RATES OF CHANGE

DEFINITIONS

RATIOS

Ratios compare two or more quantities which are measured in the same units.
E.g. If Kim has $2.40 and Taylor has 80 cents, the ratio of Taylor's money to Kim's money is 80c:$2.40 = 80:240 = 1:3. The ratio 1:3 can also be written as a fraction, $\frac{1}{3}$, and ratios are simplified in the same way as we simplify fractions. The units don't appear in the final answer.

RATES

Rates on the other hand, compare two quantities which are measured in different units.
e.g. if we travel 150km in 3 hours, what is the rate at which we travel? 150 km:3 hours = 50km/hr or 50 kph. Note we use the forward slash to stand for the word 'per'.

A **rate** compares how one quantity is changing when compared with another. Many rates measure what is happening to a quantity as time changes but there are others, like population/square km; tax payable/dollar earned; gm/litre (concentration).

http://www.virtualnerd.com/algebra-1/linear-equation-analysis/rate-of-change-definition.php

Hawker
11 Math Methods 8A
Q1, Q2, Q4, Q7

Melba
Maths Quest
Exercise 7A

This shouldn't take too long to do.

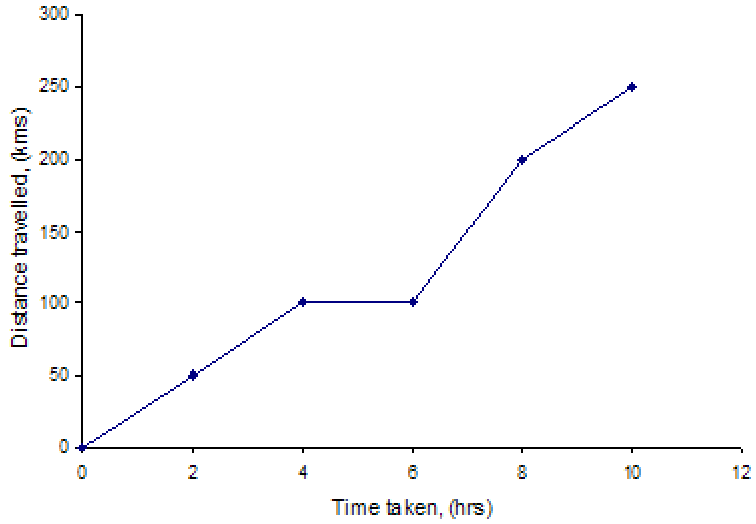
CONSTANT RATES OF CHANGE (STRAIGHT LINES)

If the change in one quantity when compared to the change in the other quantity (sometimes stated as " if the change in one quantity *with respect* to the change in the other") is always the same we have constant rates e.g. if we are travelling along the freeway at a constant speed or the rate at which a drip is delivered to a patient in hospital.

Rates of change are very important in mathematics. Take for example the 'speed' of a car. It is a measure of how far the car travels over a certain time, usually expressed in km/hr. It literally means how many kilometres the car will travel per hour. It could also be expressed in km/s or m/s or mm/day. The important thing is that it is a measure of how much the distance changes for a certain change (increase) in time.

For instance, 100 km/hr says that for every change in time of one hour, the distance travelled changes by 100 km.

If you graph the distance travelled by a car on the y-axis, and the time taken on the x-axis, for some journey you might get a graph like this:



Now, for this graph (and any graph of distance vs. time), the gradient at any point represents the *speed* of the car at that time. For instance, between hours 0 and 4, the car is travelling at 25 km/hr. Between hours 4 and 6, the gradient is 0, telling us the car is not moving. Then, between hours 6 and 8 the gradient is 50 km/hr. Finally, between hours 8 and 10 the gradient is again 25 km/hr. In general, the slope of a graph can tell us the rate of change of the y-axis variable, *with respect to* the x-axis variable.

