2020 MA3 Week 5 Investigation

The Fibonacci and Lucas Sequences

Leonardo Fibonacci of Pisa was a mathematician in the 12th century, Italy. He discovered a number series from which one can derive the Golden Mean by charting the population of rabbits. Here is the beginning of the sequence:

1, 1, 2, 3, 5, 8,13, 21, 34, …

The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842-1891). The sequence named after him is closely related to the Fibonacci sequence.

These sequences are defined recursively by:

$$F_1 = 1, F_2 = 1, F_n = F_{n-1} + F_{n-2}, \text{for } n \geq 3$$

$$L_1 = 1, L_2 = 3, L_n = L_{n-1} + L_{n-2}, \text{for } n \geq 3$$

Each number is the sum of the two proceeding numbers.

1. Write out the first 12 terms of each sequence.
2. Explain why every third term of each sequence is even and the rest are odd.
3. Write out the sequence $L_1 + F_1, L_2 + F_2, L_3 + F_3, \ldots$ and $L_1 - F_1, L_2 - F_2, L_3 - F_3, \ldots$
4. How do the two sequences relate to the Fibonacci sequence?