, Blaise Pascal (1632 – 1662).

The triangle is constructed in the following manner: in row 0 (the topmost row), there is a unique nonzero entry 1. Each entry of each subsequent row is constructed by adding the two numbers above it.

$$
\begin{array}{c}
1 \\
1 \quad 1 \\
1 \quad 2 \quad 1 \\
1 \quad 3 \quad 3 \quad 1 \\
1 \quad 4 \quad 6 \quad 4 \quad 1 \\
1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1 \\
1 \quad 6 \quad 15 \quad 20 \quad 15 \quad 6 \quad 1 \\
1 \quad 7 \quad 21 \quad 35 \quad 35 \quad 21 \quad 7 \quad 1
\end{array}
$$

For example, numbers 1 and 3 in the third row are added to produce the number 4 in the fourth row.

The numbers in Pascal's Triangle have a special relationship with the coefficients of binomial expansions (binomial coefficients) and combinations.

$(a+b)^n$	*Binomial expansion*	*Pascals Triangle as combinations* ${}^nC_r = \binom{n}{r}$
$(a+b)^0 =$	1	$\binom{0}{0}$
$(a+b)^1 =$	$a+b$	$\binom{1}{0}$ $\binom{1}{1}$
$(a+b)^2 =$	$a^2+2ab+b^2$	$\binom{2}{0}$ $\binom{2}{1}$ $\binom{2}{2}$
$(a+b)^3 =$	$a^3+3a^2b+3ab^2+b^3$	$\binom{3}{0}$ $\binom{3}{1}$ $\binom{3}{2}$ $\binom{3}{3}$
$(a+b)^4 =$	$a^4+4a^3b+6a^2b^2+4ab^3+b^4$	$\binom{4}{0}$ $\binom{4}{1}$ $\binom{4}{2}$ $\binom{4}{3}$ $\binom{4}{4}$
$(a+b)^5 =$	$a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5$	$\binom{5}{0}$ $\binom{5}{1}$ $\binom{5}{2}$ $\binom{5}{3}$ $\binom{5}{4}$ $\binom{5}{5}$
		*n is the row number (*starting at row 0) and *r* is the element in the row (also starting at 0)

Consider the expansion above of $(a+b)^n$. Particularly note the following patterns.

- For each expansion to the power of n, there is $n+1$ elements
- For each term, the sum of the exponents is n
- Powers of a decrease from left to right, from n down to 0
- Powers of b increase from left to right, from 0 up to n
- The coefficients start at 1, end at 1, **and are the terms of the relevant row from Pascal's triangle**

The pattern in the expansions observed is summaries in the formula called the **binomial theorem**.

Binomial Theorem

$$(a+b)^n = \binom{n}{0}a^n b^0 + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{n}a^0 b^n$$

$$\text{where } \binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

Any particular term in the expansion of $(a+b)^n$ can be found using $\binom{n}{r}\, a^{n-r} b^r$

Example

What is the seventh term in the expansion of $(m-2n)^{12}$?

We need to construct the seventh term from this $\binom{n}{r}\, a^{(n-r)}\, b^r$ where n is 12 and r is 6.

The coefficient $\binom{n}{r}$ where n is 12 and r is 6 is $\binom{12}{6} = 924$.

The term will have both m and $(2n)$ components. The m component would be $m^{12-6} = m^6$

The $2n$ component would be $(2n)^6 = 64n^6$.

So putting that altogether will give us $924m^6 \times 64n^6 = 59\,136m^6n^6$.

Questions

1. Use the binomial theorem to evaluate 1.05^7 correct to 2 decimal places.

2. A particular term in the expansion of $(3a^2 + \frac{p}{b})^4)$ is $\frac{96a^2}{b^3}$, for some constant p. Find the value of p.