Week 9/10
Term 3
2023

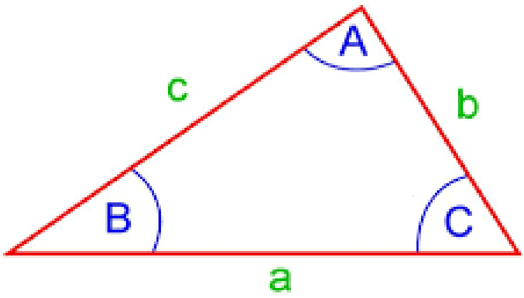
HAWKER COLLEGE

Engage | Inspire | Achieve

MA2

Applications of Trigonometry

Goals



Goals:

- solve problems involving non-right-angled triangles using the sine rule (ambiguous case excluded) and the cosine rule
- determine the area of a triangle given two sides and an included angle by using the rule *Area* = $\frac{1}{2}ab\sin C$, or given three sides by using Heron's rule, and solve related practical problems
- solve practical problems involving the trigonometry of right-angled and non-right-angled triangles, including problems involving angles of elevation and depression and the use of bearings in navigation

Theoretical Components

Resources:
PDF file: Week 9/10 Notes and Exercises

Knowledge Checklist

- Sine rule – an angle and two sides
- Sine rule – an angle and its opposite side and one other angle
- Cosine rule – two sides and the included angle
- Cosine rule – finding an angle when you know the three sides
- Finding the area of triangles using the rules: *Area* = $\frac{1}{2}ab\sin C$, or given three sides by using Heron's rule
- Navigation

Order

1. Read through the notes and examples
2. Work through the exercises
3. Complete the investigation at the end of the booklet.
4. Complete the reflection at the end of the booklet
5. Come and see your teacher and make sure you are up to date.

Practical Components

Work through the exercises and show the completed tasks to your teacher.

Be sure to ask for help as you need for the successful completion of all tasks.

Remember to regularly check Google Classroom for messages.

Sine rule
https://www.youtube.com/watch?v=r4YJuhS-1XE
Cosine rule
https://www.youtube.com/watch?v=iEWSqAk3hTw
and/or
https://www.youtube.com/watch?v=ZElOxG7_m3c

Investigation

Complete the task at the end of the booklet and submit your work for checking. ☺

QFO

MATHEMATICAL APPLICATIONS 2

WEEK 9/10 NOTES & EXERCISES

Often the triangle that is apparent or identified in a given problem is *non-right-angled*. Thus, Pythagoras' theorem or the trigonometric ratios are not as easily applied. The two rules that can be used to solve such problems are:

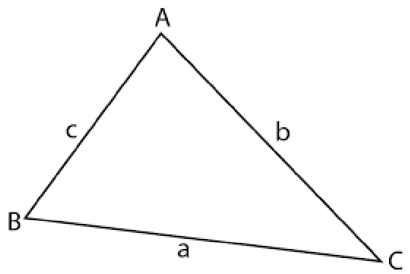
1. the *sine* rule, and
2. the *cosine* rule.

For the sine and cosine rules the following labelling convention should be used.

Angle *A* is opposite side *a* (at vertex A)

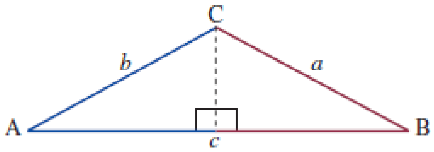
Angle *B* is opposite side *b* (at vertex B)

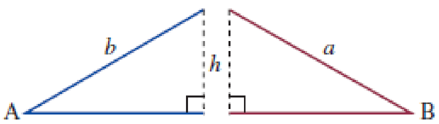
Angle *C* is opposite side *c* (at vertex C)



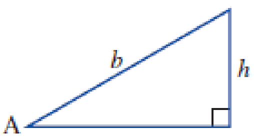
THE SINE RULE

All triangles can be divided into two right-angled triangles.



