

Review of Algebra

As in most areas of mathematics, algebra makes use of words that have a specific meaning. Some of these are;

Expression: A mathematical **expression** is any calculation or formula that involves a combination of numbers and/or variables, as well as operators eg $4x$ and $A = 2\pi r$ are both expressions.

Variable: A symbol for a number we don't know yet. It is usually a letter like x or y . Example: in $x + 2 = 6$, x is the variable.

Coefficient: The numbers in front of the variable eg the 4 in $4x$

Constant terms: Are terms that do not have a variable.

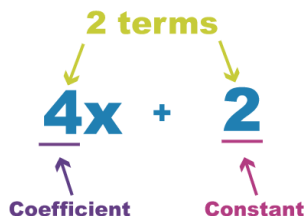
Like terms: **Like terms** are terms which have exactly the same variable factors eg $2x$ and $4x$ are like terms as they both contain x and b^2 and $\frac{1}{2}b^2$ are like terms as they both contain b^2

Note: Algebraic terms must have the **EXACT SAME** combination of variables to be like terms. $9ef^2$ and $10ef^2$ are like terms because they both have variables of e and f^2

xy and xy^2 ARE NOT like terms, however, because the powers of y are different between the two terms.

Simplify: To simplify an expression means to collect the like terms and then write as simply as possible.

This picture summarises some of this terminology:



There are a few conventions that we make when writing algebraic expressions

- When we multiply two numbers together, we use a multiplication sign, such as 2×3 . When we multiply a number by a variable, or when we multiply variables together, we leave out the multiplication sign. So 2 times $2 \times y$ is written as $2y$, for example, and $a2 \times b$ is written as a^2b .
- When we multiply a number by one or more variables, we write the number first and then the variables. For example, $p \times 3 \times q$ would be written as $3pq$.
- If we multiply one or more variables by 1, we can leave off the 1. For example, instead of writing $1x$ or $1 \times x$, we can just write x .
- We usually avoid using the division symbol, and instead write division using fractions. So instead of writing $12 \div t$, we would write $\frac{12}{t}$. This helps to avoid confusion about the order of operations in an expression.
- If we multiply a variable by itself, we usually simplify the expression by using an exponent. So if we have the expression $m \times m \times m$, we would write m^3 instead of mmm .

Examples

1.

In the algebraic expression $p - 5qr + 7$ identify the following:

- a the number of terms.
- b the coefficient of the term containing the variable p .
- c the constant term.

Solution

- a A term can be a number or a number and one or more pronumerals multiplied together. There are three terms in this expression.
- b A coefficient is the number that appears before a pronumeral within the same term. There is no number written in front of the pronumeral p . The coefficient is 1.
- c A constant is a term that contains a number only, there is no pronumeral. The constant term is 7.

2. Simplify $8x + 2y - 5x - 3y$

Rearrange the expression to group the like terms; $8x - 5x + 2y - 3y$

Collect the like terms: $3x - y$

3. Simplify $2b \times 6a \times 4c$

Multiply the numbers and pronumerals respectively: $48 \times b \times a \times c$

Write in simplest form; $48abc$

4. Simplify $28xy \div 21yz$

Write as a fraction $\frac{28xy}{21yz}$

Both 28 and 21 are divisible by 7, so $\frac{28}{21}$ becomes $\frac{4}{3}$ and $\frac{y}{y} = 1$

This gives $\frac{28xy}{21yz} = \frac{4x}{3z}$

Note: $\frac{x^2}{x} = x$ and $\frac{c}{c^2} = \frac{1}{c}$

Exercise Set 1

Q1. For the algebraic expression $3m - 4n + 6$ identify;

- a) the number of terms
- b) the coefficient of the term with variable m
- c) the constant term.

