Week 3/4
Term 3
2021

HAWKER COLLEGE
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Learning Brief
EM4

Goals

This week and next week we are going to:

- interpret commonly used probability statements including *possible*, *probable, likely, certain*
- describe ways of expressing probabilities formally using fractions, decimals, ratios, and percentages
- identify relative frequency as probability
- construct and use a sample space to determine outcomes for an experiment using tree diagrams and arrays

Theoretical Components

Resources:
PDF file: Week 3 and 4 Notes and Exercises

More theory on probability and sample space.

https://www.mathsisfun.com/data/probability.html

This clip shows you about the Monty Hall problem.

https://www.youtube.com/watch?app=desktop&v=mhlc7peGlGg

Knowledge Checklist

- The language of probability
- Outcome and sample space
- Calculating probabilities

Order

1. Work through the Week 3 and 4 booklet
2. Complete the Portfolio task
3. Complete the reflection at the end of the booklet
4. Show your teacher the completed booklet

Practical Components

Work through the exercises and show the completed tasks to your teacher.

Week 4 starts at the 'Calculating Total Number of Possibilities' section.

Portfolio Task

See the last page of the booklet

Other

Make sure you have joined the Google Classroom. If you have not, see your teacher.

ESSENTIAL MATHEMATICS 4

WEEK 3 AND 4 – PROBABILITY

Large numbers

Calculating probability often involves large numbers. For example, in Lotto there are 8,145,060 combinations of 6 numbers (in Lotto, you choose 6 numbers out of 45, i.e. from numbers 1 to 45), thus your chance of winning is very small.

How big is a billion? That depends on whether you ask an Australian or an American! However, everyone agrees on the size of a million. In the decimal system, one million is written with a 1 in the millions position.

One million is a 1 in the millions position followed by zeros in the other positions; that is, 1,000,000. Note that one million is a 1 followed by six zeros.

Example 1

1. What number is 457,000,000?
2. Write the numeral for 75 thousand.
3. Write the numeral for 43 thousand and 8.
4. Write the number 5,004,012 in words.

Solution

1. The number is 457 million.
2. 75,000
3. 43,008
4. Five million, four thousand and twelve.

Exercise 1

Write the answers to the probability questions as fractions unless you are told otherwise.

1. Match the numbers in parts **a** to **h** with the numerals listed in **i** to **viii**.

a.	8 million	i.	8,200
b.	8 thousand	ii.	80,200
c.	8 thousand 2 hundred	iii.	8,000,002
d.	82 thousand	iv.	8,000,000
e.	80 thousand 2 hundred	v.	82,000
f.	8 million 2 thousand	vi.	8,2000,000
g.	8 million 2 hundred thousand	vii.	8,000
h.	8 million and 2	viii.	8,002,000

2. Write these numbers in words.
   a. 4,000,000
   b. 49,000
   c. 800,020
   d. 4,500,000
   e. 125,000
   f. 88,060

Have you ever seen house prices written as $1.2m or a salary advertised as $52K? The media often use abbreviations to make it easier for people to read and understand large numbers. A price followed by m means ‘million’, a price followed by b means ‘billion’ and a price followed by K means ‘thousand’.

3. Match the numbers in parts **a** to **d** with the numerals listed in **i** to i**v**.

a. $85K	i. $85,000,000
b. $8.5m	ii. $85,000
c. $850K	iii. $8,500,000
d. $85m	iv. $850,000

4. Aaron is selling his house. The price is $760K. Write the price in words.

The symbols < and > and = can be used to describe the relative size of numbers.

Symbol	What it means
<	Smaller than or less than
>	Bigger than or greater than
=	Same or equal

5. Place one of the above symbols between the pairs of numbers to make the statements true.
   a. 3K 4,000
   b. 25,000,000 5m
   c. 1.3m 1,300,000
   d. 3.4m 4000K
   e. 9899 9989
   f. 9K 9009

6. Are the following statements true or false?

a. $25 \times 2500 > 7\text{K}$

b. $2 \times 3 + 5 = 2 + 3 \times 5$

c. $5\text{m} \div 200 > 25\text{K}$

d. $3\text{K} \times 1000 = 3\text{m}$

e. $8\text{K} \times 1000 = 9\text{K}$

f. $6\text{K} - 4 = 2\text{K}$

Lotteries

Gambling has been frowned upon by many people for a long time. More than 2,000 years ago, Aristotle wrote about the harmful effects of cheating and gambling. In particular, he was concerned by using unfair, biased dice. In 1661, the Government of England passed its first law against gambling and since then most other governments have passed laws against gambling in one form or another. However, many countries that otherwise disapprove of gambling allow lotteries to raise funds for such things as hospitals and education.