What this is saying is that the rate of change of a linear function can be measured or calculated, by working out the gradient, or slope, of the line. We know that for straight line (linear) functions, the gradient can be worked out using $m = \frac{rise}{run} = \frac{y2-y1}{x2-x1} = \frac{\Delta y}{\Delta x}$.

These rates of change (the ones associated with linear functions) are called constant rates of change. As the value of the gradient calculated for these is always a constant (i.e. a number).

As rates of change are linked with the gradient of these straight lines we can see that:

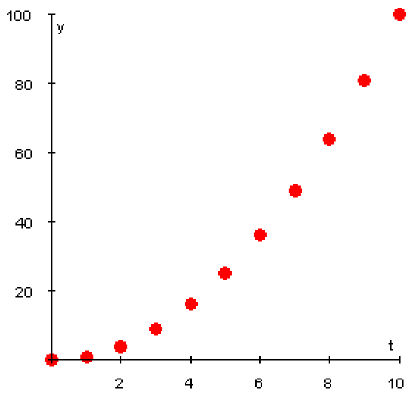
- When a rate of change is positive, the graph goes up from left to right.
- When the rate of change is negative, the graph goes down from left to right.
- When the rate of change is zero, the graph is horizontal.

Hawker
11 Math Methods 8B
Q1-3, Q6, Q8, Q12

Melba
Maths Quest
Exercise 7B

Most questions are easy but some might make you think for a while.

VARIABLE RATES OF CHANGE (CURVED LINES)



Consider the graph on the left. It may have been created by plotting the results of an experiment. By displaying these measurements in the form of a graph, we can start to ask questions about how changes in y are occurring and assign more concrete meanings to the notions of rates of change. It is much more convenient to do this on a graph than a table of values.

The reality is, that when we travel to Sydney on the highway, we don't always travel at a constant speed (rate). We slow down when we get behind some vehicles, we stop for petrol or food, we speed up to pass a slow vehicle.

Hawker
11 Math Methods 8C
Q1, Q2, Q4, Q5

Melba
Maths Quest
Exercise 7C

A fairly short exercise.

AVERAGE RATE OF CHANGE

When we have variable rates, we sometimes want to consider the *average rate of change*. This means we are considering the constant rate which would be equivalent to the variations in rate e.g. when calculating the average speed for our journey to Sydney we compare the distance travelled with the time taken (using division) – we will then know the constant speed which will cover the journey in the same amount of time.

Average rate of change of variable A with respect to variable B = $\dfrac{Change\ in\ A}{change\ in\ B}$

In the figure below, we have identified a point P on the graph, and a second point, also on the graph which will serve as example. Note that a straight line has been drawn, connecting these two points

By how much has the value of y changed between the two points?

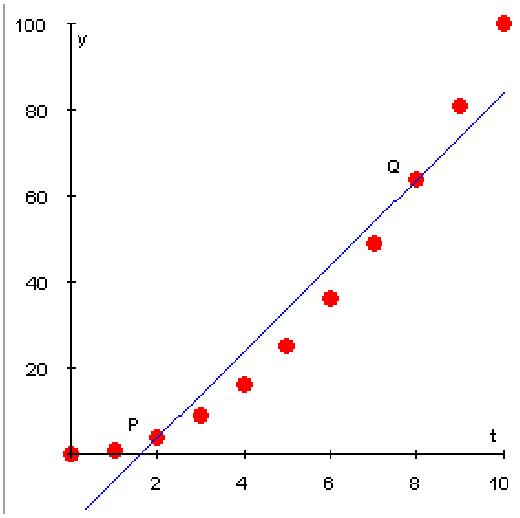
Over what period of time has this change occurred?

The ratio of these two values, i.e. (change in y)/ (change in t) is called the average rate of change in y.

Recall we used this language before... change in y/ change in x $\frac{y2-y1}{x2-x1} = \frac{\Delta y}{\Delta x}$. We are indeed finding the gradient of the line between the two points P and Q.

We call this the AVERAGE RATE OF CHANGE.

It is the rate of change of the line, connecting these two points.