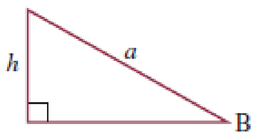


h can be evaluated for each triangle.



$$\sin(A) = \frac{h}{b}$$

$$h = b \times \sin(A)$$



$$\sin(B) = \frac{h}{a}$$

$$h = a \times \sin(B)$$

If we equate the two expressions for *h*:

$b \times \sin(A) = a \times \sin(B)$

and rearranging the equation, we obtain:

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)}$$

This means that a side divided by the *sin* of its opposite angle is equal to another side divided by the *sin* of its opposite angle

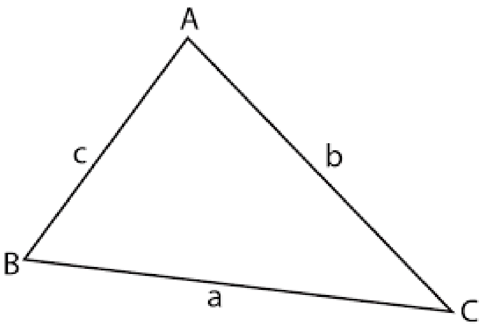
This will work for any pairs of sides and angles.

In any $\triangle ABC$

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

or

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$



The **sine rule** is used if you are given:

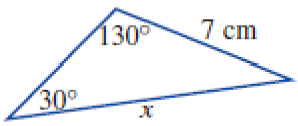
1. *two angles* and *one side*

or

2. an *angle* and *its opposite side length* (a complete ratio) and *one other side*.

Example

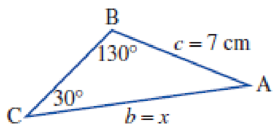
Find the unknown length, *x* cm, in the triangle below (to 1 decimal place).



Solution

Draw the triangle. Assume it is non-right-angled.

Label the triangle appropriately for the sine rule.



Confirm that it is the sine rule that can be used as you have the angle opposite to the unknown side and a known $\frac{\text{side}}{\text{angle}}$ ratio.

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

$b = x$ $\quad B = 130°$

$c = 7\text{ cm}$ $\quad C = 30°$

Substitute known values into the two ratios.

$$\frac{x}{\sin(130°)} = \frac{7}{\sin(30°)}$$

Isolate x and evaluate.

$$x = \frac{7 \times \sin(130°)}{\sin(30°)}$$

$x = 10.7246$

$x = 10.7$

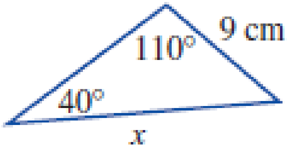
Write the answer.

The unknown length is 10.7 cm, correct to 1 decimal place.

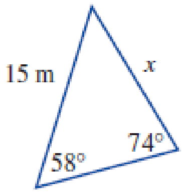
EXERCISE 1

1. Find the unknown length, *x*, in each of the following:

a)

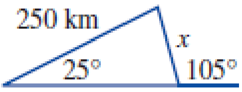


b)

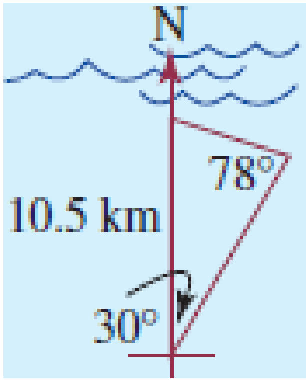


c)

d)



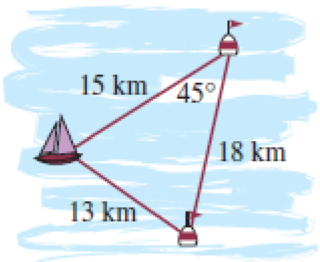
2. A sailing expedition followed a course as shown at right. Find the total distance covered in the round trip. You may need to find the third angle to complete the question.



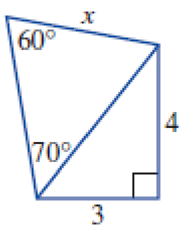
3. Steel trusses are used to support the roof of a commercial building. The struts in the truss shown are each made from 0.8 m steel lengths and are welded at the contact points with the upper and lower sections of the truss.

What is the distance (to the nearest centimetre) between each pair of consecutive welds?

