WEEK 10 NOTES & EXERCISES

Substitution in Formula

A **formula** is a rule which describes a relationship between variables. For example: Area = length × width is the formula used to calculate the area of a rectangle. It is usually written as $A = lw$. In this case A is the subject of the formula. If we know l and w, we can find the area by substitution.

Example From a height h above sea level, an observer can see a distance of d km to the horizon, where

$$d = 8\sqrt{\frac{h}{5}}$$

What distance to the nearest km, can be seen from a tower 128 m above sea level.

$d = 8\sqrt{\frac{h}{5}} = 8\sqrt{\frac{128}{5}} = 40.4771\ldots. = 40$ km to the nearest km.

Exercise Set 1

Note: Make sure you use appropriate units as part of your answer.

Q1. Calculate the volume, correct to two decimal places, of a cylinder with a base radius of 4.07 cm and a perpendicular height of 11.58 cm. Use the formula $V = \pi r^2 h$

Q2. The temperature T (in ℃) of the water in a kettle t minutes after it is switched on is given by the formula

$T = 18t + 28$. Find the temperature of the water;

a) 4 minutes after it is turned on

b $1\frac{1}{2}$ minutes after it is turned on

c) When the kettle is first turned on.

Q3. The volume of a sphere with radius r is $V = \frac{4}{3}\pi r^3$. Calculate, correct to one decimal place, the volume of the sphere with radius 14.5 cm.

Q4. The time, T seconds, it takes a swing to go back and forth once is $T = 2\pi\sqrt{\frac{l}{g}}$, where l m is the length of the swing and g is the gravitational acceleration. Find T, correct to two decimal places, if l = 2.35 and g = 10.

Q5. The speed, V m/s, required for a spacecraft to escape the Earth's gravitational pull during take-off is $V = \sqrt{2gr}$ where g is 9.8 m/s^2 and r is the radius of the Earth (6378000 m). Calculate the escape speed of a spacecraft leaving the Earth's atmosphere.

Using Formulae in Medicine

Nurses regularly use formulas in their work. They calculate such things as:

- Quantities to mix to make a solution for cleaning wounds and infections
- The amount of liquid to put in an injection
- **The number of tablets or capsules to give a patient.**

Example: To solve the following problem, use the formula:

Amount required = $\frac{R}{A} \times v$

Alan is suffering from severe kidney stone pain. His doctor prescribes 75 mg of pethidine. The ampoules of pethidine contain 100 mg (A) per 2 mL (v). Calculate the number of millilitres needed for the injection.

Amount required = $\frac{R}{A} \times v$

Substitute the values from the question: R = 75, A = 100, v = 2

Amount required = $\frac{75}{100} \times 2$

= 1.5 mL

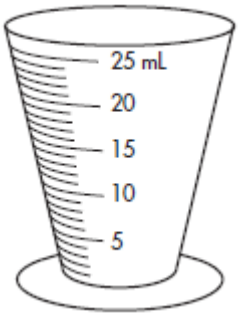
Exercise Set 2

Q1. Ten-year-old Claudia is suffering from nasal and sinus congestion. She is prescribed 12 mg of Demazin syrup three times per day. Demazin syrup contains 3.75 mg (A) in every 5 mL (v).

Amount required = $\frac{R}{A} \times v$

a) How many millilitres must Claudia be given in each dose?

b) Mark on this diagram the volume of Demazin Claudia must be given.

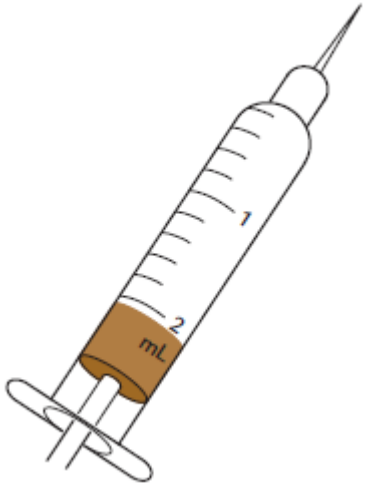


c) Claudia has the first dose at 1015 hours. When can she have the next dose?

Q2. To help thin Kim's blood after a blood clot the doctor prescribes 15000 units ® of Heptin. Each ampoule contains 25000 units (A) in 1 mL (*v*).

a) Using the formula as before calculate the dose Kim requires.

b) Mark on the diagram the level of Heptin Kim requires.