Here are the results of a special charity lottery that had 200,000 tickets.

Official Lottery Results – Special Charity Lottery

1st prize of $250,000: ticket number 34,603
'Just Wishing' syndicate, NFP, Toowoomba

One number off 1st prize: $1000 each
34,602 34,604

2nd prize of $50,000: ticket number 30,432
'New House', K. Ross, Hebbard Street, Buderim

3rd prize of $5,000: ticket number 33,800
J. Smith, Bishop Hale Road, Beaudesert

2 prizes of $1000

46,751	85,845

6 prizes of $500

1,567	58,946	64,883
72,040	92,272	99,977

12 prizes of $250

1,193	1,933	27,224	45,986
50,740	59,158	77,125	94,084
95,404	105,169	128,026	157,028

56 prizes of $100

207	30,012	68,487	95,775	144,870	164,121	179,867	191,006
984	33,393	70,475	100,027	146,892	168,345	181,246	192.203
5,941	35,062	73,533	101,188	150,052	169,266	184,711	193,777
10,655	37,011	74,321	105,932	150,459	170,020	186,023	194,210
13,511	50,484	89,875	120,073	154,292	172,113	186,554	195,442
14,409	51,644	91,147	133,534	155,414	174,998	189,225	197,533
17,529	52,222	93,899	143,617	156,541	178,778	189,892	199,804

Exercise 2

Use the lottery results to answer the questions in this exercise.

1. What is the ticket number that won first prize?

2. What is the value of the first prize?

3. ‘Just Wishing’ syndicate won first prize. What is a syndicate?

4. What do the letters ‘NFP’ after the ‘Just Wishing’ syndicate mean? (Research and choose the most appropriate one)

5. Who won second prize?

6. What do you think the winners of the second prize plan to do with the money they won?

7. Jane Smith wont third prize. What is Jane’s address?

8. Ella has ticket 72,040. What prize did she win?

9. How much did each of these people win?
   a. Ethan, ticket 172,113
   b. India, ticket 34,602
   c. Hayden, ticket 170,200
   d. Shaye, ticket 50,740

10. There were 200,000 tickets sold in this special charity lottery. Brae bought one ticket. When he bought the ticket, what was the probability that he:
    a. would win first prize?
    b. wouldn't win first prize?

11. How many prizes were there in the lottery?

12. How many 'losing tickets' (tickets that didn't win any prize) were there in the lottery?

13. What percentage of the tickets in the lottery didn't win a prize?

14. Each ticket in the lottery cost $5.
    a. Find the total value of the tickets sold in the lottery.

    b. Calculate the total value of the prize money.

    c. What percentage of the value of the ticket sales was returned in prize money?

    d. It cost $58,000 to organise and administer the lottery. How much money did the lottery make for the charity?

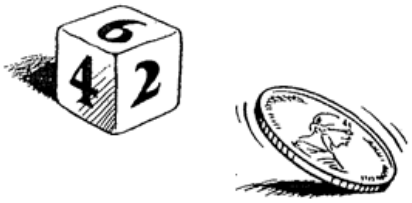
Calculating Total Number of Possibilities

You can't rely on intuition in probability. There are several ways to systematically work out the total number of possibilities in a probability problem. Drawing a grid is often a good approach, particularly when a pair of dice are involved.

Example 2

Rebecca tosses a die and a coin. What is the probability she tosses:

a. a 4 and a head?
b. a 4 or a head?



Solution

It is important to know the shorthand notation P(4 and H) is often used instead of writing 'the probability of a 4 and a head'.

When Rebecca tosses the die, it can show any one of the numbers 1, 2, 3, 4, 5 or 6. A head or a talk are the possibilities for flipping a coin.

First, we need to create a grid that shows all the possibilities for tossing a die and a coin.