4. A yacht sails the three-leg course shown. Find the largest angle between any two legs within the course, to the nearest degree.

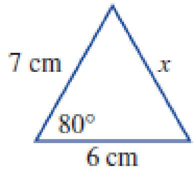


5. Find the value of *x* (to 1 decimal place) in the given figure.



THE COSINE RULE

There are situations in which the sine rule will not work. Here we do not have the 'right' information to calculate *x* with the sine rule.



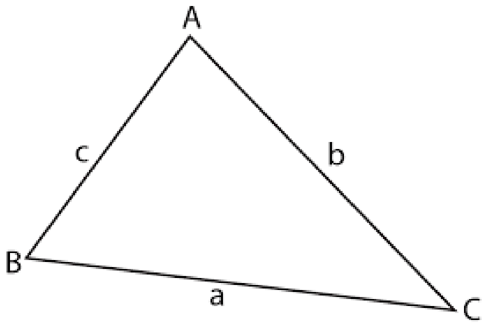
In this case we can make use of another rule, the cosine rule.

In any $\triangle ABC$

$$c^2 = a^2 + b^2 - 2ab\cos(C)$$

In any $\triangle ABC$

$$\cos(C) = \frac{a^2 + b^2 - c^2}{2ab}$$



The **cosine rule** is used to find:

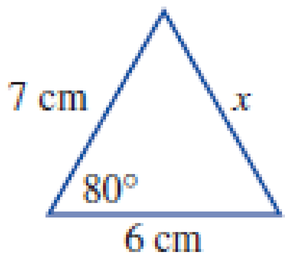
1. an *unknown length* when you have the *lengths of two sides* and the *angle in between*

   or

2. an *unknown angle* when you have the *lengths of all three sides.*

Example

Find the value of *x* in the triangle shown.



Solution

Let c = *x*, a = 7, b = 6 and C = 80°

Thus $\mathbf{c^2 = a^2 + b^2 - 2ab \times \cos(C)}$ becomes

$c^2 = 6^2 + 7^2 - 2 \times 6 \times 7 \times \cos 80°$

$c^2 = 36 + 49 - 84 \times \cos 80°$

$c^2 = 85 - 14.59$

$c^2 = 70.41$

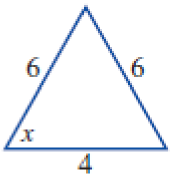
$c = \sqrt{70.41}$

$c = 8.39$

The unknown length is 8.39 cm, correct to 2 decimal places.

Example

Find the size of angle *x* in the triangle shown, to the nearest degree.



