Week 13
Term 4
2023

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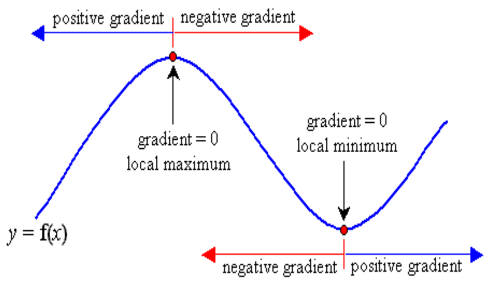
Learning Brief
SMM2

Goals



This fortnight we are:

- Locating stationary points of polynomial functions; f^' (x)=0
- Sketching graphs of polynomial functions using gradient function
- Indicating x-intercept and y-intercept on sketches
- Classifying the type of stationary points: local maximum or minimum turning point, point of inflection
- Using differentiation to find turning points and to solve problems in a practical context

Theoretical Components

Make notes on the following chapters and website:

Maths Quest 11 Mathematical Methods
- 9F - Sketching graphs containing stationary points
- 9G - Solving maximum and minimum problems

Knowledge Checklist:
- what is the x-intercept of a gradient function?
- finding gradient functions by sketching
- finding gradient functions by using the rule
- finding gradient functions using your CAS
- sketching polynomials
- applications of derivatives

Check out these videos:
- https://www.youtube.com/watch?v=H-XDX7T0ADw
- https://www.youtube.com/watch?v=HMhmC9rLzew
- https://www.youtube.com/watch?v=cdVq028miuk
- https://www.youtube.com/watch?v=YWvpnY2R9PY

Practical Components

Do the following questions:
Organise your solutions neatly in your exercise book.

You will require Chapter 9 of Maths Quest 11 Mathematical Methods (pdf – Google Classroom)
- 9F: 1c, 1e, 1h, 2c, 2e, 2h, 3, 6, 7, 9, 10, 11a, 11c, 11e, 12
- 9G: 1, 3, 6-8, 10-11, 13-14, 16

Investigation

See next page

Week 13 Investigation

Second Derivative - Calculation

All the work we have done so far has been on the derivative, although technically it has all been on the FIRST derivative. We took the derivative of the function and used it to identify features of curves, find maximums, find minimums, classify turning points.

We can take the derivative of the derivative. In fact we can keep taking the derivative if we want to.

This is what I mean.

If we start with a function like $y = x^5$

Then the FIRST derivative, $y' = 5x^4$

and we can take the derivative again. We call this the second derivative, and use the notation y''.

$y'' = 20x^3$

What does the second derivative represent?

we know that the first derivative tells us how steep the curve is, whether it is increasing or decreasing and where the gradients are equal to zero. This can help us identify turning points or stationary points of inflection.

The second derivative tells us the concavity. If the second derivative >0, then the curve is concave up (local minimum). If the second derivative <0, then the curve is concave down (local maximum).

Example: Consider the function $f(x) = 5x^3 + 10x^2$. Solve for the x-coordinate of the turning point/s for the local maximum and minimum using second derivative.

So, $f(x) = 5x^3 + 10x^2$

$f'(x) = 15x^2 + 20x$

A stationary point is a point at where f'(x)=0

Therefore, to find this x-coordinate, we have to set the expression for f'(x) equal to 0 to get an equation in terms of x.

So, $0 = 15x^2 + 20x$

$0 = x \times (15x + 20)$

Therefore, $x = 0 \; or \; x = -\frac{20}{15} = -\frac{4}{3}$

Now we know there are stationary points at $x = 0 \; or \; x = -\frac{4}{3}$, but we don't know their concavity yet, but we can find it by using second derivate.

So, $f''(x) = 30x + 20$

When $x = 0,\; f''(x) = 30 \times 0 + 20 = 20$ 20>0 therefore, local minimum

When $x = -\frac{4}{3},\; f''(x) = 30 \times \left(-\frac{4}{3}\right) + 20 = -20$ -20<0 therefore, local maximum

Question for you:

Consider the function $f(x) = -5x^4 + 5x^3$. Solve for the x-coordinate of the stationary point/s for the local maximum or minimum using second derivative. Then, explain the concavity of the function when the second derivate = 0 (is this a turning point or something else?). Graph the function to support your answer.