Q3. Fried's rule can be used to calculate an infant's dose of an adult's medicine.

Dosage = $\frac{Age\ of\ infant\ (months) \times adult\ dosage}{150}$

The adult dose of a medicine is 20 mL. Calculate the dosage for these children.

a) Cathy, aged 18 months

b) Todd, aged 2 years

Q4. The medical researchers, Young and Clark, developed rules for approximating the children's dose of adult's medicine.

Young's rule: Dosage = $\frac{Age\ of\ child\ (years) \times adult\ dose}{Age\ of\ child\ (years) + 12}$

Clark's rule: Dosage = $\frac{Weight\ of\ child\ (kg) \times adult\ dose}{70}$

Emma is 8 years old and she weighs 35 kg. She is short and heavy for her age. The adult dose of the medicine her mother wants to give her is 20mL.

a) How much medicine should Emma have according to;
   (i) Young's rule
   (ii) Clark's rule

b) The rules recommend Emma receive different amounts. Do you think Emma's mother should use the formula based on age or weight? Why?

Q5. A boy's height on his second birthday can be used to predict his adult height.

Predicted adult height (cm) = $1.75 \times h + 20$ where h = boy's height on 2nd birthday.

On his second birthday Thomas is 88 cm tall. Predict Thomas's adult height.

Solving Equations

An equation is a mathematical sentence. Equations always include an equal sign. In our previous work we simplified algebraic expressions but now we will **solve** equations.

Eg $3w - 4$ is an algebraic expression, however, $3w - 4 = 6$ is an algebraic equation.

Example Solve the equation $2x - 6 = 14$

Write the equation.	$2x - 6 = 14$
Add 6 to both sides.	$2x - 6 + 6 = 14 + 6$
Simplify both sides.	$2x = 20$
Divide both sides by 2.	$\frac{2x}{2} = \frac{20}{2}$
Simplify to find the value of x.	$x = 10$

Solve the equation $\frac{x}{3} + 2 = 7$

Write the equation.	$\frac{x}{3} + 2 = 7$
Subtract two from both sides.	$\frac{x}{3} + 2 - 2 = 7 - 2$
Simplify both sides.	$\frac{x}{3} = 5$
Multiply both sides by 3.	$\frac{x}{3} \times 3 = 5 \times 3$
Simplify to find the value of x.	$x = 15$

Solve the equation $\frac{2a\ 6}{5} = 4$

Write the equation.	$\frac{2a-6}{5}=4$
Multiply both sides by 5.	$\frac{2a-6}{5}\times 5=4\times 5$
Simplify both sides.	$2a-6=20$
Add 6 to both sides.	$2a-6+6=20+6$
Simplify both sides.	$2a=26$
Divide both sides by 2.	$\frac{2a}{2}=\frac{26}{2}$
Simplify to find the value of a.	$a=13$

Exercise Set 3

Q1. Solve these equations

a) $3d + 2 = 20$

b) $2p - 3 = 2$

c) $4x + 2 = 22$

d) $28 - 5x = 3$

e) $\frac{3h}{9} = 9$

f) $\frac{r-1}{6} = 2$

g) $12 + 2w = -18$

h) $\frac{z}{3} - 11 = 9$

i) $10 - 3a = 16$

j) $\frac{8-2b}{2} = 7$

Solving Equations With Pronumerals on Both Sides

Example Solve $2a - 5 = a + 7$

Write the equation.	$2a-5=a+7$
Subtract a from both sides.	$2a-5-a=a+7-a$
Simplify both sides.	$a-5=7$
Add 5 to both sides.	$a-5+5=7+5$
Simplify to find the value of a.	$a=12$

Exercise Set 4

Q1. Solve these equations.

a) $2x - 5 = x + 4$

b) $5k - 13 = 3k + 9$

c) $2(m - 4) = m + 12$

d) $\frac{2a-8}{5} = 2a$

Q2. Change these sentences into equations and solve.

a) Twice the number less 6 equals the sum of the number and 4.

b) Eight times the number plus 7 equals 14 less than 5 times the number.

c) Three times the difference between the number and 1 equals twice the number plus 8.