Solution

Let $a = 4,\ b = 6,\ c = 6\ and\ C = x$

$$\cos(x) = \frac{4^2 + 6^2 - 6^2}{2 \times 4 \times 6}$$

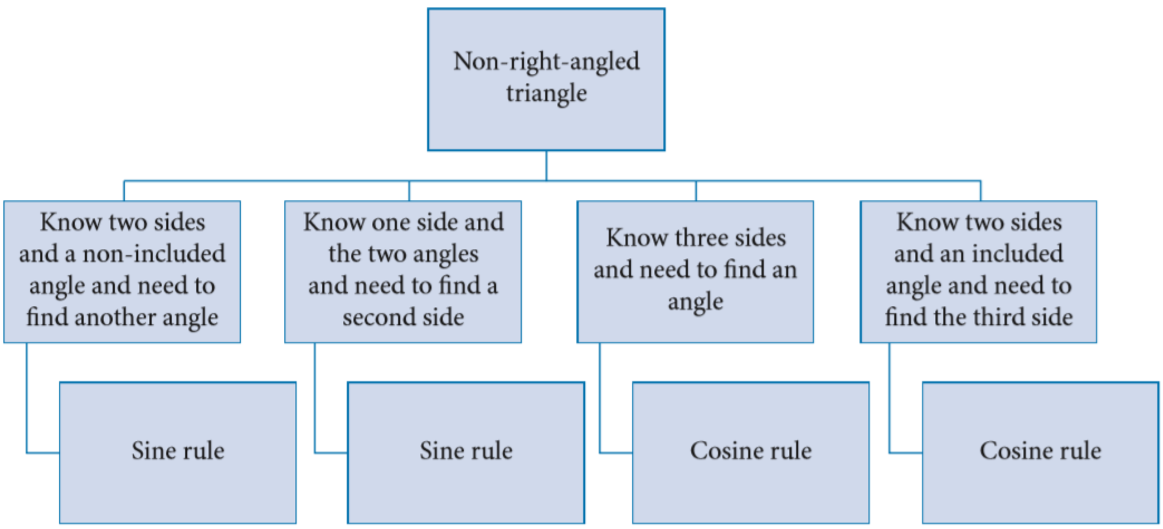
$$\cos(x) = \frac{16}{48}$$

$x = \cos^{-1}(\frac{16}{48})$

$x = 70.5°$

The angle *x* is 71°, correct to the nearest degree.

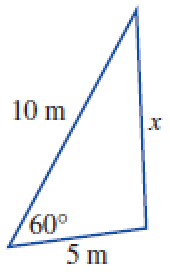
Remember:



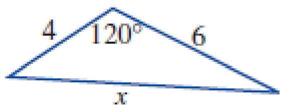
EXERCISE 2

1. Find the unknown length in each of the following (to 2 decimal places).

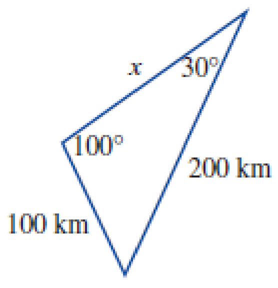
a)



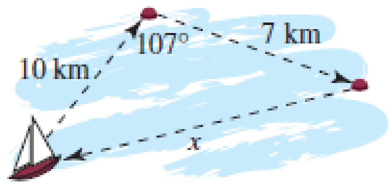
b)



c)

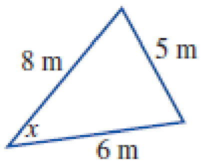


2. During a sailing race, the boats followed a course as shown. Find the length, x, of its third leg (to 1 decimal place).

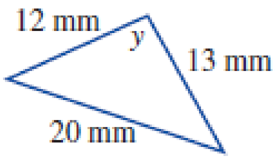


3. Find the size of the unknown angle in each of the following (to the nearest degree).

a)



b)



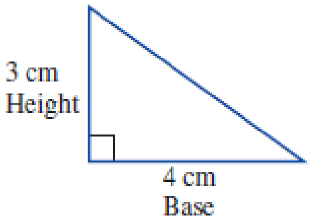
4. Consider the sailing expedition course in question Q**2**. Find the two unknown angles (to the nearest degree) in the triangular course.

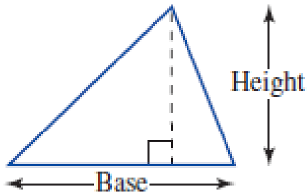
AREA OF TRIANGLE

Three possible methods can be used to find the area of a triangle:

Method 1. When the *two known lengths are perpendicular* to each other (right-angled triangle) we would use:

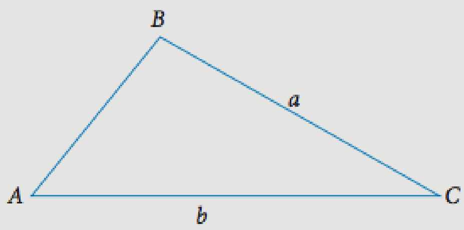
$$\text{Area}_{\text{triangle}} = \frac{1}{2} \times \text{Base} \times \text{Height}$$
$$A = \frac{1}{2}bh$$

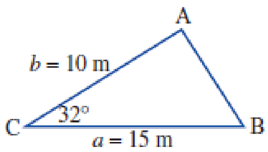


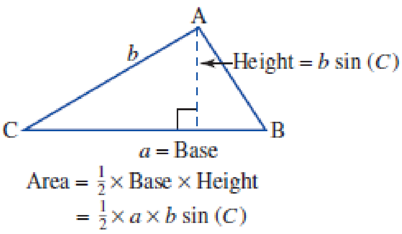


Method 2. When we are given *two lengths and the angle in between* we would use:

The area of $\triangle ABC$ is given by $A = \frac{1}{2}ab\sin(C)$ where C is the **included angle** between sides a and b.