Q2. Simplify

- a) $3x - 2y + 6x - y$
- b) $4ab - 6a - 9ab$
- c) $7b - 9a + 3b - 4$
- d) $8xy + 3x - 4xy - 6$
- e) $15x - 15 + 3x + 5$
- f) $3de - 6ed + 2d$
- g) $-5 + 7u - 10 - 3u$
- h) $3r + 4ar - 2ar - 7r$

Q2. Simplify

- a) $3x \times 5y$
- b) $-4a \times 2bc$
- c) $12mn \times -3n$
- d) $-5xy \times 3yz$

e) $4ak \times 4am$

f) $8cd \times -3d$

g) $10ab \times 2ab$

h) $-4bc \times -3cd$

Q3. Simplify

a) $\frac{12xy}{3y}$

b) $\frac{-30mn}{6np}$

c) $\frac{48st}{16rt^2}$

d) $\frac{14yx^2}{2xy}$

e) $\frac{27pr}{-9pq}$

f) $\frac{-4de}{20e}$

g) $\frac{-32abc}{-4bc^2}$

h) $\frac{45yx^2}{9x}$

Q4. If $5x^2 - 10x - 3x^2 - 12x = px^2 + qx$, find the values of p and q

Expanding and Simplifying Algebraic Expressions

Expanding an algebraic expression means that the grouping symbols (brackets) are removed. The distributive law is used which means that every term in the brackets is multiplied by the term outside the brackets.

$$\begin{aligned}a(b + c) &= a \times b + a \times c \\ &= ab + ac\end{aligned}$$

Remember that multiplication signs are omitted in algebra.
 $a \times b$ means ab

Example

Expand $-5t(3t + 8)$

Using the distributive law gives $-5t \times 3t + -5t \times 8$

which simplifies to $-15t^2 - 40t$

Exercise Set 2

Q1. Expand and simplify

a) $r(r + 5)$

b) $m(1 - m)$

c) $6y(5y - 10)$

d) $2y(7x + 4y)$

e) $y(2y + 3 - y^2)$

f) $3h(h - 7e - 4eh)$

Example

Expand and simplify $3(a - 2b) - 5(2a + b)$

Expand each bracket $3a - 3 \times 2b - 5 \times 2a - 5 \times b$

Rearrange so like terms are together $3a - 10a - 6b - 5b$

Simplify $-7a - 11b$

Q2. Expand and simplify

a) $5(x + 4) - 2(x + 3)$

b) $3(d - 4) - 2(d + 5)$

c) $(2x - 4) - 5(3x + 5)$

d) $3b(b + 5) - b(b + a)$

e) $4w(w - 7) - w(w + 1)$

f) $2(a - b) + 2(b + a)$

g) $e(3e + 5) - (2e - e^2)$

g) $p(p - q) - q(q - p)$

Substitution

Substitution occurs when we **substitute** numbers into equations in place of variables to determine a final value. We can substitute in any kinds of numbers, including whole numbers, decimals and fractions.

Example

Evaluate $6x - 4$ if $x = 3$. This means that everywhere the letter x has been written, we will replace it with the number 3.

$$6x - 4 = 6 \times 3 - 4 = 18 - 4 = 14$$

The same process applies even if there is more than one unknown value. Evaluate

(i) $2pq - r$ if $p = 2$, $q = -3$ and $r = 8$

$$(2pq - r) = 2 \times 2 \times -3 - 8 = -12 - 8 = -20$$

(ii) $5p(3q - qr)$

$$5p(3q - qr) = 15pq - 5pqr = 15 \times 2 \times -3 - (5 \times 2 \times -3 \times 8) = -90 - (-240) = -90 + 240 = 150$$

Exercise Set 3

Q1. If $m = 6$, $n = 3$ and $p = -4$ evaluate the algebraic expressions.

a) $4nm$

b) $2np - m$

c) $p^2 + mn$

d) $3mn - 2p$

e) $2n^2 + 3mp$

f) $-5p + 2mn$

g) $24mp - n$

g) $p^2 - n^2 + m^2$