	1	2	3	4	5	6
Head	H and 1	H and 2	H and 3	H and 4	H and 5	H and 6
Tail	T and 1	T and 2	T and 3	T and 4	T and 5	T and 6

This table shows that there are 12 possibilities.

a. $\mathrm{P}(4 \text{ and } \mathrm{H}) = \frac{1}{12}$ because there is only one way out of 12 possibilities that a 4 and a head can show.

b. $\mathrm{P}(4 \text{ or } \mathrm{H}) = \frac{7}{12}$ because there are seven ways this can happen. T and 4, H and 1, H and 2, H and 3, H and 4, H and 5, H and 6.

In probability, questions including the word '**or**' usually means '**one or the other or both**'.

Exercise 3

1. Jonah tosses a coin and an eight-sided die. The die has the numbers 1 to 8 on it.

   a. Complete this table showing all the possibilities.

	1	2						
Head								
Tail								

   b. What is the probability that Jonah tosses:

   i. a 7 and a tail?

   ii. a 7 or a tail?

   iii. a number greater than 5 and a head?

   iv. a head and an even number?

2. A pair of normal dice are used in a board game. Players add the two numbers showing on the dice to determine the score.

   a. Complete this grid showing all the possible totals.

	1	2	3	4	5	6
1						
2			5			
3						
4			7	8		
5						
6		8			11	

b. What is the probability of scoring a total of:
   i. 5? ii. 9?

   iii. 11? iv. 12?

c. Why is 7 considered a lucky number?

d. Which is more likely: a sum of 8 or a sum of 10?

3. A debating team consists of two students from Year 11 and three students from Year 12. If one member of the team is picked at random to be the opening speaker. What is the probability that the student:
   a. comes from Year 11?

   b. does not come from Year 11?

4. When three coins are tossed together the probability of three heads showing is $\frac{1}{8}$. What is the probability of something other than three heads will show?

Tree diagrams

Tree diagrams are another way to systematically list all the possibilities.

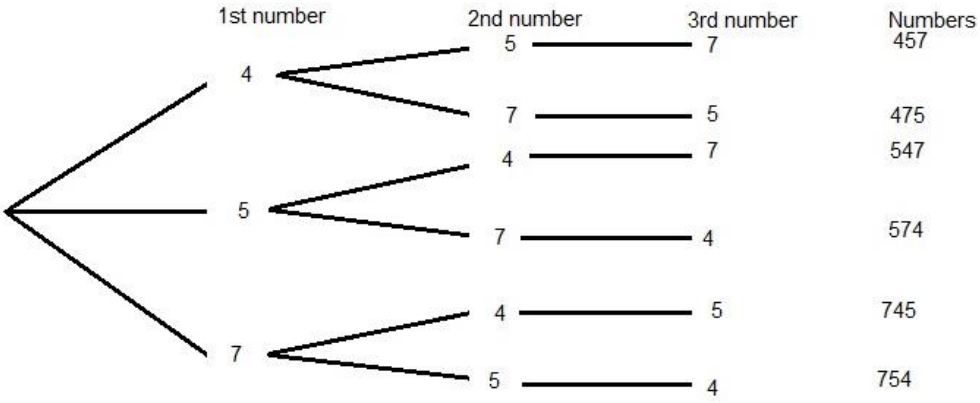
Example 3

Chloe places these three cards on the table to make a three-digit number.

a. How many three-digit numbers can she make?
b. What is the probability she will make 457?

Solution

a. From the tree diagram, we have six different three-digit numbers possible: 457, 475, 547, 574, 745, 754



b. $\mathrm{P}(457) = \frac{1}{6}$

Exercise 4

1. Three coins are tossed at the same time. Mathematically, this is the same as tossing one coin three times.
    a. Construct a tree diagram to show all possible outcomes.

b. How many possibilities are there when the three coins are tossed?

c. Calculate the probability of these events when a coin is tossed three times
   i. P(3 heads) ii. P(2 heads then a tail)

   iii. P(2 heads and a tail in any order) iv. P(at least 1 head)

2. There are four coloured discs in a bag. Two of the discs are black, one is white and the other is red. Zachary is going to take a disc from the bag at random and then take another disc from the bag at random **without** putting the first disc back.
   a. Draw a tree diagram to represent all possibilities. For the first 'branch' there should be four possibilities: black, black, white, red.

   b. What is the probability that Zachary will take:
      i. a black then a red disc ii. at least one black disc

      iii. a black and blue disc in any order

3. Two students from Laura, Jack, Lara, and Will are going to be chosen at random to ask the teacher for help.
    a. Construct a tree diagram to show that there are 12 possible outcomes of selecting two students.

    b. What is the probability of Laura being the first student chosen and Jack the second student chosen?

    c. Explain why the probability of Lara and Will being chosen is $\frac{1}{6}$.