Method 3. When *all three sides are known* we would use:

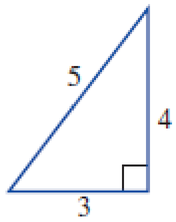
$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

where $s = \dfrac{a+b+c}{2}$ (the semi-perimeter).

This formula is known as *Heron's formula*. It was developed by Heron (or Hero) of Alexandria, a Greek mathematician and engineer who lived around AD 62.

Example

We find the area of the triangle at right to demonstrate that all three formulas provide the same result.



Method 1.

This is the most appropriate method because it is a right-angled triangle.

$$\text{Area}_{\text{triangle}} = \frac{1}{2} \times \text{Base} \times \text{Height}$$
$$A = \frac{1}{2} \times 3 \times 4$$
$$= 6 \text{ u}^2$$

Method 2

$$\text{Area}_{\text{triangle}} = \frac{1}{2} \times a \times b \times \sin(C)$$
$$A = \frac{1}{2} \times 3 \times 4 \times \sin(90^\circ)$$
$$= 6 \times 1$$
$$= 6 \text{ u}^2$$

Method 3

First calculate *s*

$$s = \frac{(a+b+c)}{2}$$
$$s = \frac{(3+4+5)}{2}$$
$$s = \frac{12}{2}$$
$$s = 6$$

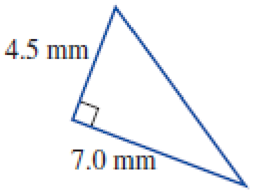
Then use this to calculate *A*

$$\text{Area}_{\text{triangle}} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$A = \sqrt{6(6-3)(6-4)(6-5)}$$
$$A = \sqrt{6 \times 3 \times 2 \times 1}$$
$$A = \sqrt{36}$$
$$= 6 \text{ u}^2$$

EXERCISE 3

1. Find the areas of the following triangles (to 1 decimal place).

a)



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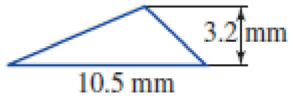

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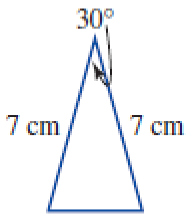

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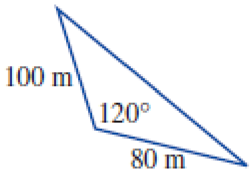
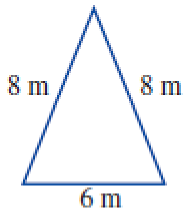


e)



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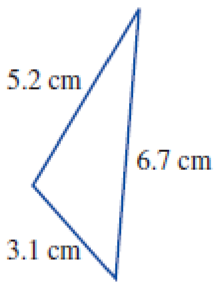
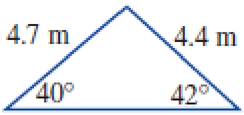
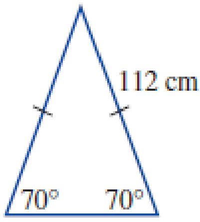


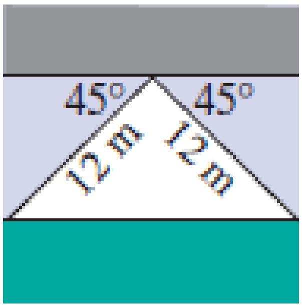
g)



h)

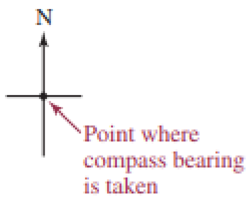


2. A triangular arch has supporting legs of equal length of 12 metres as shown in the diagram below. What is its area?



NAVIGATION AND SPECIFICATION OF LOCATIONS

In most cases when you are asked to solve problems, a carefully drawn sketch of the situation will be given. When a problem is described in words only, very careful sketches of the situation are required. Furthermore, these sketches of the situation need to be converted to triangles with angles and lengths of sides included. This is so that Pythagoras' theorem, trigonometric ratios, areas of triangles, similarity and sine or cosine rules may be used.

