# Algebra

## Linear Equations and Algebraic Modelling

In 1905, the German scientist Albert Einstein proposed a new theory of physics for small particles of matter (such as atoms) moving at very high speeds. Aged only 26 years, he hypothesised in his famous Theory of Special Relativity that matter (mass) could be converted to large amounts of energy. Einstein's mass–energy relation was summarised by the now well-known formula E = mc2 where E stands for energy, m for mass and c for the speed of light (300 000 000 m/s). Einstein's new model revolutionalised conventional laws of physics and led to the development of nuclear energy.

A criminologist studying the effects of increasing police patrols on the streets of a big city noticed the following pattern.

| **No. of police** | 50 | 150 | 250 | 300 |
|---|---|---|---|---|
| **No. of crimes** | 3100 | 2800 | 2500 | 2350 |

Is it possible for the criminologist, like Einstein, to find a general mathematical rule to fit this pattern? Algebra is the branch of mathematics that uses symbols called pronumerals to represent number patterns and relationships.

Algebraic modelling is the study of patterns and relationships occurring in nature, industry and society and the formulation of a mathematical rule or 'model' to describe such relationships.

An algebraic expression is a general statement involving pronumerals (also called variables). Pronumerals are letters of the alphabet that stand for numbers. An algebraic expression is made up of terms.
For example, 3x2 − x + 10 has 3 terms: 3x2, −x and 10.

## SOLVING EQUATIONS

An equation contains an algebraic expression and an equals (=) sign. For example, 3x − 4 is an expression, but 3x − 4 = −13 is an equation. To solve an equation, we find the value of the pronumeral that makes the equation true. The process of solving an equation requires the use of inverse (opposite) operations.

To solve an equation you can

- add the same number to both sides of the equation
- subtract the same number from both sides of the equation
- multiply each side of the equation by the same number.
- divide each side of the equation by the same number.

## One-Step Equations

The simplest equations are called one-step equations as they take one step t solve.

**Example**

Solve these equations **a** $x + 9 = 17$ **b** $p - 12 = 30$

**a**
$$x + 9 = 17$$
$$x + 9 - 9 = 17 - 9$$
$$x = 8$$

*Check:*

If $x = 8$, $x + 9 = 8 + 9 = 17$ ✓

**b**
$$p - 12 = 30$$
$$p - 12 + 12 = 30 + 12$$
$$p = 42$$

*Check:*

If $p = 42$, $p - 12 = 42 - 12 = 30$ ✓

**Example**

Solve these equations **a** $3x = 18$ **b** $7x = 42$

**a**
$$3x = 18$$
$$\frac{3x}{3} = \frac{18}{3}$$
$$x = 6$$

*Check:* If $x = 6$, $3x = 3 \times 6 = 18$ ✓

**b**
$$7x = 42$$
$$\frac{7x}{7} = \frac{42}{7}$$
$$x = 6$$

*Check:* If $x = 6$, $7x = 7 \times 6 = 42$ ✓

**Example**

Solve these equations **a** $\frac{x}{2} = 5$ **b** $\frac{p}{8} = 6$

**a**
$$\frac{x}{2} = 5$$
$$2 \times \frac{x}{2} = 5 \times 2$$
$$x = 10$$

*Check:* If $x = 10$, $\frac{x}{2} = \frac{10}{2} = 5$ ✓

**b**
$$\frac{p}{8} = 6$$
$$8 \times \frac{p}{8} = 6 \times 8$$
$$p = 48$$

*Check:* If $p = 48$, $\frac{p}{8} = \frac{48}{8} = 6$ ✓

Exercise Set 1

Q1. Solve these equations by adding or subtracting.

a) $p + 8 = 16$ b) $d - 3 = 7$ c) $a + 16 = 31$

d) $f - 28 = 36$ e) $x + 21 = 48$ f) $r + 48 = 49$

Q2. Solve these equations by dividing.

a) $5m = 25$ b) $6p = 30$ c) $18b = 90$

d) $2x = -16$ e) $5q = -25$ f) $9c = -72$

Q3. Solve these equations by multiplying.

a) $\frac{x}{7} = 2$ b) $\frac{d}{10} = 7$ c) $\frac{y}{3} = 12$

d) $\frac{m}{8} = 8$ e) $\frac{p}{3} = -4$ f) $\frac{z}{5} = -6$

## Two-Step Equations

The techniques needed to solve two step equations are the same as for one step equations except two techniques need to be applied. Always add or subtract first to get the pronumeral on one side of the equation and the numbers on the other side. Then multiply or divide to find the answer.