The average rate of change is an important quantity which we can discuss without a graph. For instance, economists are interested in the average rate of change in unemployment over the past year.

However, once we draw our data points on a graph as above, we have an appealing geometrical interpretation of the average rate of change.

Average rate of change video with example: http://youtu.be/rWt2TdXE9BA

Hawker
11 Math Methods 8D
Q1, Q2, Q3, Q7, Q8, Q9

Melba
Maths Quest
Exercise 7C

Note the importance of the gradient of the chord in the worked examples.

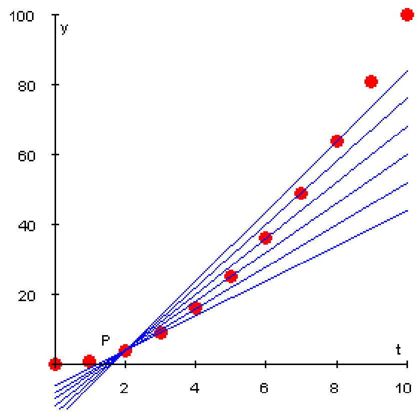
INSTANTANEOUS RATES OF CHANGE

Sometimes we want to know what the rate of change is at a particular instant in time. This relates to the gradient of the tangent of a relevant graph. At this stage we will be calculating the gradient of the tangent by estimating the position of the tangent and then calculating its gradient. Later, we will develop an accurate method for doing this calculation.

It is based on the ideas presented in http://www.youtube.com/watch?v=cLkROwEEyLM and http://www.youtube.com/watch?v=-e_kJlZZZZ8&feature=related and http://www.youtube.com/watch?v=yuEKC93wd_E .

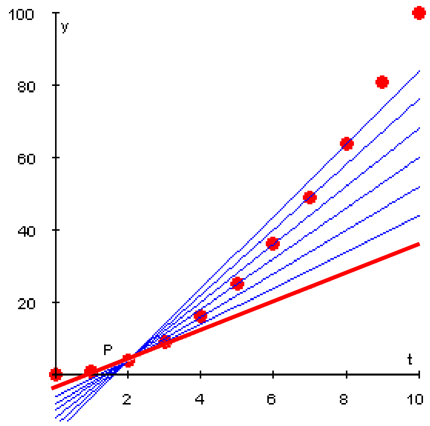
Of course, the value of the average rate that we computed depends not only on the point identified as P, but also on the second point Q on the graph. If we were to choose a different endpoint for the same calculation, i.e. a different point for Q, we would get a different average rate of change.

In the next picture, you can see what happens when the second point moves closer and closer to P. Notice that, as Q is chosen closer to P, the secant lines (shown in blue) have slopes that measure the change that take place very close to P.



Notice that the sequence of secant lines shown in the previous picture accumulate around a unique line through the point P. That line is called the *tangent line.*

It has been drawn here in red, together with the secant lines, to show their relationship.



The tangent line represents a 'limiting process' in which the average rate of change is calculated over smaller intervals around P.

Since this rate of change is obtained by measuring the average rate of change close to P, we can think of it as measuring an *instantaneous* rate of change, or the rate of change at a point.

Hawker
11 Math Methods Exercises 8E
Q1, Q2, Q5, Q6

Melba
Maths Quest
Exercise 7E

As you work through this chapter, take note of the "Remember" boxes before each set of examples.

POLYNOMIALS AND RATES OF CHANGE

This section will act as an introduction to the method we are going to introduce next week, which is used to calculate instantaneous rates of change, accurately.

We do this by finding the gradient of a secant through two very close points on the curve and using this as an approximation for the gradient of the tangent.

The worked examples and the video at http://www.youtube.com/watch?v=OGYYezEDP5s will help to explain this new idea.

Hawker
11 Math Methods Exercises 8I

Melba
Maths Quest
Exercise 7I

Make a summary of the work done in this chapter and remember to use the Chapter Review for preparation for the Test.