4. Margaret and Steve are planning to have three children.
    a. If the probability of a boy is equal to the probability of a girl, what is the probability that their first child will be a girl?

    b. Draw a tree diagram to show all possible combinations of boys and girls. Each branch should represent the gender of the child (first branch = first child).

c. Use the tree diagram to help you calculate the probability that Margaret and Steve will have:

i. three sons

ii. two sons then a daughter

iii. at least one daughter

iv. three children of the same sex

Counting the number of possibilities

There are two common methods of calculating the number of ways items can be lined up (when the order is important).

The **first method** is to systematically list all the possibilities. The **second method** uses numbers in boxes.

Example 4

How many ways can Dylan, Kai, and Noah line up in a row?

Solution

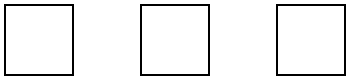
First Method:

They could line up: Dylan, Kai, Noah
Dylan, Noah, Kai
Kai, Dylan, Noah
Kai, Noah, Dylan
Noah, Dylan, Kai
Noah, Kai, Dylan

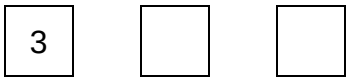
There are six different ways.

Second Method:

Use three boxes to represent the three positions in the line.



Put three in the first box to show that there could be three people who could be first.



After one of the three people is in the first position, there are two people left to fill the second position. Put two in the second box.

3	2	

There is now only one person left to fill the third position. Put one in the third box.

3	2	1

To calculate the total number of possibilities, multiply the numbers in the boxes together.

3	x	2	x	1	=	6

Exercise 5

1. Four friends are going to sit in a row at the movies.
    a. Complete the boxes to calculate the ways they can sit.

    b. How many ways can they sit?

    c. What is the probability of correctly predicting the order in which they end up sitting?

2. Sixty-five people have entered the Mt Ainslie Run Up and Power Walk fun run. Use the box method to calculate the number of ways first and second place can be filled.

3. Rihanna is going to put six new books on a shelf on her bookcase. Use the box method to calculate the number of ways she can order the books.

4. How many different ways can 10 people be arranged on 10 seats in a row? (One person per seat)

5. Eight swimmers compete for first, second and third place in the Olympics. How many ways can these three positions be filled?

6. When two friends go to their favourite Thai restaurant, they always choose three dishes to share. They choose one of the four different types of rice and noodles, one of the eight different beef dishes and one of the six vegetable dishes. How many different meals can they choose?

Portfolio Task Week 3 and 4

Standard ACT number plates have six characters. It was originally the letter Y followed by two letters then three digits. It has recently been changed to two letters, two digits and then a letter. Choose either the original or new style of number plates to answer the following questions. Make sure you show your working.

1. Things to know before you begin. You may need to have a look at the carparks.
    a. Can letters and digits be repeated?

    b. Which letters in the alphabet are allowed?

    c. Which digits are allowed?

    d. Which style did you choose?

2. How many different car number plates are there?

Y					

3. How many number plates would be possible if the system was changed to six letters and no numbers?

Y					

4. What is the probability of being given the number plate YYY 999?

5. What is the probability of being given a number plate with three Ys? (YYY ???)

MARKING RUBRIC

CRITERIA	EXPECTATIONS	POSS	MULT	GIVEN	TOTAL
Practical	Student completes practical work, including exercises of the brief to an acceptable standard set by the teacher.	**2**	**3**		**/6**
Portfolio Task	Student completes the portfolio task of the week to an acceptable standard set by the teacher.	**2**	**2**		**/4**
Reasoning and Communications	Student responses are accurate and appropriate in presentation of mathematical ideas, with clear and logical working out shown.	**4**	**-**		**/4**
Concepts and Techniques	Student submitted work selects and applies appropriate mathematical techniques to solve practical problems and demonstrates proficiency in the use of mathematical facts, techniques and formulae.	**4**	**-**		**/4**
	Submission Guidelines				
Timeliness	Student submits the exercises and portfolio tasks by the set deadline. See scoring guidelines for specific details.	**2**	**-**		**/2**
				FINAL	**/20**

Student Reflection:
How did you go with this week's work? What was interesting? What did you find easy? What do you need to work on?