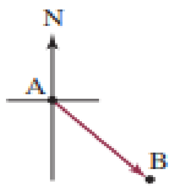


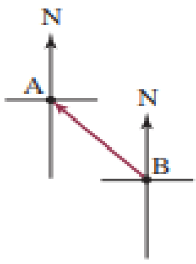
Hints

1. Carefully follow given instructions.

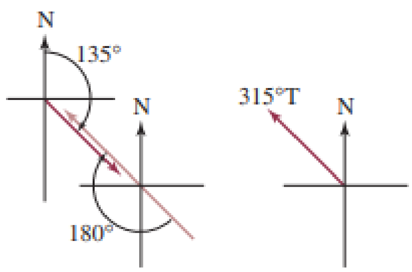
2. Always draw the compass rose at the starting point of the direction requested.

3. Key words are from and to. For example: The bearing from A to B (see diagram below left) is very different from the bearing from B to A (see diagram below right).

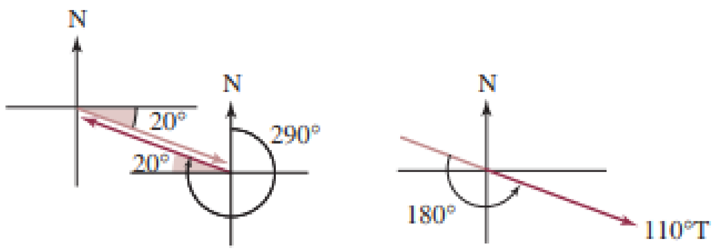




4. When you are asked to determine the direction to return directly back to an initial starting point, it is a 180° rotation or difference. For example, to return directly back after heading north, we need to change the direction to head south. Other examples are: Returning directly back after heading 135°T New bearing = 135° + 180° = 315°T

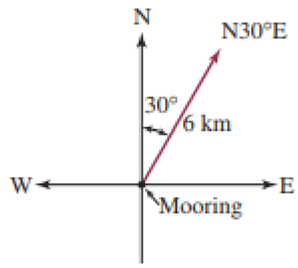


Returning directly back after heading 290°T New bearing = 290° - 180° = 110°T



Example

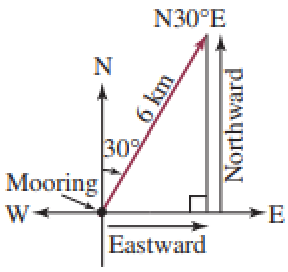
A ship leaves port, heading N30°E for 6 kilometres as shown. How far north or south is the ship from its starting point (to 1 decimal place)?



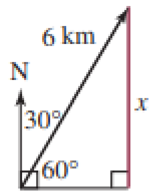
Solution

1 Draw a diagram of the journey and indicate or superimpose a suitable triangle.

a

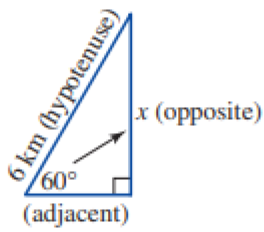


2 Identify the side of the triangle to be found. Redraw a simple triangle with most important information provided. Use the bearing given to establish the angle in the triangle, that is, use the complementary angle law.



$90° - 30° = 60°$

3 Identify the need to use a trigonometric ratio, namely the sine ratio, to find the distance north.



4 Substitute and evaluate.

$$\sin(\theta) = \frac{\text{length of opposite side}}{\text{length of hypotenuse side}} = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin(60°) = \frac{x}{6}$$