**Example**

Solve these equations **a** $5x + 7 = 37$ **b** $8x - 9 = 79$

**a**

$$5x + 7 = 37$$
$$5x + 7 - 7 = 37 - 7 \quad \{\text{subtract } 7 \text{ from both sides}\}$$
$$5x = 30$$
$$\frac{5x}{5} = \frac{30}{5} \quad \{\text{divide both sides by } 5\}$$
$$x = 6$$

$$5x + 7 = 37$$
$$(5 \times 6) + 7 = 37$$
$$30 + 7 = 37$$
$$37 = 37$$
$\therefore$ correct ✓

**b**

$$8x - 9 = 79$$
$$8x - 9 + 9 = 79 + 9 \quad \{\text{add } 9 \text{ to both sides}\}$$
$$8x = 88$$
$$\frac{8x}{8} = \frac{88}{8} \quad \{\text{divide both sides by } 8\}$$
$$x = 11$$

$$8x - 9 = 79$$
$$(8 \times 11) - 9 = 79$$
$$88 - 9 = 79$$
$$79 = 79$$
$\therefore$ correct ✓

**Exercise Set 2**

Q1. Solve the following two-step equations.

a) $3x - 9 = 3$ b) $4p + 8 = 24$ c) $6x + 7 = 61$

d) $2x - 32 = 16$ e) $10c - 30 = 100$ f) $7t - 8 = 27$

g) $4q - 21 = 115$

h) $11s + 16 = 115$

i) $7p + 33 = 117$

Q2. Solve for $x$

a) $6 - x = -5$

b) $3 - 4x = 15$

c) $11 = 3 - 2x$

d) $-1 = 2x + 6$

e) $-15 = 3 - 6x$

f) $3x + 6 = 0$

Q3. Solve for x

a) $\frac{x}{2} + 3 = 8$

b) $\frac{x}{3} - 1 = 4$

c) $\frac{x}{10} - 6 = -1$

**Example**

**a** $5x + 2 = 3x - 5$ **b** $15 - 2x = 11 + x$

**a**

$$5x + 2 = 3x - 5$$

$\therefore \quad 5x + 2 - 3x = 3x - 5 - 3x$ {subtracting $3x$ from both sides}

$\therefore \quad 2x + 2 = -5$

$\therefore \quad 2x = -7$ {subtracting $2$ from both sides}

$\therefore \quad x = -\frac{7}{2}$ {dividing both sides by $2$}

$\therefore \quad x = -3\frac{1}{2}$

**b**

$$15 - 2x = 11 + x$$

$\therefore \quad 15 - 2x + 2x = 11 + x + 2x$ {adding $2x$ to both sides}

$\therefore \quad 15 = 11 + 3x$

$\therefore \quad 15 - 11 = 11 + 3x - 11$ {subtracting $11$ from both sides}

$\therefore \quad 4 = 3x$

$\therefore \quad x = \frac{4}{3} = 1\frac{1}{3}$ {dividing both sides by $3$}

**Exercise Set 3**

Q1. Solve the following with integer solutions.

a) $6x + 2 = 2x + 18$

b) $3x + 7 = 12 - 2x$

c) $5x - 9 = 11 + 7x$

d) $9 - 2x = 3 - 5x$

Q2. Solve the following with fraction solutions.

a) $6x + 7 = 4x - 2$

b) $2x - 3 = 5x + 7$

c) $11a - 7 = 5a + 15$

d) $9x - 4 = 3 + 5x$

## Solving Literal Equations

A literal equation is an equation whose solution will be expressed in terms of another variable rather than a number.

For example:

- $2x + 3 = 13$ is a linear equation in one unknown, whose solution for $x$ is the number 5.
- $x + 2y = 6$ is a literal equation in two unknowns, whose solution for $x$ is $6 - 2y$

**Example**

If $2a - 4b = 8$, find an expression for $a$

Steps

- Add 4b to both sides of the equation

  $2a - 4b + 4b = 8 + 4b$ gives

  $2a = 8 + 4b$
- Divide both sides by 2 which gives

  $a = 4 + 2b$

If $3x + 2y = 4$ find expressions for both $x$ and $y$

For $y$

- Subtract $3x$ from both sides of the equation

  $3x + 2y - 3x = 4 - 3x$ which gives

  $2y = 4 - 3x$

- Divide both sides by 2 which gives

  $y = 2 - \frac{3}{2}x$

For $x$

- Subtract $2y$ from both sides of the equation

  $3x + 2y - 2y = 4 - 2y$ which gives

  $3x = 4 - 2y$

- Divide both sides by 3 which gives

  $x = \frac{4}{3} - \frac{2}{3}y$

**Exercise Set 4**

Q1. Find an expression for $x$

a) $x + 7y = 22$

b) $x - 4y = 11$

b) $5y = x - 12$

d) $4y = 2x + 6$

e) $3x + 5 = 2y$

f) $5y + 2x = 10$

g) $4x + 12y = -24$

h) $2x - 3y = 12$

Q2. Find an expression for $y$

a) $7x + 5y = 40$

b) $mx + y = 9$

c) $ny + mx = d$

d) $sx - ty + a = 0$