$$x = 6 \times \sin(60°)$$

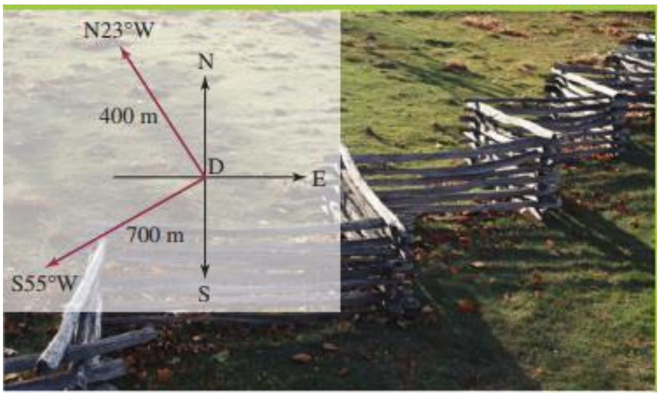
$$x = 6 \times 0.8660 = 5.196$$

5 State the answer to the required number of decimal places.

The ship is 5.2 km north of its starting point, correct to 1 decimal place.

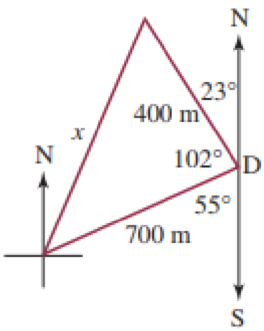
Example

A triangular paddock has two complete fences. From location D, one fence line is on a bearing of N23°W for 400 metres. The other fence line is S55°W for 700 metres. Find the length of fencing (to the nearest metre) required to complete the enclosure of the triangular paddock.

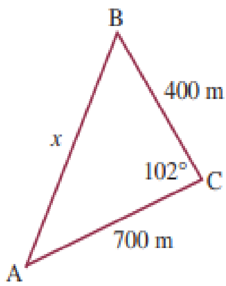


Solution

1. Identify the side of the triangle to be found. Redraw a simple triangle with the most important information provided.
2. Use the bearings given to establish the angle in the triangle; that is, use the supplementary angle law.



3. Identify the need to use the cosine rule, as two sides and the included angle are given.



$a = 400\text{ m} \quad b = 700\text{ m} \quad C = 102° \quad c = x\text{ m}$

4. Substitute and evaluate.

$c^2 = a^2 + b^2 - 2ab \times \cos(C)$
$x^2 = 400^2 + 700^2 - 2 \times 400 \times 700 \times \cos(102°)$
$x^2 = 650\,000 - 560\,000 \times {}^-0.207\,91$
$x^2 = 766\,430.55$
$x = \sqrt{766\,430.55}$
$= 875.46$

5. Answer in correct units and to the required level of accuracy.

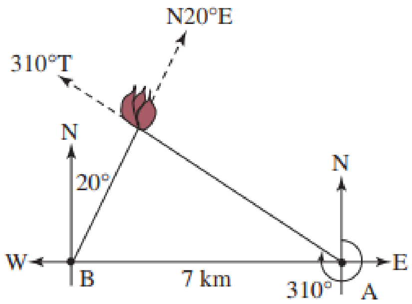
The new fence section is to be 875 metres long, correct to the nearest metre.

Example

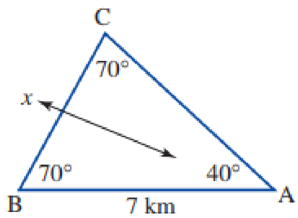
Two fire-spotting towers are 7 kilometres apart on an east– west line. From Tower A a fire is seen on a bearing of 310°T. From Tower B the same fire is spotted on a bearing of N20°E. Which tower is closest to the fire and how far is that tower from the fire (to 1 decimal place)?

Solution

1 Draw a suitable sketch of the situation described. It is necessary to determine whether Tower A is east or west of Tower B.



2 Identify the known values of the triangle and label appropriately for the sine rule.
Note: The *shortest side* of a triangle is *opposite* the *smallest angle*.



3 Substitute into the formula and evaluate.
Note: ΔABC is an isosceles triangle, so Tower A is 7 km from the fire.

$$\frac{x}{\sin(40°)} = \frac{7}{\sin(70°)}$$

$$x = \frac{7 \times \sin(40°)}{\sin(70°)}$$

$$x = 4.788\,282 \text{ km}$$

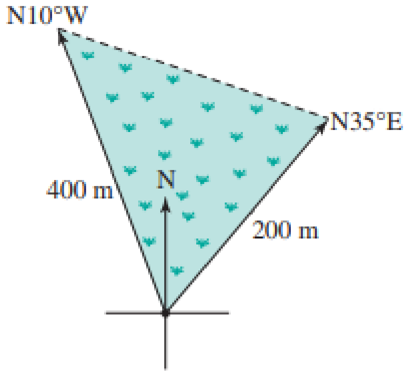
4 Write the answer in the correct units.

Tower B is closest to the fire at a distance of 4.8 km, correct to 1 decimal place.

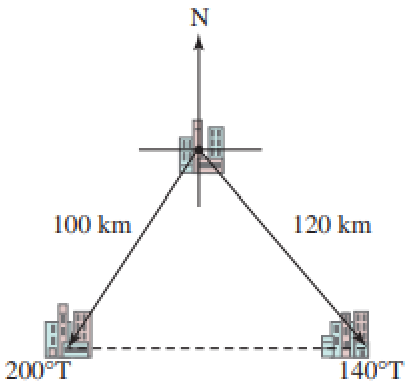
EXERCISE 4

1. Find the length of the unknown side of each of the shapes given (to the nearest unit).

a)

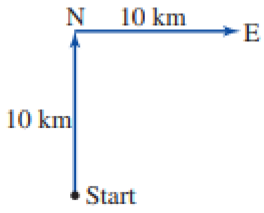


b)

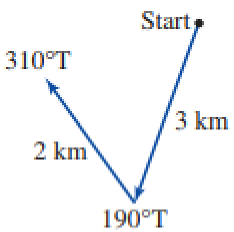


2. In each of the following diagrams, the first two legs of a journey are shown. Find the direction and distance of the third leg of the journey which returns to the start.

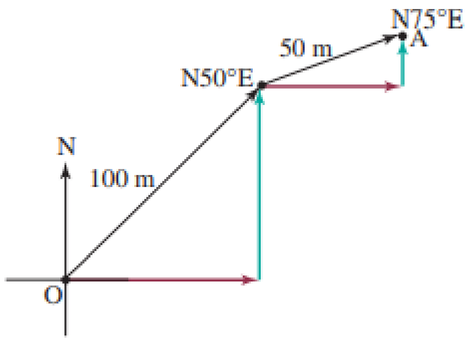
a)



b)



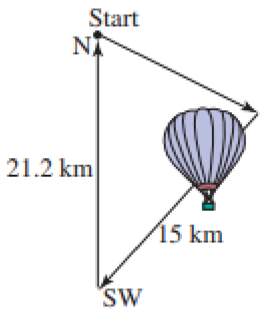
3. Find how far north/south and east/west position A is from position O



WEEK 9/10 INVESTIGATION

Task 1

A journey by a hot-air balloon is shown. The balloonist did not initially record the first leg of the journey. Find the direction and distance for the first leg of the balloonist's journey. (Show full working)



Task 2

Two fire-spotting towers are 17 kilometres apart on an east–west line. From Tower A, a fire is seen on a bearing of 130°T. From Tower B, the same fire is spotted on a bearing of S20°W. Which tower is closest to the fire and how far is that tower from the fire? (Give a diagram and show full working)

MARKING RUBRIC

CRITERIA	EXPECTATIONS	POSS	MULT	GIVEN	TOTAL
Practical	Student completes practical work of the brief to an acceptable standard set by the teacher.	**2**	**3**		**/6**
Investigation	Student completes the investigation of the brief to an acceptable standard set by the teacher.	**2**	**2**		**/4**
Reasoning and communications	Student responses are accurate and appropriate in presentation of mathematical ideas in different contexts, with clear and logical working out shown.	**4**	**-**		**/4**
Concepts and techniques	Student submitted work selects and applies appropriate mathematical modelling and problem solving techniques to solve practical problems, and demonstrates proficiency in the use of mathematical facts, techniques and formulae.	**4**	**-**		**/4**
	Submission Guidelines				
Timeliness	Student submits the exercises and portfolio task by the set deadline. See scoring guidelines for specific details.	**2**	**-**		**/2**
		FINAL			**/20**

Student Reflection:
How did you go with this week's work?

What was interesting?

What did you find easy?

What do you need to work on?